Parallel optical image addition and subtraction in a dynamic photorefractive memory by phase-code multiplexing

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We propose and demonstrate experimentally a method for utilizing a dynamic phase-encoded photorefractive memory to realize parallel optical addition, subtraction, and inversion operations of stored images. The phase-encoded holographic memory is realized in photorefractive $BaTiO_3$, storing eight images using Walsh–Hadamard binary phase codes and an incremental recording procedure. By subsampling the set of reference beams during the recall operation, the selectivity of the phase address is decreased, allowing one to combine images in such a way that different linear combination of the images can be realized at the output of the memory.

Multiple holograms may be stored in volume holographic media by a change in the reference beam angle (angle or θ multiplexing)[1–4] or the recording wavelength (wavelength or λ multiplexing)[5] or by phase encoding the reference beam (phase or Φ multiplexing).[6–8] Among these methods, phase-coded holographic storage has been shown to have several advantageous features, such as high energy efficiency, short readout times, fixed registration geometry, and fixed wavelength, that allow one to register the images without introducing moving parts into the setup. These advantages are a direct consequence of the phase-only modulation of the reference beams. Moreover, cross talk in phase-encoded multiplexing is reduced compared with that in angular multiplexing.[9,10] With this method, as many as 64 images have been stored successfully in a dynamic memory.[11,12] Taketomi *et al.*[13] showed how basic orthogonal codes can be combined to produce reference sets that permit the recall of pairwise superpositions of images. For the case of binary phase encoding, this method may also be used to recall the sum and the difference of any two of the images stored in the memory.

In this Letter we demonstrate the possibility of recalling linear combinations as well as inversions of stored data pages, using only a subset of reference beams and hence decreasing the selectivity of the phase address. This method has already been proposed by the authors.[11] The all-optical performance of the arithmetic operations is based simply on the combination of an amplitude modulator with the reference beam phase modulator. Moreover, only one recall is necessary to produce the desired operation on the whole data page, thus allowing for high-speed data-page processing. Recently a first demonstration[14] showed the possibility of realizing pairwise addition and subtraction in a memory that has stored three images in photorefractive $LiNbO_3$ recorded with a sequential procedure. Here we extend our investigations to the case of storage of multiple images by incremental recording in photorefractive $BaTiO_3$ and to the case of linear combinations of multiple image operations. Moreover, we also demonstrate analog optical inversion of stored data.

Phase-code multiplexing is based on the superposition of the image $\mathcal{A}_j = A_j \exp(ik_j\mathbf{r})\exp(-i\omega t) + \text{c.c.}$ to be stored with a set of reference beams $\mathcal{R}_j = R\sum_k^N \exp(i\mathbf{k}_k\mathbf{r})\exp(i\varphi_k^j)\exp(-i\omega t) + \text{c.c.}$, where $\exp(i\varphi_k^j)$ is a definite additional phase term added to each reference beam component. It is that phase term that represents the address of the image and allows selective retrieval of the stored images during recall. The resulting spatial refractive-index modulation in the crystal after the recording is thus given by[3]

$$\Delta n(\mathbf{r}) \propto \sum_j^N \sum_k^N \Delta\bar{n}_k^j(\mathbf{r})\exp[i(\mathbf{k}_s - \mathbf{k}_j)\mathbf{r}]\exp(-i\varphi_k^j), \quad (1)$$

where $\Delta\bar{n}_k^j(\mathbf{r})$ is the modulation depth of the index grating between the kth reference beam component and the jth signal beam.

During readout with $\mathcal{R}_{j'}$ we obtain the diffraction signal

$$\begin{aligned}\mathcal{A}_{j'}^{\text{diff}} &= \mathcal{R}_{j'}\Delta n \oplus \text{ Bragg selectivity}\\ &\propto \sum_k^N \exp\left[i\left(\varphi_k^{j'} - \varphi_k^j\right)\right],\end{aligned} \quad (2)$$

where ($\oplus$ Bragg selectivity) denotes the selectivity of the Bragg condition that has to be fulfilled for each reference beam. To reconstruct the jth image, we have to fulfill the condition[6,7]

$$\sum_{k=1}^N \exp\left[i\left(\varphi_k^{j'} - \varphi_k^j\right)\right] \equiv \sum_{k=1}^N \chi_{kj}^* \chi_{k\hat{j}} \overset{!}{=} \begin{cases} 0 & \text{for } j \neq j' \\ N & \text{for } j = j' \end{cases}. \quad (3)$$

The matrix χ_{kj} represents the additional phase terms, and k is the number of the reference beam when the jth image is written. Therefore, the phase codes of the images are given in that notation by the rows of the matrix.

For reconstruction without cross talk of any arbitrarily addressed image to be obtained, the matrix χ has to be unitary. If pure phase encoding is to be used,

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all elements of χ have to have the absolute value 1; otherwise a combination of phase and amplitude multiplexing will be obtained.[7] In our contribution, we use binary phase codes defined by the Walsh–Hadamard algorithm (elements of χ_{kj} are either 1 or -1), which is equal to a phase shift of either 0 or π.

To obtain addition (subtraction) of several images, we have to use only a subset of the reference beams having the same (different) additional reference beam phase factors. This subsampling reduces the selectivity of the phase address, permitting contributions of different addresses to be read out simultaneously. Because of the coherent nature of the readout images, they may experience constructure (destructive) interference, thus permitting addition (subtraction) of the images. To realize these cases, we have to change Eq. (3) during readout to

$$\sum_{k=1}^{N} \exp[i(\varphi_k^{j'} - \varphi_k^{j})] \equiv \sum_{k=1}^{N} \chi_{kj}^{*} \chi'_{kj'} \stackrel{!}{=} \begin{cases} 0 & \text{for } j \neq j' \\ N/m & \text{for } j = j' \end{cases}, \tag{4}$$

where m is the number of images involved in one operation, $\hat{j}'$ denotes the index corresponding to each of the images that will be added (subtracted). $\chi'_{kj'}$ is defined as a readout matrix that is different for every arithmetic operation:

$$\chi'_{kj'} = \begin{cases} \chi_{kj'} & \text{for equal (different) phase factors} \\ 0 & \text{other cases} \end{cases}. \tag{5}$$

Thus $\chi'_{kj'}$ represents the operation of an additional amplitude modulator that realizes the addressing of the dynamic memory with only a subset of the original phase codes.

In our experimental realization (see Fig. 1), an argon-ion laser (514 nm) is used to realize the signal and reference beams. In the reference arm, a binary phase modulator[11] is combined with an amplitude spatial light modulator. The computer-generated focus-array hologram is used to separate the different reference beam components, thus avoiding overlap of differently phase-modulated parts of the different reference beams. Images are fed into the image arm with a ferroelectric liquid-crystal display with 200 × 200 pixel resolution taken from a commercially available Sharp video projector. The angle between the image and the reference beams was approximately 25° inside the crystal, and the angular spacing between successive reference beam components was approximately 2.5°. During recording, the reference arm amplitude modulator was set to maximum transmission, and the images were stored with an incremental storage technique.[8] The integral storage time for each image in our experiment was approximately 10 s. After storing the images, we obtained a diffraction efficiency of approximately 3% for each image. Significant cross talk between images has not been detected. During readout, the amplitude spatial light modulator was used to select the appropriate subset of reference beam components to realize the different operations.

To demonstrate the realization of pairwise addition and subtraction, we stored eight images, using eight reference beam components. Examples of them obtained during recall with pure phase-coded reference beams are shown in Fig. 2. Figure 3 shows several representations of combined recall that result in addition or subtraction. Because we used a CCD camera to register the intensity of the images, we are able to detect only the absolute values of the desired operation. However, if phase-sensitive detection methods such as interferometric ones are used to register both amplitude and phase, the whole operation can be detected. If these arithmetic operations are realized in a setup in which the detection is combined with a supplementary threshold device, e.g., a saturable absorber as described in Ref. 15, it is possible to realize the logical operations OR for addition and XOR for subtraction.

Beyond pairwise addition and subtraction, it is possible to realize linear combinations of a larger number of images. We demonstrate the linear combination of three images. Figure 4 shows three stored images during readout with pure phase-coded reference beams (Figs. 4a–4c) as well as examples of linear combinations of two (Figs. 4d and 4e) or three (Fig. 4f) of these images. In Fig. 4f the analog character of the operations is clearly visible. Although limited by the linear dynamic range of the crystal, linear combinations including multiplicative factors of the images to be added or subtracted can be realized by use of the appropriate phase codes in the same way as multiple image combinations.

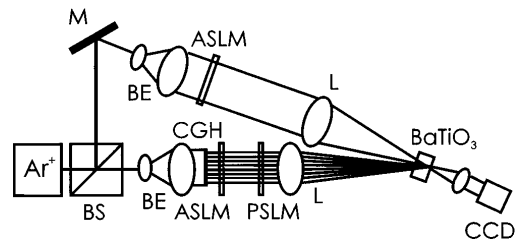

Fig. 1. Schematic of the experimental setup. M, mirror; L's, lenses; BS, beam splitter; BE's, beam expanders; ASLM, amplitude spatial light modulator; PSLM, phase spatial light modulator; CGH, computer-generated hologram.

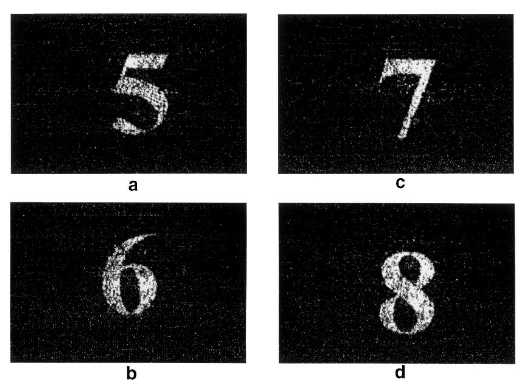

Fig. 2. Images stored with binary phase codes during readout with the complete set of reference beams.

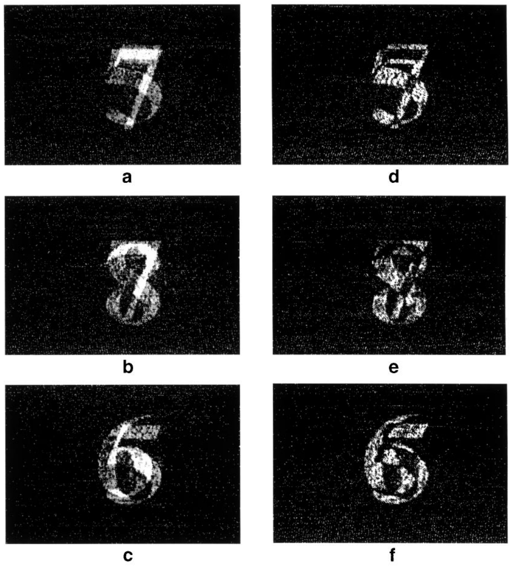

Fig. 3. Examples of addition and subtraction of pairwise images: a, $|A_5 + A_7|$; b, $|A_7 + A_8|$; c, $|A_5 + A_6|$; d, $|A_5 - A_7|$; e, $|A_7 - A_8|$; f, $|A_5 - A_6|$.

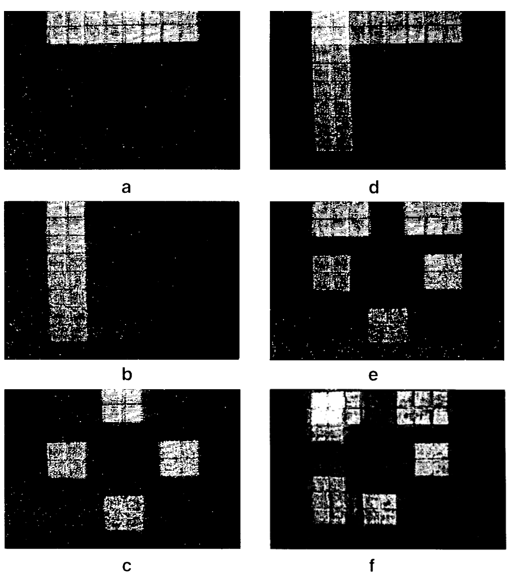

Fig. 4. a–c, Three images stored in the phase-coded memory, readout with the complete reference beam set, and their addition and subtraction by addressing the memory with subsets of the phase codes: d, $|A_1 + A_2|$; e, $|A_2 - A_3|$; f, $|A_1 + A_2 - A_3|$.

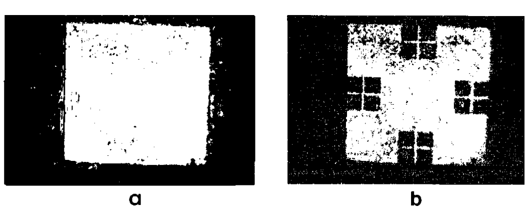

Fig. 5. For inversion of images, a plane wave is stored together with the images to be inverted in the phase coded memory (a). The inversion of the image in Fig. 4c is then equal to $|A_{\mathrm{pw}} - A_3|$) (b).

We were able to realize in a variation of the previous experiment the possibility of inverting an image by subtraction of a stored plane-wave image from the original image. An example of such an inversion is shown in Fig. 5 for the image of Fig. 4c. This procedure is equivalent to a logical NOT operation. Thus, if an appropriate threshold device is used as a supplementary element in the phase-coded memory during readout, the logical operations OR, XOR, and NOT can be realized with the method of reduced reference beam recall in phase-encoded multiplexing. Consequently, any logical operation can be performed by combination of these basic operations.

In conclusion, we presented an all-optical memory device based on phase-coded multiplexing in photorefractive volume holograms that is able to perform arithmetic and logical operations on the stored images. With only a subset of the phase codes that have been used to record and recall the images independently, the selectivity of the phase address is decreased, permitting parallel readout of coherent, phase-shifted images. Thus operations such as addition, subtraction, and inversion of stored images can be performed with a single recall operation, allowing for high-speed page-oriented data processing.

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