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Disorder-induced localization of light in one- and two-dimensional photonic lattices

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Abstract
We study Anderson localization of light in one-dimensional (1D) and 2D photonic lattices. The influence of nonlinearity and disorder on Anderson localization in both systems is studied numerically. A sharp difference between localization in the linear and the nonlinear regimes is observed. Localization in the linear regime is more pronounced in the 2D lattice than in the 1D lattice. In the nonlinear regime, the coexistence of nonlinearity and disorder leads us to different conclusions. There are different nonlinear regimes in which 1D localization is more pronounced than 2D localization.

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(Some figures may appear in colour only in the online journal)

1. Introduction

Anderson localization is the absence of extended free (ballistic) propagation of waves in a disordered medium. There has recently been noted an increased interest in the study of Anderson localization in disordered systems, even more than 50 years after its discovery. Originally proposed for electrons and one-particle excitations in solids [1], it was soon observed in many other fields, such as acoustics [2], Bose–Einstein condensates [3] and optics [4, 5]. It is a general wave phenomenon that applies to the transport of electromagnetic, acoustic, quantum, spin and other waves. Anderson localization is realized experimentally as transverse localization in two-dimensional (2D) [6] and 1D [7] random lattice potentials.

In this paper, we analyze the effect of lattice dimension on the Anderson localization of light, specifically localization in 1D and 2D photonic lattices. A systematic quantitative study of the dependence of both the strength of disorder and the strength of nonlinearity on the Anderson localization in such systems is presented. While in the linear regime Anderson localization is more pronounced in 2D lattices, in the nonlinear regime this is not the case. We study localization in the nonlinear regime with both focusing and defocusing nonlinearity, and compare them with localization in the linear regime. There are focusing nonlinear regimes where the localization is more pronounced in 1D than in 2D lattices. However, in defocusing regimes the localization is always less pronounced than in the linear regime. We also investigate Anderson localization in the intermediate-dimensionality regime. The localization of intermediate states is less pronounced than both the 1D and 2D cases in the linear regime. But, in the nonlinear regime, the localization depends on the strength of the nonlinearity.

2. Theoretical model and system geometry

We study localization of light in optically induced photonic lattices and describe the propagation of a beam along the z-axis using the effective nonlinear Schrödinger equation for the slowly varying electric field amplitude A:

$$i\frac{\partial A}{\partial z} = -\Delta A - \gamma |A|^2 A - VA, \tag{1}$$

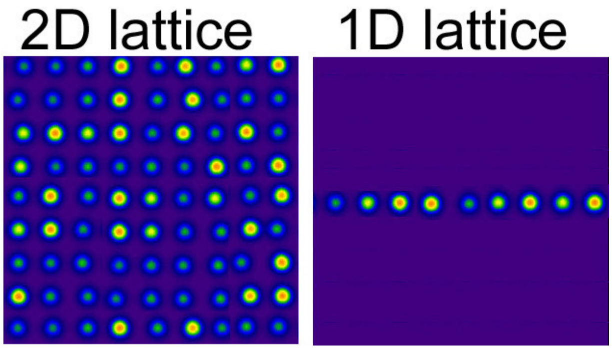

Figure 1. Problem geometry with 2D and 1D disordered lattices.

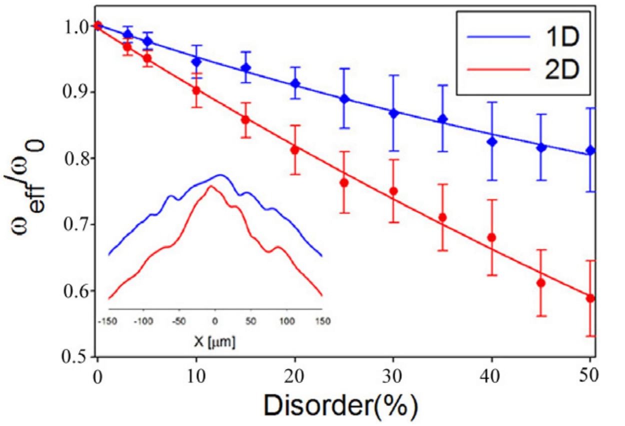

Figure 2. Comparison between 1D and 2D Anderson localization in the linear regime. Effective beam width at the lattice output versus disorder level. The widths are normalized to their input values. Error bars depict the spread in values coming from statistics. The inset depicts averaged beam profiles at the exit face of the crystal for 50% disorder level, for both 1D and 2D cases. Physical parameters: the crystal length $L = 20$ mm, input lattice intensity $V_0 = 1$, lattice period $d = 15\,\mu$m, input beam intensity $|F_0|^2 = 0.5$ and input beam FWHM $= 13\,\mu$m.

where Δ is the transverse Laplacian, γ is the dimensionless coupling constant and V is the transverse lattice potential, defined as a sum of Gaussian beams, with peak intensity V_0. A scaling $x/x_0 \to x$, $y/x_0 \to y$, $z/L_D \to z$ is utilized for the dimensionless equation, where x_0 is the typical full-width at half-maximum (FWHM) beam waist and L_D is the diffraction length. The propagation equation is solved numerically by employing a numerical approach developed earlier [8]. To study the Anderson localization effects, we realize disorder using random lattice intensity and also random lattice period. Random lattice intensity V_{0r} takes values from the region $1 - Nr < V_{0r} < 1 + Nr$ (r is the random number generator in the interval [0, 1]; N determines the degree of disorder) and the random lattice period values are in the range $[d \pm 0.5d]$.

We investigate localization effects in the 1D disordered photonic lattice, and compare them with localization in the 2D photonic lattice. We start from the 2D photonic lattice and keep the lattice period along one transverse direction fixed, but increase the period along the other transverse direction. In such a case, one can switch from a 2D to the 1D lattice and investigate localization effects in a system with crossover dimensionality. Figure 1 presents examples of the 1D and 2D lattices with one realization of the disorder.

3. Anderson localization in the linear regime

First, we investigate localization effects in the linear regime, in both 1D and 2D lattices. To observe the effect of Anderson localization, we gradually increase the level of disorder. For quantitative analysis we use the standard quantities describing the phenomenon of Anderson localization: the inverse participation ratio $P = \int I^2(x, y, L)\mathrm{d}x\,\mathrm{d}y / \{\int I(x, y, L)\mathrm{d}x\,\mathrm{d}y\}^{-2}$ and the effective beam width $\omega_{\mathrm{eff}} = P^{-1/2}$. To compare 1D and 2D Anderson localization, we measure the effective beam width at the lattice output for different disorder levels. Many realizations of disorder are needed to measure such quantities. In our numerics, different disorder realizations are realized by starting each simulation with different seeds for the random-number generator. We take 100 realizations of disorder for each disorder level.

Figure 2 presents a comparison between the 1D and 2D localizations in the linear regime. Averaged effective widths normalized to the corresponding input values are presented as functions of the disorder level. The effective beam width decreases as the level of disorder is increased in both 1D and 2D cases, but the localization is more pronounced in the 2D than in the 1D lattice. The effective beam width decreases faster in the 2D lattice, as compared to the 1D lattice, as the level of disorder is increased. The inset in figure 2 presents the averaged intensity beam profiles for 50% disorder level, measured at the exit face of the crystal. The output intensity beam profile narrows down, with exponentially decaying tails, as a direct indication of strong localization, as the level of disorder is increased. Corresponding localization lengths are measured by fitting the averaged intensity profiles to an exponentially decaying profile $I \sim \exp(-2|r|/\xi)$. Comparing 1D and 2D Anderson localization, we observed that the localization length in the 1D case is larger than in the 2D case. For the cases presented in figure 2, $\xi_{1D} = 73.4\,\mu$m and $\xi_{2D} = 48.5\,\mu$m.

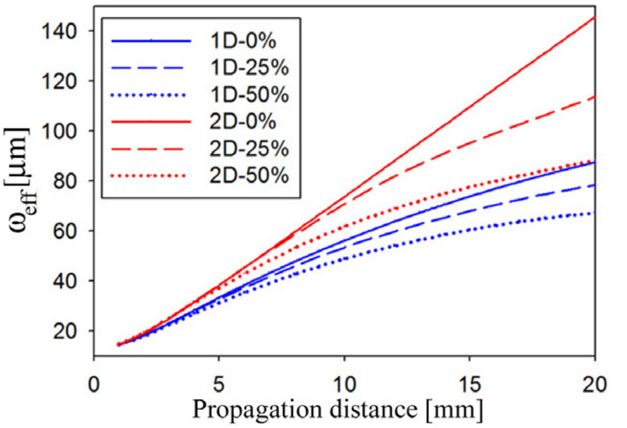

Figure 3. Effective beam width as a function of the propagation distance for different disorder levels. Blue curves are for the 1D localization, red curves are for the 2D case. Parameters are the same as in figure 2.

Next, we investigate Anderson localization along the propagation distance, and again compare 1D and 2D systems. Figure 3 presents the corresponding effective beam widths as functions of the propagation distance, for different disorder

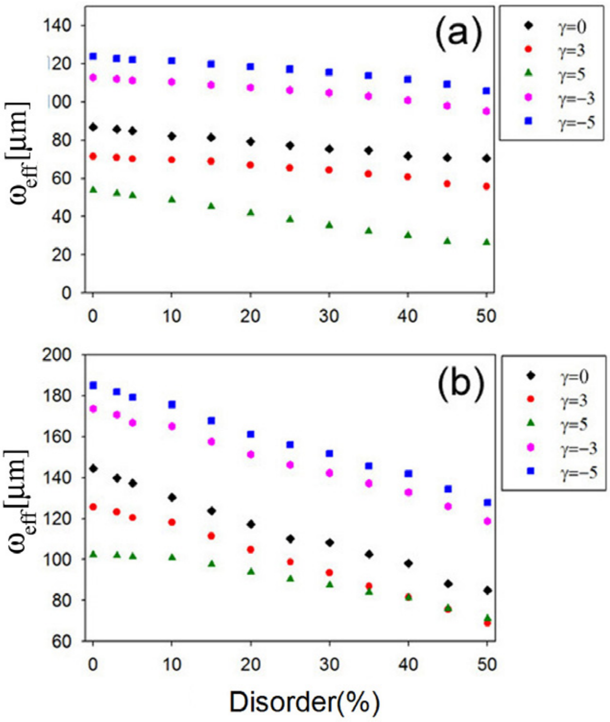

Figure 4. Linear versus nonlinear localization. Effective beam width at the lattice output versus the disorder level for linear, focusing and defocusing nonlinearities: (a) 1D lattice and (b) 2D lattice. Parameters are the same as in figure 2.

levels. In the case of no disorder there is diffusive broadening of the beams in both 1D and 2D lattices, but more pronounced in the 2D case. The broadening is suppressed with the inclusion of disorder in both systems. Comparing the corresponding disorder levels, a more pronounced suppression in 2D lattice is observed.

4. Anderson localization in the nonlinear regime

It is of general interest to consider localization effects in the nonlinear regime and investigate the influence of nonlinearity on the Anderson localization. We study both the 1D and 2D cases, in different nonlinear regimes. We investigate localization for focusing as well as defocusing nonlinearity and compare them with the linear regime localization.

Figure 4 presents the comparison between localization in the linear and nonlinear regimes. One can see that the effective beam widths have greater values in the defocusing regime, followed by the linear and then the focusing regime. Increasing the level of disorder, the effective beam widths get smaller for both focusing and defocusing nonlinearities, in both 1D and 2D lattices, similar to the linear regime.

To compare the corresponding cases, we measure normalized effective beam widths in the linear and nonlinear regimes, for both 1D and 2D lattices (figure 5). As we increase the strength of the focusing nonlinearity in both 1D and 2D lattices, the localization gets more pronounced than in the linear regime. In the case of defocusing nonlinearity the localization is less pronounced than in the linear regime, for both 1D and 2D cases. However, for stronger focusing nonlinearities the coexistence of nonlinearity and disorder effects leads to different conclusions.

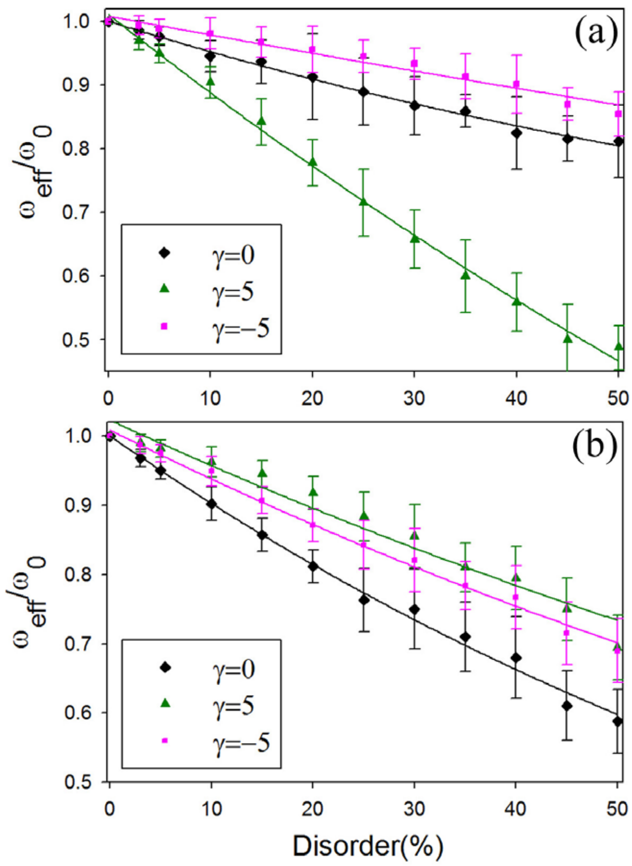

Figure 5. Normalized effective beam width at the lattice output versus the disorder level for the (a) 1D lattice and (b) 2D lattice. Parameters are the same as in figure 2.

With a further increase in the focusing nonlinearity, disorder produces no significant influence on the localization. There exists a strong nonlinearity threshold, above which disorder presents no predominant effect in the localization process; this threshold is different for the 1D and 2D cases. For the parameters we used, these threshold values are $\gamma \approx 5$ for the 2D case and $\gamma \approx 9$ for the 1D case. In both 1D and 2D cases, we depict the focusing nonlinear regime for $\gamma = 5$. In such a focusing regime, 1D localization is more pronounced than 2D localization, and even the defocusing localization is more pronounced than the focusing one for the 2D case (figure 5(b)). This sharp difference between the linear and nonlinear regimes is a consequence of the competition between disorder and nonlinearity effects in the localization process. While in the linear regime only the strength of disorder exerts an influence on the localization, in the nonlinear regime the situation is more complex. Besides the disorder level, an important role is also played by the strength of nonlinearity. This fact changes the strength of localization in the system with different dimensionality.

Finally, we discuss the intermediate-dimensionality regime, and localization effects in such a regime. We start from the 2D photonic lattice and increase the lattice period along one transverse direction, until we reach the 1D lattice. The localization in the intermediate states is less pronounced than in both the 1D and 2D cases in the linear regime. However, in the nonlinear regime the localization depends on the strength of nonlinearity. For a quantitative description of the dimensionality crossover in the beam localization, we use the localization length ξ. We establish the existence of two different localization lengths along two transverse directions in the system with dimensionality crossover [9] (not shown).

5. Conclusions

We have analyzed numerically the influence of the system's dimension on the Anderson localization of light. We have observed the coexistence of and competition between the strength of nonlinearity and the strength of disorder in such a system. In the linear regime, the localization is more pronounced in the 2D than in the 1D photonic lattice. But in the nonlinear regime the localization effects depend on the strength of the nonlinearity. We have also investigated the localization in the defocusing regime and have compared it with the focusing and linear regime. We have found a gradual transition in the system with the dimensionality crossover, and observed two different localization lengths.

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