Unitary matrices for phase-coded holographic memories

Xinzheng Zhang, Gernot Berger, Mathias Dietz, and Cornelia Denz

Institut für Angewandte Physik, Westfälishe Wilhelms-Universität Münster, Corrensstrasse 2-4, 48149 Münster, Germany

Received December 23, 2005; accepted January 10, 2006; posted January 23, 2006 (Doc. ID 66904)

We propose a novel type of unitary matrix for phase-code multiplexed holographic memories, which could be quickly generated from geometric sequences. Our analysis shows that the phase-code matrices are unitary rather than orthogonal. The new matrices have complex elements. The order of unitary matrices can be any positive integer, so that we can accommodate the available spatial light modulators to obtain the maximum possible storage capacity. The cross-talk noises in phase-encoded memories with unitary matrices and with Hadamard matrices are of the same order of magnitude, which are much lower than those in holographic memories with wavelength multiplexing or angle multiplexing. © 2006 Optical Society of America

OCIS codes: 090.4220, 210.2860, 090.7330.

The Bragg selectivity of volume gratings allows multiple holograms to be recorded inside the same volume of the recording medium. Holograms can be superimposed through angle,[1] phase-code,[2,3] wavelength,[4] peristrophic,[5] shift,[6] and correlation multiplexing techniques.[7] Among them, deterministic phase-code multiplexing has been widely investigated because of its several advantages.[8–15] This method allows the recording and retrieving of data pages without introducing moving parts or frequency-shifting elements into the setup, which results in a fast access speed.[2,8] The cross-talk analyses have shown that the signal-to-noise ratio (SNR) of phase-code multiplexing is significantly higher than those of wavelength multiplexing and angle multiplexing.[11,12] In addition, phase-code multiplexing offers the potential of performing parallel optical operations of stored images[13] and the opportunity of powerful associative search.[14,15]

In phase-code multiplexing each reference beam consists of a set of plane waves, which are modulated by a phase spatial light modulator (SLM), as shown in Fig. 1. The addressing mechanism is the orthogonal phase-code set of the reference beam. The commonly used phase codes are Walsh–Hadamard codes, which have only two possible values, 1 and −1, corresponding to phase delays 0 and π, respectively.[16] The lowest order of a Hadamard matrix (H matrix), $N=2$, has the form $H^2=[1,1;1,-1]$. The H matrices whose orders are a power of 2 could be easily generated from the Kronecker product of H^2.[8] Yang *et al.*[17,18] reviewed various methods and described an algorithm for the construction of H matrices. However, the orders of H matrices are still restricted to $N=4m$ (with m a positive integer). Furthermore, the construction of some special types of H matrix is time consuming.

In this Letter we propose a novel type of matrix–unitary matrices (U matrices) for phase-code multiplexed holographic memories. The order of U matrices can be any positive integer, and a U matrix of any order could be quickly constructed from geometric sequences. The cross-talk noises in phase-coded memories with U matrices are of the same order of magnitude as those with H matrices.

Holograms are superimposed within a holographic medium; the mth hologram is recorded by interfering the signal beam S_m with a reference beam R_m. One set of adjustable phases P_m represents the address of the mth hologram. For the storage of N holograms, N phase codes have to be used; i.e., the reference beam consists of N plane-wave components. The complex amplitude of the mth reference beam with the lth component being phase modulated by P_{ml} can be written as[11]

$$R_m = \sum_{l=0}^{N-1} P_{ml} \exp(j\mathbf{k}_l \cdot \mathbf{r}). \quad (1)$$

We assume that the resultant change of the permittivity of the storage medium is linearly related to the intensity in the interference pattern. During readout, the recorded medium is illuminated by one probe beam R_n with the phase address P_n. In practice, two

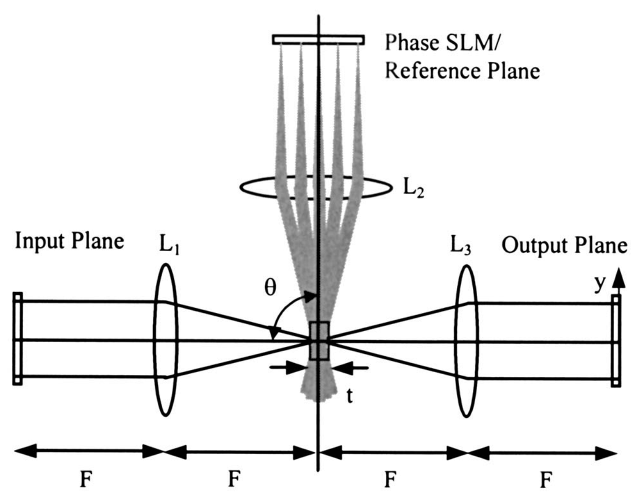

Fig. 1. Recording and readout geometry for phase-code multiplexing.

adjacent plane-wave components of the reference beam are separated by the angle Bragg selectivity, so that the lth plane-wave component cannot read out the subholograms recorded by the other plane-wave components.[8] Suppose only one hologram is recorded inside the medium by reference beam R_m and signal S_m; the diffracted field can be written as

$$E_n \approx \left(\sum_{l=0}^{N-1} P_{nl} P_{ml}^{*} \right) S_m, \tag{2}$$

where P_{ml}^{*} denotes the complex conjugate of P_{ml}. Therefore, if the phase-code modulations P_n are consistent with P_m, then the original signal beam is reconstructed intact. On the other hand, P_n can be selected such that $\Sigma_{l=0}^{N-1} P_{nl} P_{ml}^{*}=0$, then the reconstructions interfere destructively resulting in zero intensity. Hence, the phase addresses can be considered as the row or column vectors of one matrix fulfilling the unitary condition

$$P\tilde{P}^{*} = NI, \tag{3}$$

where $\tilde{P}$ denotes the transpose of matrix P and I is the identity matrix. This relation means that P is a unitary matrix instead of orthogonal matrix. To the best of our knowledge, only H matrices are involved in deterministic phase-encoded holography so far. H matrices with orders of a power of 2 are symmetrical; therefore the relation has the simplified form $PP^{*} = NI$.[16] Furthermore, the elements in H matrices have only two possible real values, so that they are both orthogonal and unitary. As a result, the relation could be written as $P\tilde{P}=NI$.[18]

In this Letter we present one novel solution to Eq. (3). The entry of the U matrices can be easily calculated from geometric sequences

$$U_{ml}^{N} = \exp\left(j \frac{2\pi ml}{N} \right), \tag{4}$$

which corresponds to the phase delay $\varphi_{ml}=2\pi ml/N$ of the lth plane wave of the mth address for storage of N holograms. By substituting Eq. (4) into Eq. (3), we can calculate the unitarity of U matrices by means of the geometric series

$$\sum_{l=0}^{n-1} U_{nl} U_{ml}^{*} = \frac{1-\exp[j2\pi(n-m)l]}{1-\exp\left[j\frac{2\pi(n-m)l}{N} \right]} = \begin{cases} N & n=m \\ 0 & n \neq m \end{cases}. \tag{5}$$

The order of U matrices can be any positive integer. For example, the U matrix U^2 of order 2 is identical to the H matrix H^2 of order 2, and the U matrix of order 4 has the form

$$U^4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix}. \tag{6}$$

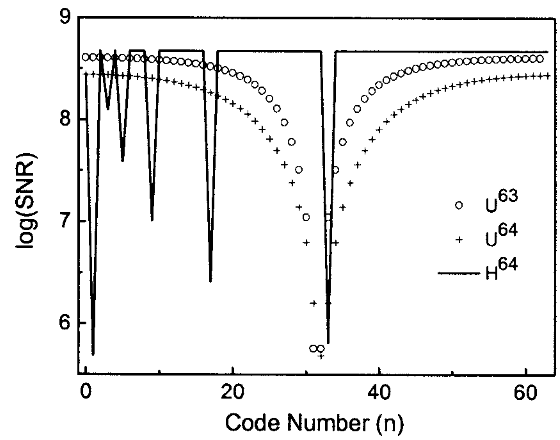

Fig. 2. Log(SNR) versus hologram code number.

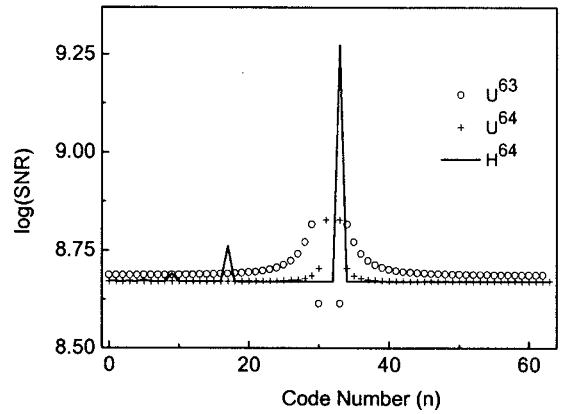

Fig. 3. Log(SNR) versus hologram code number without bad codes. U^{63} without codes 31, 32; U^{64} without code 32; H^{64} without code 1.

The signal beam is usually a Fourier transform of the object instead of a plane wave in a practical setup, which will cause cross-talk noise between different holograms. Curtis and Psaltis[11] deduced the expression for the noise-to-signal ratio (NSR) in phase-coded memories with H matrices in 1993.[11] Under the same assumptions, the average NSR with U matrices can be expressed as

$$\mathrm{NSR} = \frac{1}{N^2} \sum_{m=-q}^{q} \left| \sum_{k=-q}^{q} \sum_{l\neq k, l=-q}^{q} U_{n,k+q} U_{m+q,l+q}^{*} \times \mathrm{sinc}\left[k-l+\frac{y\lambda}{2Ft}(l^2-k^2)+\frac{\lambda^3}{8t^3}(l^2-k^2)^2 \right] \right|^2, \tag{7}$$

where $q=(N-1)/2$ and $n=0,1,\ldots,N-1$. As illustrated in Fig. 1, y is the coordinate at the output plane, F is the focal length of all three lenses L_1, L_2, and L_3; t is the thickness of the recording medium; and λ is the wavelength of the recording beams. The angle θ between the normal to the reference plane and the optical axis of the input plane is equal to 90°.

To compare the results from U matrices and H matrices, we adopt the parameters from Ref. 11 with $F = 30$ cm, $t=1$ cm, $y_{\max}=1.5$ cm, and $\lambda=500$ nm. Figure

2 shows the worst-case SNR versus phase-code number n for U^{63}, U^{64}, and H^{64} by setting y to its maximum value. The log(SNR) values are on the same scale; baseline log(SNR) values are about 8.5, with the worst 5.7. The H matrices of an order of a power of 2 only have one bad code word 1.[11] For U matrices, the center code words have a high frequency, and thus a large NSR. U matrices of odd order have two bad code words, $(N-1)/2$ and $(N+1)/2$, while U matrices of even order have one bad code word, $N/2$. The SNR can be improved by taking out these bad codes, as shown in Fig. 3. The SNR from U matrices is of the same order of magnitude as those from H matrices, which is much higher than those of wavelength multiplexing and of angle multiplexing. In addition, the SNR decreases with increasing N, i.e. with increased storage capacity. Notice that the SNR is very high, for it is estimated based only on cross-talk. It might become lower because of other system imperfections in practice.

In conclusion, we propose a novel type of unitary matrix for phase-code multiplexed holographic memories, which could be simply generated from geometric sequences. The order of unitary matrices can be any positive integer, which is limited only by the resolution of the phase spatial light modulator. The SNRs in phase-coded memories with unitary matrices and with Hadamard matrices are of the same order of magnitude.

We acknowledge support from the Deutsche Forschungsgemeinschaft, the National Natural Science Foundation of China (grant 60208003), and the Alexander von Humboldt Foundation. Xinzheng Zhang (zxz@nankai.edu.cn) is a Humboldt research fellow from TEDA Applied Physics School, Nankai University, China.

References

1. L. d'Auria, J.-P. Huignard, and E. Spitz, IEEE Trans. Magn. **9**, 83 (1973).
2. J. T. LaMacchia and D. L. White, Appl. Opt. **7**, 91 (1968).
3. C. Denz, G. Pauliat, and G. Roosen, Opt. Commun. **85**, 171 (1991).
4. G. A. Rakuljic, V. Leyva, and A. Yariv, Opt. Lett. **17**, 1471 (1992).
5. K. Curtis, A. Pu, and D. Psaltis, Opt. Lett. **19**, 993 (1994).
6. D. Psaltis, M. Levene, A. Pu, G. Barbastathis, and K. Curtis, Opt. Lett. **20**, 782 (1995).
7. K. Curtis and W. L. Wilson, U.S. Patent 5,719,691 (February 12, 1998).
8. C. Denz, G. Pauliat, G. Roosen, and T. Tschudi, Appl. Opt. **31**, 5700 (1992).
9. C. Alves, G. Pauliat, and G. Roosen, Opt. Lett. **19**, 1894 (1994).
10. C. Denz, K.-O. Mueller, T. Heimann, and T. Tschudi, IEEE J. Sel. Top. Quantum Electron. **4**, 832 (1998).
11. K. Curtis and D. Psaltis, J. Opt. Soc. Am. A **10**, 2547 (1993).
12. Z. Wen and Y. Tao, Opt. Commun. **148**, 11 (1998).
13. C. Denz, T. Dellwig, J. Lembcke, and T. Tschudi, Opt. Lett. **21**, 278 (1996).
14. G. Berger, C. Denz, S. S. Orlov, B. Phillips, and L. Hesselink, Appl. Phys. B **73**, 839 (2001).
15. G. Berger, M. Stumpe, M. Hoehne, and C. Denz, J. Opt. A Pure Appl. Opt. **7**, 567 (2005).
16. C. Denz, K.-O. Mueller, F. Visinka, and T. Tschudi, in *Holographic Data Storage*, H. J. Coufal, D. Psaltis, and G. T. Sincerbox, eds. (Springer-Verlag, 2000).
17. X. Yang, Y. Xu, and Z. Wen, Opt. Lett. **21**, 1067 (1996).
18. X. Yang, Z. Wen, and Yu Xu, in Proc. SPIE **2849**, 217 (1996).