# Pattern control and mode interaction in a photorefractive single feedback system

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We experimentally investigate pump beam frequency detuning as a control technique in a photorefractive single-mirror feedback system exhibiting spontaneous self-organized transverse pattern formation with a regime of multistability. Deterministic switching between bistable patterns is demonstrated experimentally. An experimental stability analysis uncovers the effect of frequency detuning on the thresholds of unstable modes. The interaction of independent modes is found to be responsible for the different pattern symmetries observed in this system. © 2007 Optical Society of America

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## 1. INTRODUCTION

An intriguing property of many extended nonlinear nonequilibrium systems is the spontaneous formation of self-organized periodic spatial patterns,[1] a topic for which nonlinear optics has become one of the prime research fields[2] in physics. Pattern formation has been demonstrated and investigated in a variety of systems, using such diverse nonlinear media as atomic vapors,[3,4] liquid crystals,[5,6] organic films,[7] or photorefractives.[8] Control of pattern formation has recently received increased attention, through endeavors to stabilize active modes, to manipulate pattern orientation and symmetries, to inhibit growth of instabilities, and to select specific solutions out of several simultaneously available ones.[9–12]

In nonlinear optics various control methods are readily available, such as seeding, or invasive and noninvasive spatial frequency filtering.[13] Some control methods are generally applicable to a large number of systems while many depend on the characteristics of a specific system under consideration.

Pattern forming systems that take advantage of photorefractive wave mixing demonstrate a rich variety of patterns[14] with several bistabilities and multistabilities and hence lend themselves to the investigation of control techniques. Photorefractive wave mixing depends on the interference between coherent pump beams and is therefore sensitive to a small frequency detuning of the pumps.[15–17] In this paper, we experimentally examine pump beam frequency detuning as a method for controlling pattern formation in a single-mirror feedback system. Such detuning affects the photorefractive wave mixing by inducing longitudinal motion of the refractive index grating[15] and was observed to lead to transitions between bistable patterns.[16] Although a linear stability analysis demonstrated the effects of moving gratings due to the frequency detuning, the resulting transitions between patterns of different symmetries are unexplained. No analytical or numerical works exist at the time of this writing that predict the pattern symmetries as observed in experiment beyond the hexagonal patterns.

In the following, we will investigate experimentally how the fundamental unstable modes of the system are influenced and the symmetry of the generated patterns is determined by the pump beam detuning, and investigate the suitability of pump beam detuning as a control method for the specific system and in a general context. In Section 2 we will introduce the experimental system and demonstrate the existence of a bistability between two-dimensional transverse patterns. Section 3 presents spontaneous transitions at the bistability and introduces frequency detuning as a control method to reliably induce pattern transitions. Finally, Section 4 will uncover the mechanism through which frequency detuning affects the symmetry of patterns, leading to its suitability as a method for pattern control in this experimental system.

## 2. SPONTANEOUS PATTERN FORMATION

At the heart of the photorefractive single feedback setup (Fig. 1) is an iron-doped potassium niobate crystal ($Fe{:}KNbO_3$) of dimensions $(a,b,c)=5$, 5, and 6 mm, in which two focused laser beams (cw 532 nm, focus diameter 320 $\mu$m) interfere and form a reflection grating.[18] One beam is directly obtained from the laser source with a beam power of 13 mW incident on the crystal. The second beam consists of the first beam's output fed back into the photorefractive material by a mirror. Instead of placing a real mirror at the end of the crystal, a virtual mirror is created by a $4f$ setup,[14] which enables access to the Fourier plane and allows for negative virtual mirror positions. To avoid internal reflections at the crystal surfaces, the crystal is tilted by a few degrees.

As a result of the diffusive charge carrier transport in potassium niobate and the crystal's $c$ axis being aligned with the propagation direction, the refractive index modulation is phase shifted by $\pi/2$ with respect to the intensity modulation leading to energy coupling of the beams, such that the incident beam is weakened and the reflected beam is amplified.

Experimentally, the coupling strength $\gamma l$ can be set by changing the polarization of the incident beam, thus varying the effective electro-optic coefficient up to a maximum value of $\gamma l_{\max}=5.5$ determined using two beam coupling.

Above a threshold coupling strength that varies with the mirror position $d$, transverse modulations are observed to grow in the output beams. The basic mechanism at work is a selective amplification of beams propagating at certain small angles (around 1°) to the pump beams. The angular selectivity is due to diffraction during the round trip propagation to the external feedback mirror and to a lesser degree due to diffraction during propagation within the bulk nonlinear medium. In both cases diffraction allows any part of the initial pump beam to be correlated with a surrounding area of the reflected pump beam by diffractive spreading. The competition of scales imposed by these two feedback mechanisms is responsible for the complex patterns characteristic to this system. Azimuthal selectivity, which is necessary for the generation of patterns with broken rotational symmetry, stems from further nonlinear interaction of the initial unstable modes.

The growth of a periodic two-dimensional pattern gives rise to spatial sidebands that form the characteristic far-field patterns (Fig. 2). The far-field is also accessible in the Fourier plane of the 4*f* feedback assembly, which we exploit to perform spatial frequency filtering in order to determine the active modes.

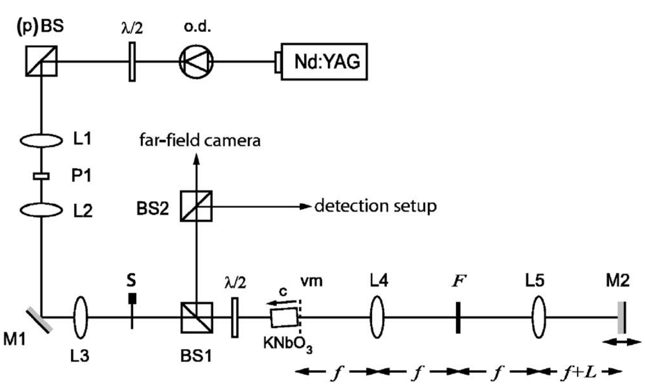


Fig. 1. Experimental setup for investigation transverse pattern formation. L, lenses; (p)BS, (polarizing) beam splitter; M1, mirror; M2, piezomounted mirrorr; vm, virtual mirror plane; S, shutter; P, pinhole; *F*, Fourier plane of the feedback arm.

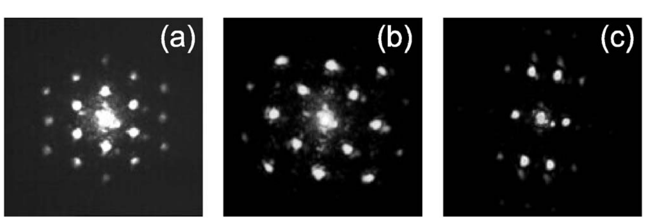


Fig. 2. Far-field patterns observed without control. Central in each image is the output pump beam while the lateral spots correspond to the modes constituting the transverse modulation of the actual near-field beams. (a) Common hexagonal pattern, (b) square pattern, (c) squeezed hexagonal pattern.

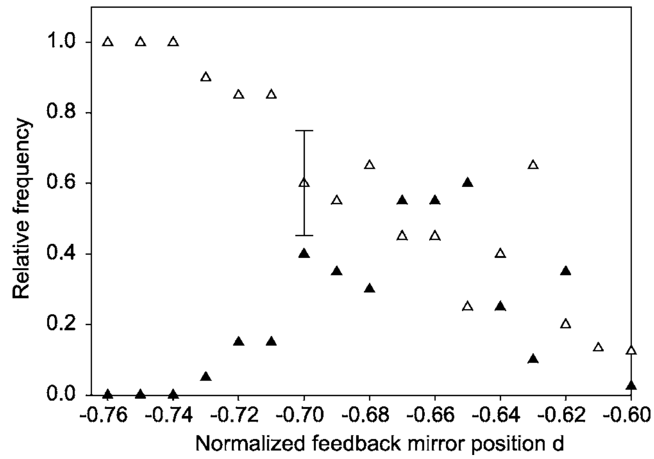


Fig. 3. Relative stability of two particular patterns, measured by their frequency to be generated from noise, depending on the location of the feedback mirror. Open triangles, squeezed hexagonal pattern; filled triangles, square pattern. Exemplary error bar gives the expected statistical error. Squeezed hexagon is practically always observed at mirror positions around $d=-0.75$. Both patterns have about equal probability of being generated at $\approx d=-0.67$.

Scale and symmetry of the patterns depend on the position $d$ of the (virtual) mirror,[14] given normalized to the length of the crystal $l=6$ mm

$$d=n_0 L/l, \tag{1}$$

where $L$ is the true distance from the crystal's back face to the virtual mirror plane and $n_0 \approx 2.33$ is the refractive index of the medium.

The scale of patterns is recorded using the normalized transverse wavenumber

$$k_d l=\frac{k_0 \theta^2}{2n_0} l \tag{2}$$

of the modulation where $k_0$ is the wavenumber of the light and $\theta$ is the angle between the corresponding spatial sideband and the pump beam. The mirror position influences the transverse correlation across the beams due to diffraction and thereby determines the growth rates of different spatial modes.

While hexagonal patterns are observed over most of the available parameter space, a range of mirror positions inside the photorefractive crystal $-0.8<d<-0.2$ offers different symmetries and has therefore been called the multiple pattern region.[14] Here, a bistability between patterns of different symmetries (square and squeezed hexagonal) allows for an investigation of the effect of pump beam detuning on transverse pattern formation.

This bistable region is determined by measuring the frequency with which each pattern class is spontaneously observed for a given position of the virtual feedback mirror. Equal frequency of occurrence is taken as an indication of equal relative stability. To this end, the symmetry class of 500 spontaneously formed patterns is recorded over the entire multiple pattern region. A subset of the data for squeezed hexagonal and square patterns is shown in Fig. 3. At the virtual mirror position $d=-0.75$, the pattern observed always has a squeezed hexagonal symmetry. As the mirror is moved towards the center of the medium, the probability of observing a square pattern rises until both have equal probabilities of being observed at $d=-0.68$. Thus we assume that at this mirror position both patterns are approximately equally stable, and select $d=-0.68$ as a working point for investigating controlled pattern transitions.

## 3. PATTERN SWITCHING

The bistability between the two pattern symmetries is additionally indicated by the observation of spontaneous transitions, which are initiated by noise immanent to the

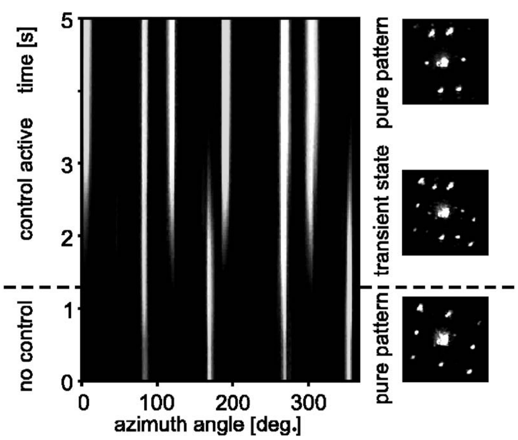


Fig. 4. Pattern transition induced by negative frequency detuning. Azimuth angle of the camera image (right) is projected onto the Cartesian $x$ axis while the temporal evolution is plotted on the $y$ axis. After activation of the control signal, the square pattern changes into a squeezed hexagon via a transient state in which the old and new pattern share a single mode.

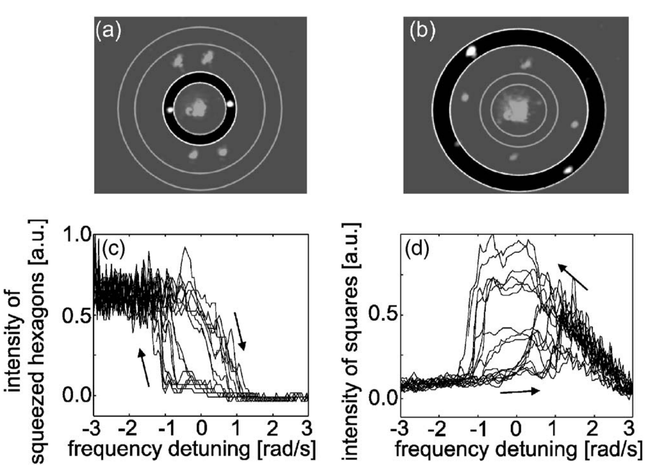


Fig. 5. System state curves tracking the response to an oscillating control signal. Intensity of far-field spots characteristic of two pattern classes is filtered by masks (a), (b) in the transverse Fourier space and detected. Intensities of two patterns, (c) squeezed hexagon, (d) square, during multiple transitions induced by a triangular control signal. Pattern selected by large control signals remains as the control diminished, showing hysteretic behavior and indicating a symmetric bistability. Paths seemingly crossing through the center of the hystereses stem from slightly sheared square patterns as well as occasional lack of one pair of the square pattern spots. As a result of some spots lying outside of the discrimination window or having lower than normal intensity, only half of the intensity is registered.

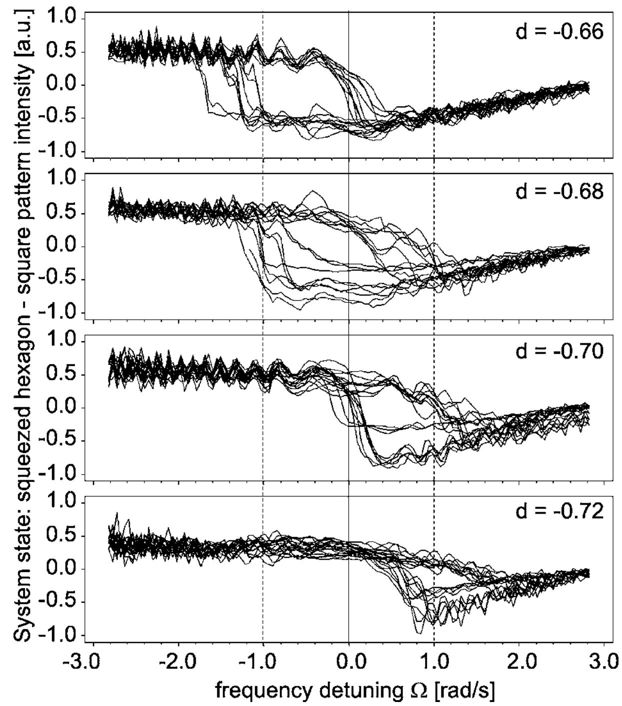


Fig. 6. Hystereses for several feedback mirror positions: (a) $d=-0.66$, (b) $d=-0.68$, (c) $d=-0.70$, (d) $d=-0.72$. In contrast to Fig. 5, the $y$ axis now denotes the difference of squeezed hexagon and square pattern intensities, yielding a direct measure for the actual system state. As the squeezed hexagonal pattern grows in stability, the hysteresis is moved to farther positive control signal values and finally no square pattern is observable without control signal. Paths seemingly crossing the hystereses' centers again stem from variations in the square pattern (compare Fig. 5).

system. As demonstrated by Schwab *et al.*,[16] pattern transitions can be externally induced by detuning the frequency of the pump beams. The mechanism through which pump beam detuning leads to pattern transitions as well as the question of whether this control method is limited to the photorefractive nonlinearity or may be generalized were left open.

Experimentally, we obtain pump detuning by slowly moving the feedback mirror in a longitudinal direction and taking advantage of the Doppler shift.[15] To achieve continuous detuning while at the same time holding the mirror position constant within the limit of $\Delta d=0.01$, the mirror is jumped back to its initial position with maximum speed and in phase such that the reflection grating is not disturbed by the jump after having traveled half a wavelength. This is facilitated by the slow response of the photorefractive medium owing to diffusion time constants on the order of several tens of milliseconds. Transitions from square to squeezed hexagonal pattern are induced by a negative frequency detuning while inverse transitions are the result of a positive frequency detuning.

Figure 4 gives an exemplary transition from a pattern with square symmetry to a squeezed hexagon under pump detuning. Initially, a square pattern is formed, which changes to a squeezed hexagon via a transient state consisting of both patterns. Observe that both patterns share a component mode that is sustained during the transition. This signifies that the transition is in fact an alternation between equally stable patterns and does not result from a disturbance erasing the reflection grating, which would affect all modes. The initial assumption of bistability is justified by the observation that the addressed pattern remains after the control signal is removed. Hence we expect that it should be possible to observe a hysteresis in the system response to a periodic control signal, giving detailed information about the actual control signal strengths required for inducing transitions.

Figure 5 illustrates the system response to a triangular control signal oscillating between ±3 rad/s. The system state is quantified in terms of the combined intensity of all pattern spots with wavenumbers specific to each of the two patterns. Their intensities are detected using Fourier space masks permitting only modes characteristic to a specific pattern. As the control signal is varied, the state is tracked, resulting in an approximate phase space footprint. We identify areas of stability of one specific pattern and a bistable region in between, as signified by the hystereses. For control signals $|\Omega| \ll 1$ rad/s, no transitions can be induced. Reliable switching between both patterns is observed for $|\Omega| > 1$ rad/s while a larger control signal guarantees the observation of the addressed pattern, indicating relative instability of the other pattern. Note that for large positive control signal strengths, pattern formation is generally suppressed as indicated by the diminishing square pattern signal. For negative control signals, the squeezed hexagon remains strongly visible. This behavior is consistent with earlier observations and the result of a strongly asymmetric influence of pump beam detuning on the growth rates of unstable modes.[16]

We noted above that the relative frequency of the patterns' occurrence depends on the position of the virtual feedback mirror. Under the assumption that a change in the frequency of observation is indicative of one pattern gaining relative stability over the other, we expect that if we increase the relative stability of one pattern by changing the feedback mirror position, a stronger control signal will be required to switch to the weaker pattern. At the same time, the transition to the dominant pattern should require a smaller control signal.

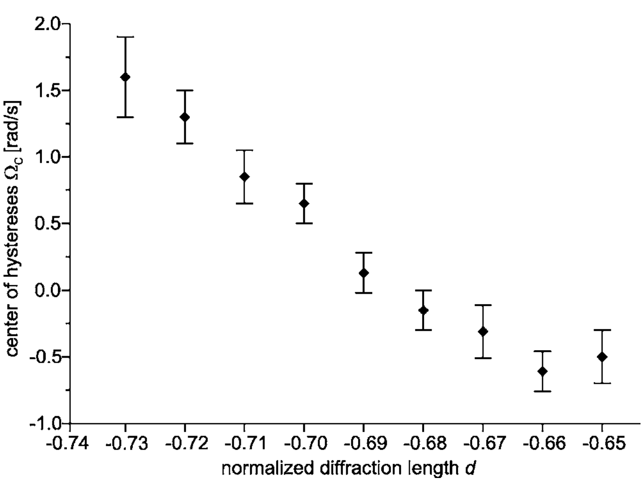


Fig. 7. Center of gravity of the hystereses as a function of the feedback mirror position (normalized diffraction length), which changes the relative stability of the two patterns involved.

This dependence of the control signals required for transitions is investigated by repeating the experiment for different mirror positions. Results are given in Figs. 6 and 7 and confirm the expectation. As the mirror position is moved towards the dominant squeezed hexagonal pattern region, the hysteresis shifts towards larger control signals for induction of square patterns. In fact, for the mirror position $d=-0.72$, a squeezed hexagon is always observed without control signal and the generation of a square pattern requires a large positive pump detuning.

So far we confirmed that pump beam detuning is a viable means of switching between patterns at or near a virtual mirror position where the system offers multiple patterns with different symmetries. We have confirmed the existence of a bistability by the observation of hysteretic behavior and controlled the relative stability of patterns by varying both the feedback mirror position and the pump beam detuning.

## 4. EXPERIMENTAL STABILITY ANALYSIS

To investigate the mechanism by which pump beam detuning affects the pattern formation process, we consider the influence on unstable modes in a system with one-dimensional feedback. Employing an experimental stability analysis,[19,20] we systematically determine the threshold of modes associated with one-dimensional patterns, essentially recovering a function $\gamma l_{\mathrm{thresh}} = \gamma l_{\mathrm{thresh}}(d, k_d l)$. We will call all such one-dimensional modes for which the system is above threshold at a given point in parameter space *fundamental*. If we now consider two-dimensional feedback, we expect the two-dimensional patterns to consist of fundamental modes in the following way: assuming an initial isotropy, an active fundamental mode should initially grow from noise with all azimuthal wave vectors approximately equally present. This is in fact observed and gives rise to so-called target patterns that only exist for a brief transient in the system considered here. Beyond that isotropic transient, nonlinear interaction of the modes leads to an azimuthal symmetry breaking again arising from fluctuations.[21] Experimentally, we find that the two-dimensional pattern consists of fundamental modes that are very close to the one-dimensional experiments or of geometric additions of those.[20] Why the available fundamental modes are combined to create patterns of the different symmetries we observe is still a largely open question. Knowledge of the influence of the control signal on the fundamental modes will provide insight into the selection or suppression of a specific two-dimensional pattern.

The experimental method is extensively described in a previous publication[20] and is only briefly summarized here: a slit mask is inserted into the Fourier plane of the feedback setup to reduce the system to a single transverse dimension. An additional mask allows only modes with three specific wavenumbers to propagate through the feedback: the central Gaussian mode $k_d l = 0$ and a narrow, selectable wavenumber band centered around $\pm k_d l_{\mathrm{sel}}$. As a result, only a stripe pattern with a wavenumber within the selected range is able to grow; all remaining modes are suppressed by the applied Fourier control. Subsequently, the coupling strength is slowly increased from the minimum value until the onset of the selected mode is observed, or until the maximum available coupling strength is reached. Iteration over different wavenumbers yields the complete threshold curve for the given parameters (i.e., mirror position and pump beam detuning value). For comparison of threshold minima and patterns observed beyond threshold, we assume an anticorrelation between threshold levels and growth rates of the associated modes. This assumption was previously justified by experimental observation.[20]

As before, we consider the feedback mirror position that allows switching between stable square and squeezed hexagonal patterns. Figure 8 illustrates results at a mirror position of $d=0.66\pm 0.01$ for three different pump beam detuning values ($\Omega=0$, $\Omega=+2.46$ rad/s, $\Omega=-2.46$ rad/s). Without a control signal (open circles), three distinct modes can be identified ($k_d l=3$, $k_d l\approx 7.5$, and $k_d l=12.5$) of which the central mode is dominant due to the low threshold. The wavenumber of this mode directly corresponds to the pattern component that square and squeezed hexagonal patterns share and that persists through the pattern transitions. The mode at $k_d l=3$ corresponds to the smaller wavenumber of the squeezed hexagonal pattern. The mode at $k_d l=12.5$ may be responsible for the stabilization of square over normal hexagonal patterns, as the normalized wavenumber is about twofold of the dominant mode [recall that $k_d l\propto (k_\perp)^2$] and thus should promote square (or near square) geometry over the hexagonal one. The mode necessary for the squeezed hexagon is too weak to stabilize this pattern, which explains the dominance of the square pattern at this mirror position (see Fig. 6).

A strong positive frequency detuning (triangles) completely inhibits the satellite modes while also significantly narrowing the dominant mode. Until the satellite modes are suppressed, the square pattern persists. However, this pattern vanishes as only the strong central mode remains, strengthening the conclusion that the auxiliary mode is a required support of the square pattern.

A strong negative control signal (filled circles) weakens the central mode even more and also weakens the mode at $k_d l=12.5$ somewhat without completely inhibiting it. The relevant observation is that the mode around $k_d l=3$ is not at all inhibited. Therefore, a negative control signal equalizes the thresholds, and hence the growth rates, of the first two modes. As approximately equal thresholds for these modes are responsible for generation of the squeezed hexagonal pattern,[20] the observed tendency towards this pattern is a consequence of the changed mode stabilities.

The specific influence of pump beam detuning on the one-dimensional modes' thresholds is direct evidence for the mechanism by which the symmetry of the patterns is influenced by this control method. Changes to the thresholds and hence growth rates of the fundamental modes change the interaction scenarios forming two-dimensional patterns and ultimately modify their stability leading to the possibility to induce controlled transitions between the patterns considered.

To summarize, the questions posed at the beginning of this paper are now resolved: pump beam detuning as a control method for this experimental system depends completely on the way the fundamental modes are influenced. Thus, this mechanism may not be generalized to other experimental scenarios.

However, the observation that a control technique may change the threshold of specific modes need not be a unique feature of the photorefractive nonlinearity. Therefore, control of two-dimensional transverse pattern formation by shaping individual modes is a viable control method that can be explored in all optical pattern forming systems, subject to the availability of a technique for shaping the modes.

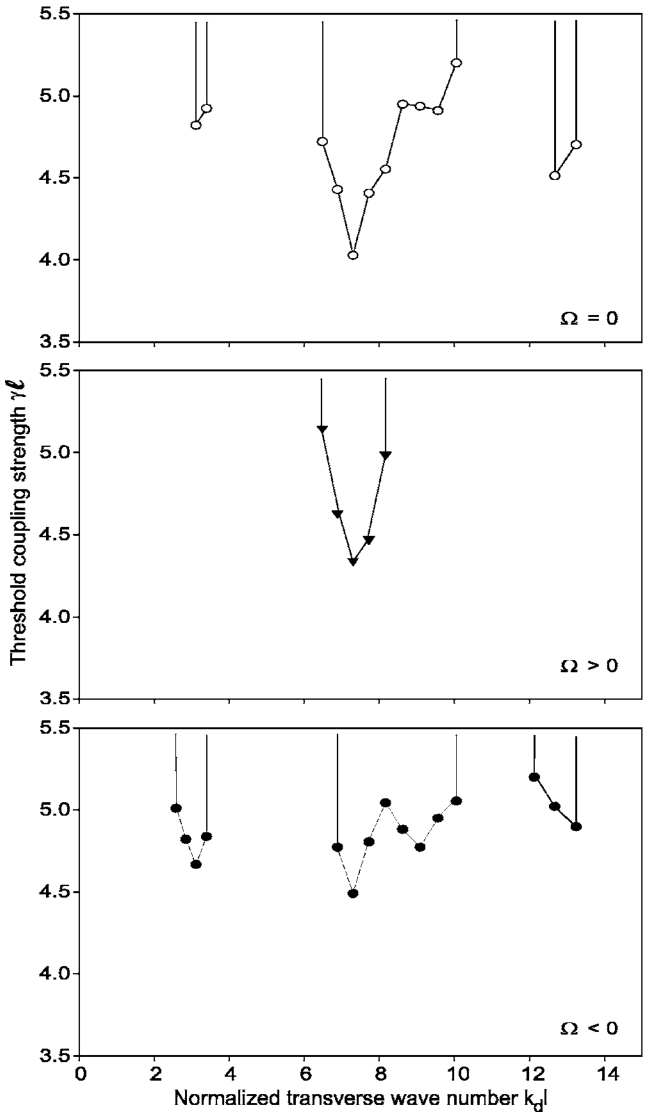


Fig. 8. Experimental threshold curves for zero, positive, and negative control signals at the squeezed hexagon–square pattern bistability. A positive control signal inhibits most modes except for the one at $k_d l=7.5$. A negative control signal weakens the broad mode around $k_d l=7.5$ but leaves the mode at $k_d l=3$ unchanged, possibly slightly strengthened. Lines are drawn as a guide to the eye. Quantitative coupling strength values are derived from two-beam coupling experiments (maximum value) and the beam polarization angle (effective electro-optic coefficient).

## 5. CONCLUSION

Detuning of the pump beams influences the geometry of patterns formed in a single feedback system with photorefractive nonlinearity. We demonstrated switching between two patterns through a transient state consisting of both patterns competing. Transitions were reliably induced by the control signal, exhibiting a bistability marked by hysteretic system response. We determined the effect of pump beam detuning on the threshold of modes under one-dimensional feedback and thereby identified inhibition of modes corresponding to individual wavenumber as the mechanism through which pump beam detuning affects the generation of two-dimensional transverse patterns. The initial existence and controlled suppression of these modes was seen to cause the change in pattern symmetry, thereby indicating two-dimensional interaction between these independent modes to be the source for the existence of nonhexgonal patterns in this system.

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