Overloaded phase-code multiplexing for volume holographic storage

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Overloaded phase codes for volume holographic data storage are introduced. In contrast to any previous phase-code design, overloaded phase codes enable multiplexing of a number of data pages that exceeds the number of utilized reference beams. In this way the achievable data capacity can be augmented. Overloaded codes are generated by extending multilevel phase codes based on the discrete Fourier transform. We demonstrate multiplexing of 70 analog pages by means of 64 reference beams. The analysis of reconstructed digital data pages suggests that a capacity gain of up to 15% is reasonable. © 2008 Optical Society of America

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Volume holographic data storage has a great potential to become the basis of next generation optical storage devices, since it provides high storage densities and high data transfer rates [1,2]. In this context, the employed multiplexing technique plays a key role. Phase-code multiplexing is an advanced variant of angular multiplexing [3]. It does not require moving components for superposition of holograms and provides a signal-to-noise ratio (SNR) 2 orders of magnitude higher than pure angular multiplexing [4–6]. So far several implementation concepts of phase-code multiplexing have been discussed. Owing to its straightforward implementation a popular method has been random phase encoding, which utilizes random phase distributions in the reference wave (e.g., [7]). It can be realized by means of a collimated laser beam that illuminates a (1-D) random phase plate in the reference arm. The phase-modulated transmitted light is made to intersect the signal wave inside the storage medium. Between subsequent recordings the phase plate is shifted to provide different unique phase codes. The achievable SNR of this method is strongly dominated by Bragg-matched cross talk. It can be estimated as $\mathrm{SNR}_{\mathrm{random}} \approx N/M$ [6], where N is the number of individually phase-modulated segments of the reference wave and M denotes the number of data pages that can be multiplexed ($M,N \in \mathbb{N}$). A reasonable SNR can only be achieved if the number of phase elements very much exceeds the number of data pages ($M \ll N$). In 1991 deterministic orthogonal phase-code multiplexing based on binary Hadamard matrices (H_N) was proposed [3]. In this method the number of phase-shifting elements N equals the number of data pages M that are multiplexed ($M=N$). Noise is theoretically only composed by light diffracted at non-Bragg-matched gratings. SNRs exceed that of random phase-code multiplexing by several orders of magnitude. The order of the phase codes is restricted to values of $4m$ ($m \in \mathbb{N}$) at best. Recently, deterministic multilevel phase codes have been presented that are based on the discrete Fourier transform (DFT) [8]. Phase-code sets of any order can be generated that allow multiplexing at error rates similar to that of Hadamard related codes [9]. According to these phase-code multiplexing techniques it is a generally accepted limit that a reference wave segmented into N parts allows, at a maximum, the recording of $M=N$ data pages at a reasonable error rate. Here we show that this constraint can be overcome by deterministic overloaded phase-code multiplexing. Based on the codes proposed in [8] we present the construction and application of overloaded, nonorthogonal phase codes that enable multiplexing of a number of data pages M by means of an N-fold segmented reference wave with $M>N$.

Overloaded phase-code sets are constructed by means of rectangular Vandermonde-type matrices whose elements are computed by

$$v_{kl} = \exp\left((-1)^p 2\pi i \frac{(k-1)(l-1)}{M}\right), \tag{1}$$

where $k \in [1,2,\ldots,M]$, $l \in [1,2,\ldots,N]$, and $M>N$. M and N denote the number of multiplexed data pages and the number of employed reference beams, respectively. p is usually set to 1 according to DFT theory. Phase-code matrices $\Phi_{M\times N}$ are generated by assigning each v_{kl} to a phase delay

$$\varphi_{kl} = \frac{2\pi}{M}\left[\left(-i\frac{M}{2\pi}\ln(v_{kl})\right)\mathrm{mod}\, M\right] \tag{2}$$

of the lth reference beam of the kth phase code. If $M>N$, $\Phi_{M\times N}$ is a nonorthogonal matrix that provides M phase codes for recording M data pages by means of N reference beams. When employing an overloaded set of phase codes the incorporated phase modulator needs to be capable of displaying at least M equidistant phase delays in the interval $[0,2\pi-(1/M)]$. That is, the number of required phase steps is determined by the number of multiplexed data pages. In some situations the actual required number of phase steps can be reduced by combining valid code matrices by means of the Kronecker product. For instance, to multiplex 36 pages by the use of 32 reference beams corresponding phase codes can be generated by means of $H_2 \otimes H_2 \otimes V_{9\times 8}$, which requires 18 instead of 36 different phase steps.

The utilization of nonorthogonal matrices gives rise to considerable cross talk. To study the theoretical noise characteristics the SNR has to be derived. This can be done by using the noise-to-signal ratio (NSR) derived in [9], extending the expression to the present case and determining its inverse. If t is the hologram thickness in the z direction, λ is the wavelength, x_2 denotes the x coordinate of a diffracted wave in the output plane, and F is the focal length of the lenses used to focus the signal and reference wave into the storage material and of the lens used to image the reconstructed wave to the output plane, then the NSR when employing rectangular matrices can be written as

$$\mathrm{NSR}_{n'} = \frac{1}{N^2}\sum_m \left| \sum_k \sum_{\substack{l \\ l\neq k \text{ if } m=n'}} v'_{n'k}(v'_{ml})^* \operatorname{sinc} \xi \right|^2,$$

with

$$\xi = \left[(k-l) + \frac{\lambda x_2}{2Ft}(l^2-k^2) + \frac{\lambda^3}{8t^3}(l^2-k^2)^2 \right]. \quad (3)$$

The indices k and l are elements of $[-N/2, -N/2+1, -N/2+2, \ldots, N/2-1]$, and the indices n' and m are elements of $[-M/2, -M/2+1, -M/2+2, \ldots, M/2-1]$ with $N<M$ and $M,N \in \mathbb{N}$. M is the number of data pages, and N denotes the number of reference beams. If M or N are odd, the corresponding indices are not integer values. $v'_{ab} = v_{(a+M/2+1)(b+N/2+1)}$ takes the modified range of the indices into account. Index n' is related to an index n that denotes the row of a phase-code matrix, bearing the phase shifts for recording or reconstructing data page n, by $n=n'+M/2+1$, i.e., $n \in [1, \ldots, M]$. The theoretical NSR computed by Eq. (3) takes all code related cross-talk noise into account, which arises from terms that do not contribute to the desired, addressed data page during readout. When utilizing overloaded phase codes, cross-talk noise due partly to reconstruction of nonaddressed pages (Bragg-matched cross talk) is naturally much stronger than noise arising from diffraction at non-Bragg-matched gratings. Therefore, it seems to be appropriate to neglect the latter effect. That is, l can be set equal to k and the sinc function in Eq. (3) disappears. After substituting the ($'$) quantities, the desired SNR related to phase-code n can be derived as

$$\mathrm{SNR}_n \approx \left[\frac{1}{N^2} \sum_{\substack{m=1 \\ m\neq n}}^{M} \left| \sum_{k=1}^{N} \exp\left(2\pi i k \frac{m-n}{M} \right) \right|^2 \right]^{-1} = \frac{N}{M-N}. \quad (4)$$

The resultant SNR is independent of n and is equal to the reciprocal relative capacity gain. That is, the theoretical SNR is constant when recording rM data pages by means of rN beams for any r. This argumentation is valid only when neglecting experimental limitations.

Numerical comparison of the SNR according to Eqs. (3) and (4) indicates that only a small error is accepted by the above assumption that neglects diffraction at non-Bragg-matched gratings. The distribution that takes the sinc function into account exhibits the same behavior as discussed in [9] for phase codes related to unitary matrices. However, the difference of the highest and lowest SNR values is marginal. For instance, for a $V_{70\times 64}$ the maximal and minimal SNRs are 10.6571 and 10.6668. Equation (4) computes an SNR of 10.6667. Hence, setting $k=l$ is reasonable. A systematic investigation reveals that the tolerated error remains below 1% for reference beam numbers up to $N=90$, independent of the capacity gain or M.

Based on Eq. (4) the overall SNR characteristics are examined when increasing the number of overloaded phase codes. Figure 1 shows the SNR versus the total number of multiplexed pages M by use of 64 reference beams (upper horizontal axis) and versus the plain page related capacity gain in percentages (lower horizontal axis). Capacity overloads of 10% or 20% cause a deterioration of the achievable SNR down to ≈ 10 and ≈ 5, respectively. Admittedly, these values are small in comparison to the maximal theoretical SNR for an orthogonal phase-code set. However, the idea of overloaded phase-code multiplexing is to exploit the fact that noise always arises in real systems. Hence, the practical value of overloaded phase-code multiplexing has to be experimentally estimated. An increase of the overall SNR might be accomplished by taking the cross-talk characteristics into account. Owing to the construction rule, i.e., evaluating the geometric sequence (1), the cross talk has to follow a sinc^2 function. That is, pages that are recorded by phase codes adjacent to the addressing code always produce the strongest cross talk. By adapting the data layout in the signal arm, cross talk might be reduced.

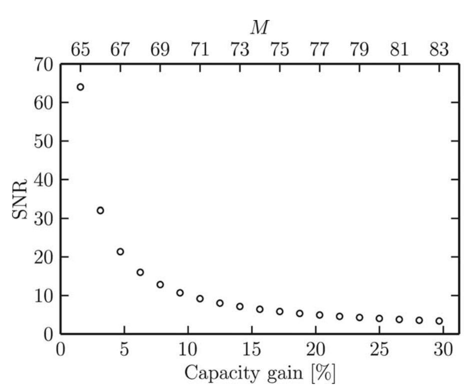

Fig. 1. SNR versus M and versus the capacity gain in percentages, when multiplexing M pages ($M \in [65, \ldots, 83]$) using $N=64$ reference beams.

Fig. 2. Reconstructed analog data pages of a set of 70 pages that are recorded by means of 64 reference beams.

The experimental applicability of overloaded phase-code multiplexing is investigated in a 90° configuration using a cw laser at $\lambda = 488$ nm and employing $LiNbO_3$:Fe (0.01 mol. % Fe) as storage material. In the reference arm a diffractive optical element splits the laser beam into 128 beams whose phases can be individually adjusted by a phase-only modulator (128 pixels, 2 mm long, and 97 μm wide). An aperture allows one to reduce the number of employed reference beams. By grouping reference beams into multiple pixels that are evenly distributed across the aperture, the effective intensity noise is kept below 4%. The phase is adjusted with an accuracy of $\approx 1\%$. The diameter of the reference beams incident on the media is about 1.5 mm. In the signal arm a liquid crystal array with a resolution of 800×600 pixels is used for amplitude modulation. A data page displayed by the spatial light modulator is imaged through the media onto a camera using a telescope arrangement (focus lengths: 80, −50, and 80 mm). Multiplexing is performed using an incremental recording schedule [10].

At first, the storage performance when employing overloaded phase-code multiplexing is to be qualitatively examined. For that purpose 70 analog pages are multiplexed using 64 reference beams, which corresponds to a page-related capacity gain of 9.4%. Each of the 70 pages displays an analog number that corresponds to the number of the phase code used to record the particular page. Hence, the number of ON pixels in the incorporated data pages is much less than for typical digital data pages. The experiment demonstrates that all of the 70 reconstructed pages, of which three are illustrated in Fig. 2, show a clear image. It turns out that pages recorded with higher phase-code numbers, on average, show gradually less noise. This is supposed to be caused by a very low number of recording cycles conducted by the incremental recording schedule in the experiment.

To investigate the storage of digital data, $M = 22$ and 23 data pages are recorded by means of $N = 21$ reference beams, corresponding to capacity gains of 4.76% and 9.52%. The employed pages are modulated by a 9:12 modulation code arranged in 3×4 channels per block (50% ON pixels). The bit error rates (BERs) are computed to 1.8×10^{-6} ($V_{22\times21}$) and 1.0×10^{-4} ($V_{23\times21}$). For the latter case, Fig. 3 shows the ascertained channel histograms. Recording 21 pages by means of 21 beams yields under the same conditions a BER of 4.2×10^{-10}. The experimental SNRs degraded from 9.2 ($V_{21\times21}$) down to 6.3 ($V_{22\times21}$) and 4.8 ($V_{22\times21}$). These experiments verify the capability of overloaded phase-code multiplexing, but they also reveal that the actual SNR values are lower than the

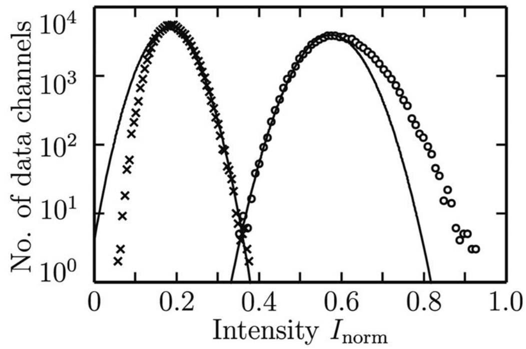

Fig. 3. Channel histograms of data pages multiplexed by overloaded phase codes $V_{23\times21}$(I_{norm}=normalized intensity after block decoding).

theoretical values given by Eq. (4). This is caused by experimental limitations such as unwanted intensity variations in the signal arm and unwanted amplitude and phase variations in the reference arm. Typically, it is the aim to maximize the capacity as far as possible by adapting experimental parameters until a BER of 10^{-3} is reached, which can effectively be improved down to 10^{-12} by suitable error-correcting codes (ECC) (e.g., Reed–Solomon codes). In all performed experiments, employing reference beam numbers of $N = 16$, 20, 32, 64, this target BER has been exceeded for an overload of more than $\approx 15\%$ at the latest.

In conclusion, we introduced overloaded phase codes for multiplexing in volume holographic storage systems. Experiments prove that these codes are suitable to augment the storage capacity. The implementation of the technique is straightforward in any phase-encoded system that employs a phase modulator capable of displaying the required number of phase steps. Overloaded phase-code multiplexing refers only to the coding technique realized in the reference arm. It is independent of the actual data format and can always be applied additionally. The present experimental analysis suggest that the utilization of nonorthogonal phase codes is worthwhile for accomplishing capacity gains of up to $\approx 15\%$.

References

1. H. J. Coufal, D. Psaltis, and G. T. Sincerbox, eds., *Holographic Data Storage* (Springer, 2000).
2. L. Hesselink, S. S. Orlov, and M. C. Bashaw, Proc. IEEE **92**, 1231 (2004).
3. C. Denz, G. Pauliat, G. Roosen, and T. Tschudi, Opt. Commun. **85**, 171 (1991).
4. C. Denz, G. Pauliat, G. Roosen, and T. Tschudi, Appl. Opt. **31**, 5700 (1992).
5. K. Curtis and D. Psaltis, J. Opt. Soc. Am. A **10**, 2547 (1993).
6. M. C. Bashaw, J. F. Heanue, A. Aharoni, J. F. Walkup, and L. Hesselink, J. Opt. Soc. Am. B **11**, 1820 (1994).
7. V. N. Morozov, Sov. J. Quantum Electron. **7**, 961 (1977).
8. X. Zhang, G. Berger, M. Dietz, and C. Denz, Opt. Lett. **31**, 1047 (2006).
9. X. Zhang, G. Berger, M. Dietz, and C. Denz, Appl. Phys. B **85**, 575 (2006).
10. Y. Taketomi, J. E. Ford, H. Sasaki, J. Ma, Y. Fainman, and S. H. Lee, Opt. Lett. **16**, 1774 (1991).