Self-pumped phase conjugation of light beams carrying orbital angular momentum

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Abstract: We investigate the properties of angular momentum carrying vortex beams, reflected by a phase-conjugating mirror. It is shown that a self-pumped photorefractive phase-conjugating mirror is suitable to produce stable, high-fidelity phase conjugation of vortex beams. We prove that the topological charge of the vortex beam is maintained, and thus the angular momentum in the laboratory frame of reference is reversed, as it is expected by the time reversal property of the phase-conjugating mirror. The three dimensional interference pattern in front of the phase-conjugating mirror is studied and applications in optical traps are suggested.

OCIS codes: (190.5040) Phase conjugation; (190.5330) Photorefractive optics; (260.6042) Singular optics; (050.4865) Optical vortices; (140.7010) Laser trapping.

References and links

1. I. Basistiy, M. Soskin, and M. Vasnetsov, "Optical wave-front dislocations and their properties," Opt. Commun. **119**, 604–612 (1995).
2. K. Ladavac and D. G. Grier, "Microoptomechanical pumps assembled and driven by holographic optical vortex arrays," Opt. Express **12**, 1144–1149 (2004).
3. L. Allen, M. W. Beijersbergen, R. J. C. Spreeuw, and J. P. Woerdman, "Orbital angular momentum of light and the transformation of Laguerre-Gaussian laser modes," Phys. Rev. A **45**, 8185 – 8189 (1992).
4. A. Y. Okulov, "Angular momentum of photons and phase conjugation," J. Phys. B: At. Mol. Opt. Phys. **41**(10), 101001 (2008).
5. R. Hellwarth, "Generation Of Time-Reversed Wave Fronts By Nonlinear Refraction," J. Opt. Soc. Am. **67**(1), 1–3 (1977).
6. G. S. He, "Optical phase conjugation: principles, techniques, and applications," Prog. Quantum Electron. **26**(3), 131–191 (2002).
7. J. Feinberg, "Self-Pumped, Continuous-Wave Phase Conjugator Using Internal-Reflection," Opt. Lett. **7**(10), 486–488 (1982).
8. A. Ashkin, "History of optical trapping and manipulation of small-neutral particle, atoms, and molecules," IEEE J. Sel. Top. Quantum Electron. **6**(6), 841–856 (2000).
9. R. Beth, "Mechanical Detection and Measurement of the Angular Momentum of Light," Phys. Rev. **50**, 115–125 (1936).
10. J. Leach, M. J. Padgett, S. M. Barnett, S. Franke-Arnold, and J. Courtial, "Measuring the orbital angular momentum of a single photon," Phys. Rev. Lett. **88**(25), 257901 (2002).
11. He, Friese, Heckenberg, and Rubinsztein-Dunlop, "Direct observation of transfer of angular momentum to absorptive particles from a laser beam with a phase singularity." Phys. Rev. Lett. **75**(5), 826–829 (1995).
12. G. Gibson, J. Courtial, M. J. Padgett, M. Vasnetsov, V. Pas'ko, S. M. Barnett, and S. Franke-Arnold, "Free-space information transfer using light beams carrying orbital angular momentum," Opt. Express **12**(22), 5448–5456 (2004).
13. R. A. Fisher, ed., *Optical Phase Conjugation* (Academic Press, Inc., 1983).
14. P. V. Polyanskii and K. V. Fel'de, "Static holographic phase conjugation of vortex beams," Opt. Spectrosc. **98**(6), 913–918 (2005).

#116646 - $15.00 USD Received 3 Sep 2009; revised 5 Oct 2009; accepted 22 Oct 2009; published 30 Nov 2009

15. I. G. Marienko, M. S. Soskin, and M. V. Vasnetsov, "Phase conjugation of wavefronts containing phase singularities," in *International Conference on Singular Optics*, M. S. Soskin, ed., 3487, pp. 39–41 (Proceedings of SPIE, 1998).
16. J. Feinberg, "Continuous-Wave Self-Pumped Phase Conjugator With Wide Field Of View," Opt. Lett. **8**(9), 480–482 (1983).
17. F. C. Jahoda, P. G. Weber, and J. Feinberg, "Optical Feedback, Wavelength Response, And Interference Effects Of Self-Pumped Phase Conjugation In $BaTiO_3$," Opt. Lett. **9**(8), 362–364 (1984).
18. V. Y. Bazhenov, M. S. Soskin, and M. V. Vasnetsov, "Screw Dislocations In Light Wave-Fronts," J. Mod. Opt. **39**(5), 985–990 (1992).
19. T. A. Nieminen, A. B. Stilgoe, N. R. Heckenberg, and H. Rubinsztein-Dunlop, "Angular momentum of a strongly focused Gaussian beam," J. Opt. A: Pure Appl. Opt. **10**(11), 115005 (2008).
20. M. P. MacDonald, L. Paterson, K. Volke-Sepulveda, J. Arlt, W. Sibbett, and K. Dholakia, "Creation and manipulation of three-dimensional optically trapped structures," Science **296**(5570), 1101–1103 (2002).
21. A. Jesacher, S. Fürhapter, S. Bernet, and M. Ritsch-Marte, "Size selective trapping with optical "cogwheel" tweezers," Opt. Express **12**, 4129–4135 (2004).
22. J. P. Gordon, "Radiation Forces And Momenta In Dielectric Media," Phys. Rev. A **8**(1), 14–21 (1973).
23. V. G. Shvedov, A. S. Desyatnikov, A. V. Rode, W. Krolikowski, and Y. S. Kivshar, "Optical guiding of absorbing nanoclusters in air." Opt. Express **17**(7), 5743–57 (2009).
24. M. Bhattacharya, "Lattice with a twist: Helical waveguides for ultracold matter," Opt. Commun. **279**(1), 219–222 (2007).

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1. Introduction

Light beams with screw wavefront dislocations are known to carry optical orbital angular momentum [1]. An important field of applications for these beams is optical tweezers where angular momentum is transferred to microscopic samples e.g. to drive micro machines [2]. A screw wavefront dislocation may also be called an optical vortex. It possesses a topological charge, equal to the integer m, where m is defined by the $2\pi m$ phase change on any closed circuit around the dislocation center. The topological charge also indicates the optical orbital angular momentum, which is given as $m\hbar$ per photon [3]. The sign of m is defined by the handiness of the screw-like surface of fixed phase in space. It is important to note that the sign of m thus is always given in the frame of reference of the beam, while optical angular momentum conveniently is given in the laboratory frame of reference. It is a well known and often used fact that the sign of the topological charge of a vortex beam is reversed when it is reflected by a mirror [1, 4]. Since the direction of propagation also reverses in normal reflection, orbital momentum is maintained. The situation is different for a phase-conjugating mirror. Due to its time reversal property, the incident and reflected wavefront surfaces match perfectly [5]. As a result, the topological charge does not change sign and the optical orbital angular momentum is reversed. Hence, the difference in angular momentum of $2\mathrm{m}\hbar$ per photon needs to be transferred to the phase-conjugating mirror [4].

Established methods for experimental realization of optical phase conjugation are based on stimulated Brillouin scattering (SBS) or degenerate four-wave mixing in nonlinear media [6]. SBS mirrors require high laser power, while four-wave mixing in photorefractive crystals is particularly suited for optical phase conjugation at low power levels. However, experimental implementation of four-wave mixing may be challenging due to the high requirements with respect to the alignment and quality of the reference beams and stability of the interferometer.

In this contribution we utilize a self-pumped photorefractive phase-conjugating mirror [7] to investigate the fundamental characteristics of phase conjugation of vortex beams. It is shown that this implementation of a phase-conjugating mirror is suitable to produce very stable, high-fidelity phase conjugation of vortex beams. We directly compare the reflection properties of a conventional mirror to that of a phase-conjugating mirror. Furthermore the standing light field in front of the phase-conjugating mirror is studied and possible, powerful applications in optical trapping are proposed.

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2. Reflection of light beams carrying (orbital) angular momentum

Light beams can carry linear momentum, spin angular momentum and orbital angular momentum. Linear momentum, $p = \hbar\omega/c$ per photon, is widely used in optical tweezers [8], where it is transferred to microscopic particles. Spin angular momentum is strongly related to light polarization, resulting in a value of $|\vec{S}| = \pm\hbar$ per photon for circularly polarized light, where the sign is given by the chirality. An experimental proof of this relation was shown in the famous experiment of Beth [9].

Orbital angular momentum is related to a tilt of the wavefront. In case of a screw wavefront dislocation with $\exp(im\varphi)$ azimuthal phase dependence, also called an optical vortex, the pitch of the screw defines the topological charge m. The orbital angular momentum then is given as $m\hbar$ per photon [3, 10]. A direct experimental validation of this relation was done with optical tweezers quite recently [11]. Recent applications of orbital angular momentum include drive of micro machines with optical tweezers [2] and transfer of information encoded in orbital angular momentum states of light beams [12].

A first-order Laguerre-Gaussian (LG) beam is the experimentally most easily realized beam with orbital angular momentum. Mathematically, the LG beam is a free-space solution of the paraxial wave equation in the cylindrical system of coordinates [4].

$$E(z,r,\varphi,t) \propto \frac{E_0 \exp\left(i\left(kz-\omega t\right)+im\varphi\right)}{\left(1+iz/\left(kD^2\right)\right)^2} r^m \exp\left(-\frac{r^2}{D^2\left(1+iz/\left(kD^2\right)\right)}\right) \qquad (1)$$

Here, z, r, φ are coordinates in the cylindrical system of coordinates, m is the topological charge, D the diameter of the beam waist (FWHM) at $z = 0$ and E_0 is the maximal electrical field amplitude.

First, consider normal reflection of a laser beam that propagates in $+z$ direction at a conventional mirror. The beam carries spin angular momentum depending on its polarization and orbital angular momentum depending on its topological charge. It is important to note that the topological charge is usually defined in the frame of reference of the beam, while the sign of the angular momentum conveniently is given in the laboratory frame of reference. A circularly polarized photon will change from right-hand circular polarization to left-hand circular polarization and vise versa. As the direction of propagation is also reversed, the spin angular momentum is not changed and no angular momentum is transferred to the mirror. Similarly, the topological charge is reversed [1], while orbital angular momentum is not changed and thus not transferred to the mirror.

If the reflection is performed by a phase-conjugating mirror instead of the conventional mirror, polarization behaves equally and spin angular momentum is maintained. On the other hand, orbital angular momentum behaves significantly differently. An optimal phase-conjugating mirror reverses the wave-fronts exactly [13] and thus maintains the handiness of the screw-like planes of fixed phase of a Laguerre-Gaussian beam. The topological charge is also maintained, since its modulus is given by the pitch of the screw which does not change and its sign is given by the handiness of the screw. This is confirmed by previous experimental studies [14, 15]. However, since the direction of propagation is reversed, the orbital angular momentum changes by $|\Delta\vec{L}| = 2m\hbar$ per photon. The law of conservation of angular momentum requires that the phase-conjugating mirror itself accepts this difference.

A most interesting fact is that the incident and phase-conjugated wave are automatically overlapping perfectly. These counter-propagating waves thus form a stable interference pattern along the propagation direction of the waves.

For the case of reflection at a conventional mirror, the intensity of the interference pattern of

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two counter-propagating first-order LG beams of equal intensity can be written as [4]

$$I \approx 2|E_0|^2 \left(1+\cos\left(-2kz\right)\right) r^m \exp\left(-\frac{2r^2}{D^2\left(1+z^2/\left(kD^2\right)^2\right)}\right). \quad (2)$$

If a phase-conjugating mirror is used, the reflected beam carries opposite orbital angular momentum. The interference pattern changes significantly [4]:

$$I \approx 2|E_0|^2 \left(1+\cos\left(-2kz+2m\varphi\right)\right) r^{2m} \exp\left(-\frac{2r^2}{D^2\left(1+z^2/\left(kD^2\right)^2\right)}\right). \quad (3)$$

The expression now explicitly contains the azimuthal angle φ. Figure 1 shows isosurface plots of both cases.

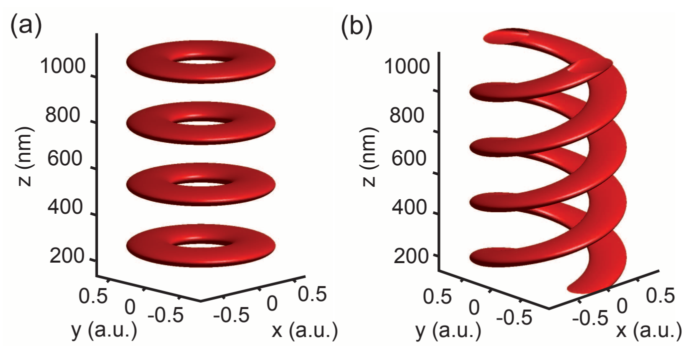

Fig. 1. Interference pattern of a first order LG beam and its reflection. The reflection is performed by a conventional (a) and a phase-conjugating mirror (b), respectively. A wavelength of $\lambda = 532\,\mathrm{nm}$ is assumed. The drawn isosurfaces correspond to a fraction a of the peak intensity where $a = 0.82$ is chosen for best visibility of the structures.

Phase conjugation is often implemented by stimulated Brillouin scattering. The underlying working principle causes a frequency shift between incident and reflected wave. As a consequence, the double helix interference pattern in Fig. 1 (b) rotates with a frequency Ω equal to the frequency difference of incident and reflected wave [4]. This connection could also be used to easily measure a possible frequency difference. Photorefractive phase-conjugating mirrors are not expected to introduce a frequency shift and thus should yield a stable interference pattern.

Basic phase conjugation of vortex beams has been performed with static holograms [14]. In this case, no interference pattern between the incident and the phase-conjugated wave can be expected at all, since the writing and the read-out of the hologram is separated in time by the process of hologram development.

3. Experimental setup for self-pumped phase conjugation

We use a photorefractive phase-conjugating mirror in order to prove the theoretical predictions and study the reflection behaviour of angular momentum carrying beams. Photorefractive nonlinear materials have widely been used to implement phase conjugation by four-wave mixing [5, 13]. Important reasons for the choice of a photorefractive phase-conjugating mirror are the low laser power that is required for its operation, the high reflectivity and the relatively simple implementation. We decided to use a self-pumped variation, where the pump beam is

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generated within the photorefractive material by beam fanning and internal reflection [7]. As a consequence, all interferometric parts of the phase-conjugating four-wave mixing setup are enclosed inside the photorefractive material. This allows high-fidelity and stable phase conjugation of arbitrary input beams [16].

Our setup is shown in Fig. 2. The employed laser light source is a frequency-doubled Nd:YAG solid-state laser, emitting a single mode at $\lambda = 532\,\mathrm{nm}$ with a maximum output power of $P = 100\,\mathrm{mW}$. A nominally undoped photorefractive $BaTiO_3$ crystal with six polished surfaces is used as the phase-conjugating mirror. It is used in self-pumped geometry with an incident angle of $\Theta \approx 48^\circ$ with respect to the crystal $\vec{c}$-axis, measured outside the crystal. The incident light is extraordinarily polarized. With this geometry and input powers of $P = 15-30\,\mathrm{mW}$, phase conjugation of a stationary input field is obtained within a time in the order of $t_{\mathrm{pc}} \approx 3\,\mathrm{min}$. The optical isolator is mandatory in any setup with phase conjugation, because phase conjugation exactly reverses the whole incident light field and thus focusses it back into the laser, even through a complex optical train or pinholes. The result is unwanted feedback into the laser resonator and intensity or mode instabilities of the laser output [17].

First-order LG beams with topological charge of $m = \pm 1, \pm 2, \pm 3$ are generated by computer-generated holograms [18], which are read out by a TEM_{00} plane wave approximation. The desired diffraction order of the hologram is selected by an iris diaphragm. The LG vortex beam then is split, where one part is reflected by the phase-conjugating mirror, and the other part is reflected by a conventional mirror M1. This mirror provides a reference with respect to the phase-conjugating mirror. The setup is chosen such that it is highly symmetric and each part undergoes the same number of reflections, until it is detected by the video camera. This enables us to compare the reflection at the phase-conjugating mirror directly with the reflection at a conventional mirror.

Lens L1 in the optical path of the conventionally reflected vortex is useful to compensate the widening of the beam during propagation. Its position can be varied, including one position where the vortex is imaged from its origin directly behind the hologram plate onto the video camera. For the further interpretation of our results, it is important to state that lens L1 does not influence the angular momentum content of the vortex beam in the paraxial case [19]. The lens can be left out, if identical size of both vortices at the camera plane is not desired. This was verified by performing all experiments where angular momentum is considered with and without lens L1. The vortices reflected by the two different mirrors can be observed spatially separated on the camera or alternatively overlaid. Additionally their phase structure can be investigated interferometrically with a plane reference wave.

4. Phase conjugation of beams carrying orbital angular momentum

At first, the simplest case of an optical vortex with a topological charge of $m = 1$ is considered. The vortex is split by the beam splitter BS1 to be able to compare the reflection at the phase-conjugating mirror with the reflection at the conventional mirror. One part of the vortex beam is reflected by the phase-conjugating mirror PCM, the other part is reflected by the conventional reference mirror M1. Both vortices are detected by the camera in a spatially separated way. In order to determine their relative orbital angular momentum, their topological charges are measured. For this purpose, a coherent plane wave, tilted by an angle Θ, is overlaid with the vortices. Figure 3 shows the resulting interference patterns. By counting the bright lines entering and leaving the marked circles and taking the difference, it can be clearly seen that the modulus of the topological charge is $|m| = 1$ in each case as expected. With the adequate care - the tilt angle Θ has the same sign for both vortices and the deviation of the plane reference wave's phase from an ideal plane wave is small with respect to the phase wedge introduced by the overall tilt [1]- it can be followed unambiguously that the sign of the topological charge is

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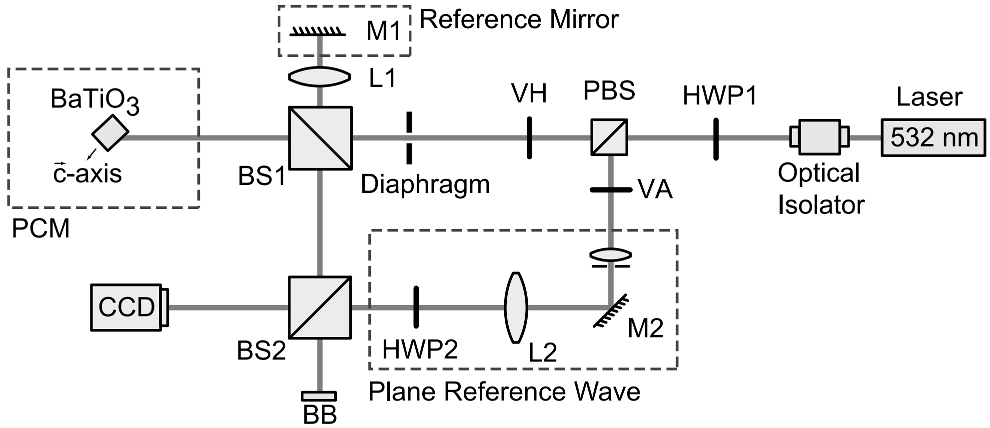

Fig. 2. Experimental setup for investigation of reflection properties of phase-conjugating mirrors. PCM, phase-conjugating mirror; VH, vortex hologram plate; (P)BS, (polarizing) beams splitters; HWP, half wave plates; L, lenses; M, mirrors; VA, variable attenuator.

reversed [18]. Since the vortex beams' direction of propagation is the same at the video camera plane, it is clear that they posses opposite orbital angular momentum. The setup is symmetric, the only difference being the conventional and the phase-conjugating mirror, respectively. Consequently, due to the conservation of angular momentum, the phase-conjugating mirror needs to absorb the difference of $|\Delta\vec{L}| = 2\hbar$ per photon. The polarization state of the vortices is maintained, and thus no spin angular momentum is transferred at all.

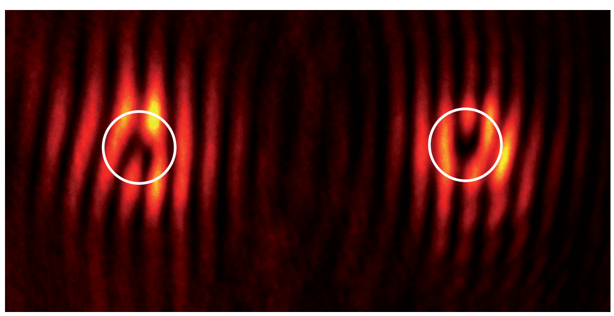

Fig. 3. Interference pattern of a plane wave and a vortex reflected by a phase-conjugating mirror (left) and a conventional mirror (right), respectively. It is clearly seen that the topological charge of the vortex is reversed. The circle indicates the center of the phase dislocation.

For a fundamental understanding of the phase conjugation of vortex beams it is important to have experimental access to the standing light field in front of the phase-conjugating mirror. Therefore, both vortices are overlaid at the video camera plane. The result is shown in Fig. 4(a). As expected, the interference of two co-propagating vortices with opposite topological charge is an optical dipole. This again indicates that the phase-conjugating mirror reverses the angular momentum in the laboratory frame of reference.

However, there is another interpretation of this optical dipole pattern. In the experimental setup (Fig. 2) the double helix interference pattern [Fig. 1(b)] is expected to exist between the

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hologram plane VH and the phase-conjugating mirror PCM. Direct experimental observation of the complete three dimensional structure seems ambitious, although not impossible, as the axial structure size is approximately the resolution limit of optical systems. However, a good approximation of one plane of the structure is accessible. Therefore, a fraction of each of the counter-propagating waves that constitute the three dimensional interference pattern needs to be branched off at one plane within the structure and both abstracted waves are overlaid elsewhere in a co-propagating regime. In this interpretation, the beam splitter BS1 branches off a part of the wave coming from right, reflects it towards the mirror M1 that guides it to the video camera. A part of the wave reflected by the phase-conjugating mirror and coming from left is also branched off by beam splitter BS1 and guided directly towards the video camera.

The observed optical dipole thus can be seen as an approximation to the intensity distribution of one plane inside the double helix interference pattern [Fig. 1(b)] and the position of the plane is determined by the beam splitter BS1. It is thus evident that there indeed is a double helix interference pattern and furthermore its rotation frequency Ω can be measured as the rotation frequency of the dipole. In our experiments we observed only stochastical rotation in the order of $\Omega < 1\,\text{mHz}$, which can completely be explained by a very slight (thermal) drift in the experimental setup. We thus conclude that a self-pumped photorefractive phase-conjugating mirror introduces no frequency shift into the reflected beam.

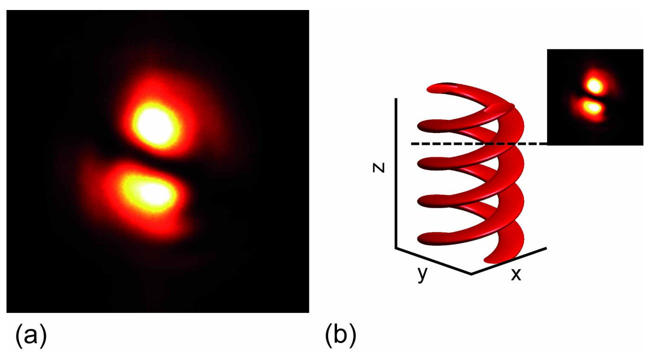

Fig. 4. Interference pattern of two vortices, reflected by a phase-conjugating mirror and a conventional mirror, respectively (a). Interpretation of the dipole pattern as one plane of the three dimensional helical interference pattern in front of the phase-conjugating mirror (b). The isosurface plot is adopted from Fig. 1.

Similar experiments are performed with vortices of higher topological charge $m = \pm 2, \pm 3$. The results are shown in Fig. 5. In all cases, the spin angular momentum is conserved, while the orbital angular momentum is reversed by the phase-conjugating mirror. The double helix interference pattern in front of the phase-conjugating mirror changes into a $2|m|$ fold helix interference pattern and the corresponding measurements show $2|m|$-poles instead of a dipole.

These multipole interference patterns [Fig. 5(b), 5(d)] look similar to that used commonly in optical tweezers to trap and rotate multiple particles [20, 21]. However, up to now mostly co-propagating vortex beams with opposite topological charge have been used in optical traps. The three dimensional structure of these interference patterns is significantly different to our experiments, where a light field is established by counter-propagating vortex beams of identical topological charge, generated by optical phase conjugation. Any lateral intensity profile of the co-propagating $2|m|$ multipole light field is similar to a lateral profile of the $2|m|$ fold helical light field considered in our experiments, apart from a certain rotation angle ϕ. Unlike the helical light field though, the interference pattern of the co-propagating vortex beams has no twist at all. A second major difference is the relative orientation of the Poynting vectors

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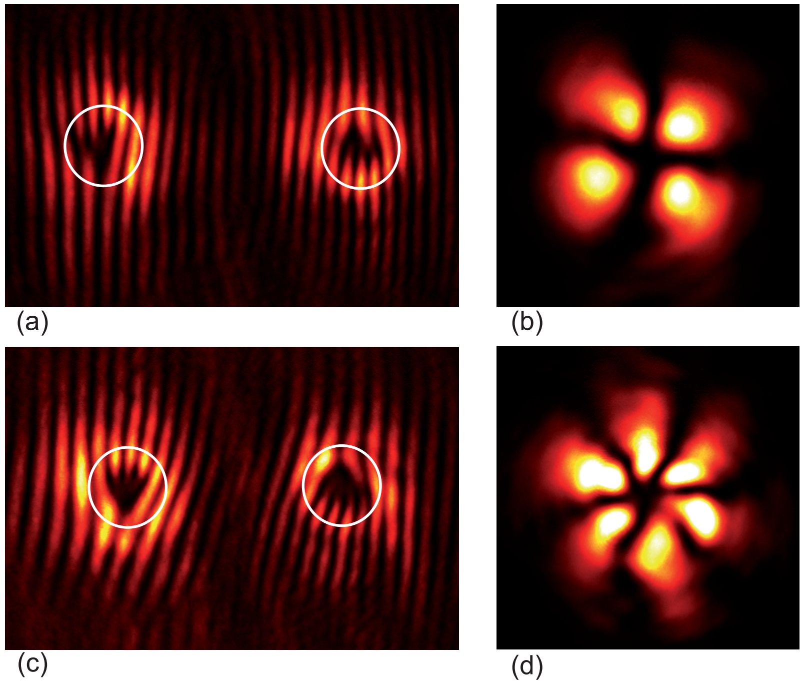

Fig. 5. Interference pattern of $|m| = 2$ vortices (a,b) and $|m| = 3$ vortices (c,d). The left column shows a comparison between vortices reflected by a phase-conjugating (left) and a conventional mirror (right). At the right column, both vortices are interfering, yielding a $2|m|$ multipole interference pattern.

of the constituting beams. In the co-propagating configuration, the axial components of both Poynting vectors are parallel, yielding a resulting scattering force in optical traps. The counter-propagating beams with antiparallel Poynting vectors do not posses a resulting scattering force in axial direction (given the beams have equal intensity) [22], thus allowing for a highly symmetric axial trapping potential.

Quite recently, there have also been successful demonstrations of optical trapping with counter-propagating vortex beams of opposite topological charge [23]. The resulting three dimensional interference pattern is completely different (cf. Fig. 1(a) for the case that both beams are mutually coherent) and enables trapping of absorbing particles.

The helical interference pattern of two identical counter-propagating vortex beams is of highly topical interest and has been proposed recently to trap and guide ultracold matter optically [24]. The main challenges for the actual experimental implementation of the proposed standing light field are the required overlap and mutual phase stability of the constituting vortex beams. We believe that the demonstrated implementation with self-pumped photorefractive phase conjugation is well suited to provide the desired trapping geometry. In contrast to conventional counter-propagating vortex beams, the phase conjugation ensures that both constituting beams are phase-locked and the helical interference pattern is highly stable. Even more important is the fact that both constituting beams overlap to a very high degree; in the limit of a perfect phase-conjugating mirror the overlap as well would be perfect.

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5. Summary

A self-pumped photorefractive phase-conjugating mirror is implemented to investigate the reflection properties of angular momentum carrying first order Laguerre-Gaussian vortex beams. It is shown that during reflection spin angular momentum is conserved and orbital angular momentum is reversed as expected by recent theoretical investigations [4]. Conservation of total angular momentum requires the phase-conjugating mirror to accept the difference in orbital angular momentum. It could be shown that a stationary three dimensional interference pattern exists in front of the phase-conjugating mirror. The incoming and reflected waves are automatically overlapping, phase-locked and the frequency is maintained, in contrast to phase-conjugating mirrors based on stimulated Brillouin scattering. These features make photorefractive phase-conjugating mirrors a promising solution for the implementation of three dimensional helical geometries [24] for optical trapping.

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