Defect-controlled transverse localization of light in disordered photonic lattices

Dragana M. Jović,[1,2,*] Milivoj R. Belić,[3] and Cornelia Denz[2]

[1]*Institute of Physics, University of Belgrade, P.O. Box 68, Belgrade 11001, Serbia*
[2]*Institut für Angewandte Physik and Center for Nonlinear Science (CeNoS), Westfälische Wilhelms-Universität Münster, Münster 48149, Germany*
[3]*Texas A&M University at Qatar, P.O. Box 23874, Doha, Qatar*
**Corresponding author: jovic@ipb.ac.rs*

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We study numerically Anderson localization of light in a disordered photonic lattice containing vacancy defects of different length. The influence of Kerr nonlinearity and disorder level on the transverse localization of light in different triangular lattice geometries is discussed. We demonstrate both suppression and enhancement of light localization in the presence of defects of different size, depending on the disorder level and the strength of the nonlinearity. We find that, in the linear regime, localization is more pronounced in the lattice with the simplest defect type—the single vacancy. In a strongly focusing nonlinear regime, the presence of all defect kinds enhances localization, as compared to the case with no defects. In the defocusing nonlinear regime, a suppression of localization in the presence of all defect types is demonstrated, as compared to the localization in the lattice without defects. In the end, the effect of input beam width on various regimes of Anderson localization is discussed.

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1. INTRODUCTION

Anderson localization of light has recently attracted much attention, both theoretically and experimentally [1–4]. It has become an important part of recent investigations of random photonic lattices and has been studied experimentally in both one-dimensional [5] and two-dimensional (2D) [6] systems with randomized lattice potentials. These experiments motivated numerous theoretical studies, including the analysis of wave spreading in nonlinear disordered systems and lasing in photonic crystals. In the studies of Anderson localization in nonlinear systems, it is of interest to know how nonlinearity affects the localization process. An interplay between nonlinearity and disorder was investigated experimentally in fiber arrays [7], in disordered 2D photonic lattices [6], near boundaries of disordered photonic lattices [8], and at the interface between linear and nonlinear dielectric media [9]. The past decade has also witnessed progress in random lasing, which is based on light localization in random gain media [10,11].

Recent years have also witnessed an increased interest in the study of defect states in photonic lattices [12–17] and the formation of defect channels [18,19]. These defects disrupt the periodicity of the crystal, creating optical states within the otherwise forbidden bandgap frequencies. In addition, insertion of point-like or line-like defects into photonic crystals or waveguide arrays provides an additional physical mechanism for light confinement, and a possibility to control light propagation in photonic lattices. Light can be localized within defect regions and manipulated by engineering the defect geometry and placement. This topic has attracted special attention, owing to its novel physics and its variety of potential applications, including high-quality waveguide structures, complex integrated circuits such as beam splitters and optical routers, and laser cavities [18,20].

We want to put these concepts together—to study Anderson localization of light in a disordered lattice containing different types of defects. The system of interest is very simple: a lattice, a defect, and a propagating beam. Although defects may be considered as an extreme form of lattice disorder, one should clearly distinguish light localization around defects from the Anderson localization in a disordered lattice. We accomplish this by first considering Anderson localization in a lattice without defects and then adding defects of different type. Since there is an infinite number of different defect types, we confine our attention to vacancies of different length. Even though this may seem to be a severe restriction, there is still a wealth of results to report, as will be seen below.

Thus, to extend on the ideas of these concepts, in this paper we analyze transverse light localization in a disordered photonic lattice that contains vacancies of different length. We confine our attention to the triangular 2D lattice, although extension to the square lattice is easy. The lattice is uniform in the third dimension, but of finite extent. We analyze how lattice defects modify Anderson localization of light. Specifically, the differences in localization in a photonic lattice with defects of three different lengths are considered and compared to localization in the lattice with no defects. The lattice geometry is shown in the top row of Fig. 1. The bottom row depicts typical intensity distributions at the exit lattice face of a narrow Gaussian beam launched into the medium, centered on the defect. It is seen that even this simple defect geometry yields significantly different distributions. Different realizations of disorder could lead to different intensity distributions for the same disorder level, so we perform quantitative

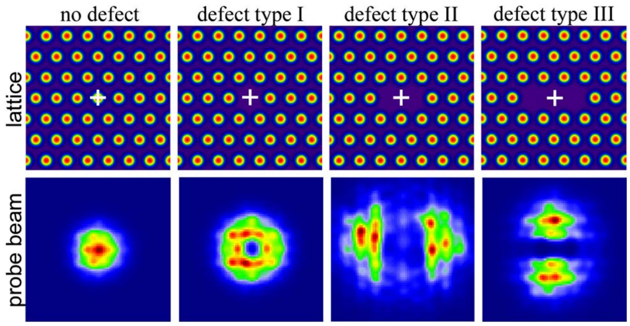

Fig. 1. (Color online) Top row: sketch of the lattice geometry of the triangular photonic lattice with vacancy defects of different length. The crosses mark the location of input Gaussian beams. Bottom row: intensity distributions of the localized modes in the linear regime for 20% disorder level. A narrow Gaussian beam, with FWHM equal to the lattice spacing, is launched into the medium.

statistical analysis to describe phenomena of Anderson localization in such geometries. We investigate how the transverse localization of light in such a geometry depends on both the strength of the disorder and the strength of the nonlinearity in the system. The character of localization is nontrivial, depending on the joint influence of all parameters: the defect geometry, the level of disorder, and the strength of nonlinearity.

While in the linear regime localization is most pronounced in the lattice with the simplest defect type—the single vacancy—in the nonlinear regime it depends on the strength of nonlinearity. We consider two nonlinear regimes: focusing and defocusing. In both of these, localization in defect geometries is more pronounced for defect type I, followed by type II and then type III (for the types of defects, check Fig. 1, first row). However, compared to the case with no lattice defects, the presence of all defect kinds in the focusing regime enhances the localization. On the other hand, in the defocusing nonlinear regime, the suppression of localization in the presence of all defect types is demonstrated, as compared to the lattice with no defects.

We also investigate the effect of different input beam widths on the various regimes of Anderson localization. In the case of a broad input beam, the inclusion of defects enhances localization in both linear and nonlinear regimes, as compared to the lattice with no defects. In addition, higher disorder level is needed to reach the same localization as for the narrow input beam, i.e., to observe the same localization length.

The paper is organized as follows. In Section 2 we introduce the theoretical model, which describes the propagation of light in optically induced disordered photonic lattices with defects. Section 3 summarizes our numerical results for different defect geometries in the linear medium. In Section 4 we study localization in the same geometries but in different nonlinear regimes, and compare them to localization in the linear regime. Section 5 presents results concerning the effect of different input beam widths on localization. Finally, Section 6 concludes the paper.

2. MODELING OF LIGHT LOCALIZATION IN DISORDERED PHOTONIC LATTICES

We study localization of light in an optically induced disordered photonic lattice containing different types of defects. The lattice is induced in a nonlinear dielectric medium with Kerr nonlinearity. The lattice peaks have a typical Gaussian shape. The propagation of a light beam along the z axis, perpendicular to the lattice, is described using the scaled nonlinear Schrödinger equation for the optical electric field amplitude E:

$$i\frac{\partial E}{\partial z} = -\Delta E - \gamma |E|^2 E - VE, \tag{1}$$

where Δ is the transverse Laplacian, γ is the dimensionless strength of the nonlinearity, and $V(x, y)$ is the transverse lattice potential, with the peak intensity V_0. Thus, we consider the influence of simple Kerr nonlinearity on the localization process in the presence of defects. A scaling $x/x_0 \to x$, $y/x_0 \to y$, $z/L_D \to zc$ is utilized for the dimensionless equation, where x_0 is the typical FWHM beam waist and L_D is the corresponding diffraction length. The propagation equation is solved numerically by employing a beam propagation method developed earlier [21,22]. We launch narrow Gaussian beams positioned at the defect center into the lattice, perpendicular to the input crystal face, and observe how they spread in propagation.

To study Anderson localization effects, one has to use a random lattice potential V_r, instead of the perfect periodic potential V. The disorder is realized using a randomized lattice peak intensity, V_{0r}, which takes the values from the interval $V_0(1 - Nr) \le V_{0r} \le V_0(1 + Nr)$, where r is the random number generator and N determines the degree of disorder. They both may take values from the interval [0,1]. The disorder level is quantified by the ratio between the intensity of the random lattice and the intensity of the periodic lattice. Thus, the position of randomized peaks is regular; only their intensities vary. Then, different types of defects are introduced, by eliminating one, two, or three lattice sites. In the end, Gaussian beams are launched.

The study of Anderson localization involves statistical description of different realizations of a randomized system. While in each realization one observes localization, the quantitative measures of localization may vary from one realization to the next. To obtain more precise measures—such as the effective beam width—one has to produce many realizations and form an appropriate ensemble statistics. In our numerics they are realized by starting each run with a different seed of the random number generator and then forming ensemble averages. Typically, we produce 100 realizations per simulation.

3. TRANSVERSE LOCALIZATION IN THE LINEAR REGIME

First, we investigate localization in the linear regime ($\gamma = 0$). To estimate how the lattice defects affect the localization process, we compare the corresponding localized modes in the disordered photonic lattice with and without lattice defects. Typical examples of localized modes in the linear regime are presented in Fig. 1, second row. To compare Anderson localization in different lattice geometries, we perform quantitative analysis by measuring the standard quantities characterizing the localization: the inverse participation ratio, $P = \int I^2(x, y, L)\mathrm{d}x\mathrm{d}y/\{\int I(x, y, L)\mathrm{d}x\mathrm{d}y\}^2$, the effective beam width $\omega_{\mathrm{eff}} = P^{-1/2}$, and the localization length. In all lattice geometries with and without defects, one observes Anderson localization by increasing the level of disorder.

The effective beam width is measured at the lattice output for different disorder levels; typical results are summarized in Fig. 2. The general trend for all cases (with and without

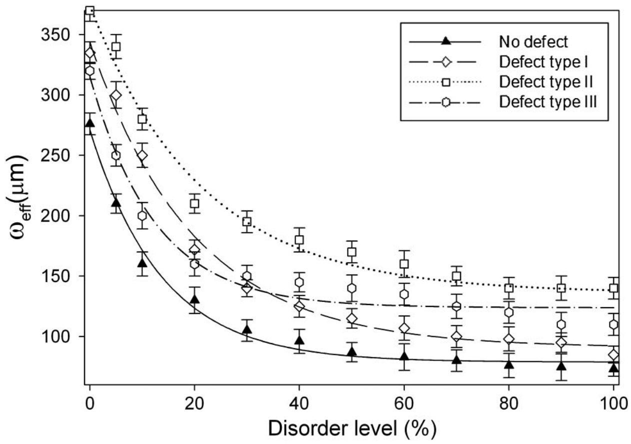

Fig. 2. Transverse localization of light in the linear regime. Effective beam width at the lattice output is shown as a function of the disorder level, for different defect types. Points are ensemble averages and lines are the least-square fits through the points. Error bars depict the spread in values coming from statistics. Physical parameters are crystal length $L = 50$ mm, input lattice intensity $V_0 = 1$, lattice period 11.2 μm, input beam intensity $|E_0|^2 = 0.5$, and input beam FWHM $= 10$ μm.

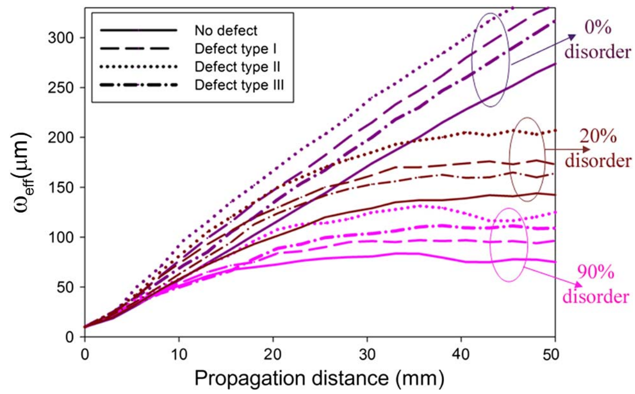

Fig. 3. (Color online) Averaged effective beam width versus propagation distance, for different disorder levels (0%, 20%, and 90%), and for different defect types. Parameters are as in Fig. 2.

defects) is similar: the effective beam width decreases as the level of disorder is increased, or, in other words, disorder always suppresses transport. But, for the lattice with the simplest defect (defect type I), the reduction in the effective width is much steeper than in the other cases, for higher disorder levels. The effective beam widths for defect types I and III are similar; at a disorder level of 30% they even cross each other. But for broader beams, crossing of curves disappears (check Fig. 6). It is seen that the effective beam width for the lattice with defect type II always has the largest value, whereas it is the smallest for the lattice without defects. While one would expect that the presence of defects would enhance localization, in the linear regime this is not always so [check also Fig. 4(a)]. On the other hand, the localization effects are closely connected to the defect shape, the number of neighboring lattice sites, and as the anisotropy in the system, induced by the inclusion of defects. The distance between the input beam and the nearest lattice site has an important role in the localization process; it is shorter for defect type II. Linear beam broadening is also closely connected with the input beam position; the input beam is launched at the lattice site in the geometry without defect and leads to stronger localization than in defect geometry.

Next, we study the influence of medium length on beam localization. This is important for making sure that the regime of Anderson localization is reached. Again, we investigate three different defect types, as well as the lattice with no defects. Figure 3 presents the averaged effective width as a function of the propagation distance. For a perfectly periodic lattice (no disorder), the beam broadens linearly while propagating in all lattice geometries, as expected. It is interesting to note the influence of defects on this linear broadening: the broadening of the beam is more pronounced for defect type II, followed by the type I, and then type III. The lattice with no defects is the least broadened.

At weak disorder (20%), the beams first expand diffusively, before localization sets in, and with the same ordering of widths of different defect types as in the case without disorder. However, when the stronger disorder (90%) is introduced, the behavior changes: localization is reached after a shorter propagation distance, but with the beam widths of the defect types III and I interchanged. This difference is a sign of competition between the disorder and defect influence on the localization process. Also, for longer propagation distances, i.e., for larger crystal thicknesses, the localized modes are observed at lower disorder levels than for shorter propagation distances. This applies to all defect types. Finally, it should be noted that at the distance of $L = 50$ mm, the Anderson localization region is safely reached; this is why in all our simulations, we pick $L = 50$ mm as the medium thickness.

4. LOCALIZATION IN THE NONLINEAR REGIME

With intense laser light illuminating crystals, it is not difficult to reach nonlinear regimes of beam propagation. It is of great interest to know how nonlinearity affects the localization process in different defect geometries. Therefore, we study next the influence of the medium nonlinearity on localization, and how the simultaneous existence of nonlinearity, defects, and disorder affects Anderson localization. Again, the effective beam width is measured at the lattice output, but to compare the localization in different regimes, averaged effective widths are normalized to the corresponding values without disorder. We use such normalized quantity to indicate the strength of localization. It is a geometric criterion, in that if the effective beam width is more reduced, the localization is more pronounced. We use the strength of nonlinearity $\gamma = 5$ as representative for the focusing case, and $\gamma = -5$ for the defocusing case.

In the linear regime [Fig. 4(a)], localization is most pronounced in the lattice with defect type I; the effective beam width decreases the fastest in this geometry, as compared to the other. This figure should be compared with Fig. 2, as it offers the same information but in a slightly different format. In the geometries with defect types II and III, the localization is less pronounced than in the lattice with no defects. However, the inclusion of the medium's nonlinearity drastically changes these conclusions.

For the focusing nonlinear regime [Fig. 4(b)], localization in all geometries with defects is more pronounced than in the lattice without defects. In such a highly nonlinear regime, the disorder effects become less predominant and further increase in disorder does not produce significant influence on localization in the lattice without defects [23]. On the contrary,

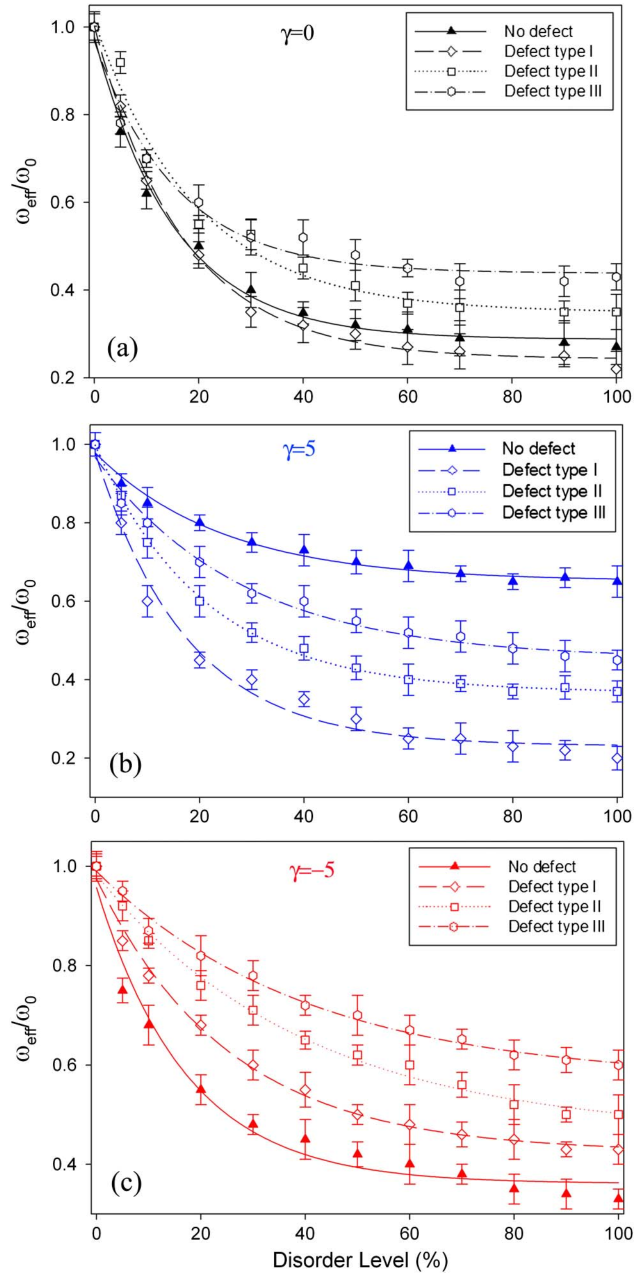

Fig. 4. (Color online) Comparison between Anderson localization in the linear regime and in different nonlinear regimes. (a) Linear case. (b) Focusing nonlinearity. (c) Defocusing nonlinearity. The normalized effective beam width at the lattice output is depicted as a function of the disorder level. The widths are normalized to their values without disorder. Points are ensemble averages and lines are least-square fits through the points. Parameters are as in Fig. 2.

in the defocusing nonlinear regime [Fig. 4(c)] localization is more pronounced in the geometry with no defects, at all levels of disorder.

To further quantify the effects of nonlinearity on localization, we measure the localization length ξ, by fitting the averaged intensity profile to an exponentially decaying profile $I \sim \exp(-2|x|/\xi)$. In a 2D geometry, there exist two localization lengths, one along the x axis and the other along the y axis. In our case, they are close to each other. When fitting to the exponential profile, we choose the error in the estimate to be up to 10%. Different defect geometries, as well as different

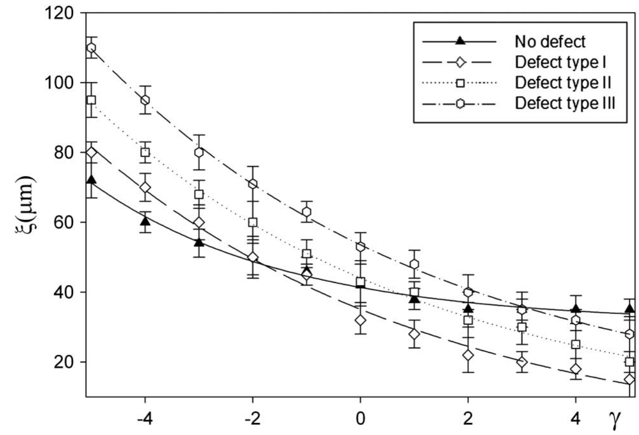

Fig. 5. Localization length as a function of the strength of nonlinearity for the 30% disorder level and different defect types. Physical parameters are as in Fig. 2.

nonlinear regimes, are compared in Fig. 5. Similar conclusions to Fig. 4 are drawn, concerning the behavior of localization between different cases.

5. EFFECT OF INPUT BEAM WIDTH ON LOCALIZATION

Finally, we analyze in some detail the localization effects for broader input beams. We study how the input beam width influences the localization process in different defect geometries. We confine our attention to the linear regime. As before, we perform a quantitative analysis, measuring the standard quantities and comparing Anderson localization in different lattice geometries. Figure 6 presents averaged effective beam width at the lattice output, for different defect types in the linear regime, as a function of the disorder level. Under the same conditions, broad beams diffract less than narrow beams, and turn out to be narrower upon reaching the output face. Other than this, the general behavior is similar—however, with some specific differences.

Effective beam widths in the geometries with defects are close to each other, but with a steeper reduction than in the case with no defects. For the lattice with defect type III, the effective beam widths always have the largest value, as compared to the other defect geometries; this is different from the case of narrow beams. However, if we use normalized effective beam width to indicate the strength of localization, it is observed that the localization is more pronounced for defect type II, followed by the types I and III, and in the end is the case with no lattice defects. Similar conclusions follow for localization in different nonlinear regimes (not shown).

If still broader beams are used, the conclusions remain the same. Also, for larger L one observes localization at lower disorder levels than for shorter L. While for the narrow input beam the localization depends on the joint influence of the defect type and nonlinearity, in the case of broad input beams the inclusion of defects always enhances localization in both the linear and nonlinear regimes.

To compare the localization of narrow and broad input beams, we use the averaged intensity profiles obtained for the 30% disorder level, in the geometry with defect type I (Fig. 7). Increasing the level of disorder, the output intensity beam profile narrows down and the linearly decaying exponential tails

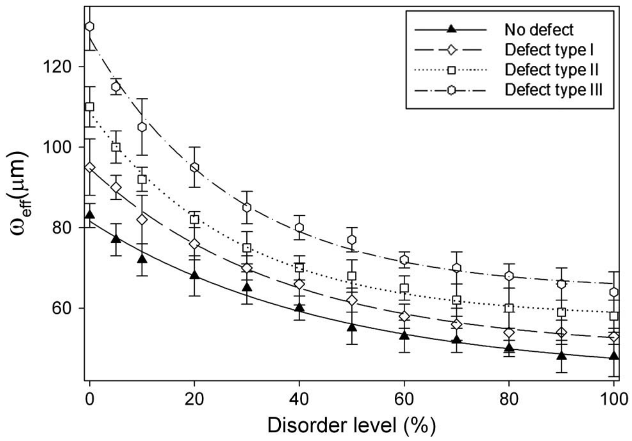

Fig. 6. Localization of the broad beam in the linear regime. The effective beam width at the lattice output is presented as a function of disorder level, for different defect types. Points are ensemble averages and lines are least-square fits through the points. Physical parameters are input beam FWHM = 50 μm; other parameters are as in Fig. 2.

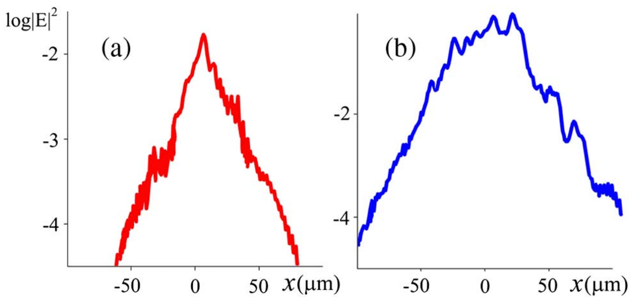

Fig. 7. (Color online) Ensemble-averaged intensity profiles (on the logarithmic scale) at the lattice output in the linear regime for narrow (a) and broad (b) beams for 30% disorder level.

become a direct indication of localization. It is seen that for 30% disorder level, the localization regime for the broad beam is not yet fully reached, as the average beam profile is not yet linearly decaying exponential. This means that a higher disorder level is necessary to observe the localization of the broad beam under the same conditions (for the same parameters) than for the narrow beam. If we measure the localization length ξ, by fitting the averaged intensity profile [Fig. 7(a)] to an exponentially decaying profile $I \sim \exp(-2|x|/\xi)$, we determine the value $\xi = 32.4$ μm. For a broader beam, such a value of the localization length is observed at ≈70% disorder level.

Along the propagation direction, a more pronounced broadening of the narrow beam is observed, especially at lower disorder levels. But in the case of a broad input beam there is no such beam broadening, even in the absence of disorder [24]. This is also the reason for the lower values of the effective beam width at the crystal output for broader beams, as compared to the narrow beams (see Fig. 6). Broad Gaussian beams do not diffract along propagation as do narrow beams, so Gaussian shape is visible especially for lower disorder levels [Fig. 7(b)]. But for very long propagation distance and for higher disorder levels, Gaussian shape disappears.

6. CONCLUSIONS

In conclusion, we have considered various aspects of Anderson localization in photonic crystals with line defects of different length. We have analyzed numerically how the presence of different defect types modifies Anderson localization of light. We have demonstrated that it could suppress or enhance light localization, depending on the strength of nonlinearity, as well as on the strength of disorder in the system. In the linear regime, the simplest defect type enhances the localization of light most. But in the nonlinear regime, localization effects depend on the strength of nonlinearity differently, for different defect types. Analyzing the effect of input beam width on Anderson localization, a more pronounced localization in the lattice geometries with all defect types for the broader input beam has been observed, as compared to the lattice with no defects. Higher disorder level is needed to reach the same localization as for the narrow input beam. Our study on light localization in photonic crystals with different defect types might be of interest for realization of novel light-manipulating optical circuits in future optical telecommunication and sensing networks.

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