Multicomponent dipole-mode spatial solitons

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We study (2 + 1)-dimensional multicomponent spatial vector solitons with a nontrivial topological structure of their constituents and demonstrate that these solitary waves exhibit a symmetry-breaking instability, provided their total topological charge is nonzero. We describe a novel type of stable multicomponent dipole-mode solitons with intriguing swinging dynamics. © 2002 Optical Society of America

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Recent progress in the study of spatial optical solitons and their interaction, as well as the extensive experimental demonstrations of stable self-focusing of light in different types of nonlinear bulk media, has opened the road for new concepts in controlling the diffraction of optical beams and designing new devices for optical switching and storage.[1] Many novel fundamental concepts in the physics of spatial optical solitons that were recently suggested are associated with vectorial interaction and multicomponent soliton beams that mutually self-trap in a nonlinear medium. Such composite multimode solitons can have complex structures and, in many cases, their total intensity profile exhibits multiple humps.[2]

In a bulk medium, vector solitons exist in different forms, and, as was recently shown for two-component self-trapped beams, many types of multipole vector solitons can be predicted and analyzed for an isotropic nonlinear bulk medium with saturable nonlinearity.[3] Recently, an important generalization of this concept to the case of N-component two-dimensional vector solitons was suggested for an example of a thresholding nonlinearity.[4–7] In particular, Musslimani *et al.*[8] predicted the existence of multihump N-component composite spatial solitons that carry different topological charges (spins) and, therefore, can provide exciting possibilities for spin-dependent interactions of self-trapped optical beams.[9,10]

The purpose of this Letter is twofold. First, we study in more detail the dynamics of multicomponent spatial solitons carrying topological charges in different components and demonstrate that, in contrast to the conjecture of their stability made in Ref. 8, these vector solitons demonstrate a symmetry-breaking instability in all cases in which their total angular momentum is nonzero. Second, based on earlier studies of two-component vector solitons[3] and the conceptual approach developed in Ref. 8, we propose a novel type of stable multicomponent vector solitons consisting of two perpendicular dipole components trapped by the soliton-induced waveguide. These vector solitons are studied here for the case of $N = 3$ components, which are shown to be the building blocks for the solitons composed of N incoherently coupled dipole-mode beams.[11] Additionally, we demonstrate numerically that these novel vector solitons are very robust for a broad range of their parameter space, and they demonstrate intriguing swinging dynamics outside the stability domain, resembling long-lived excitations and vibrations of molecules.

We consider the interaction of N mutually incoherent (2 + 1)-dimensional optical beams propagating in a bulk saturable medium, described by the normalized equations ($j = 1, 2, \ldots, N$),

$$i \frac{\partial E_j}{\partial z} + \Delta_\perp E_j - \frac{E_j}{1 + \Sigma |E_j|^2} = 0\,, \tag{1}$$

where $\Delta_\perp$ is the transverse Laplacian and z is the propagation coordinate. Equations (1) describe, in a rather simplified isotropic approximation, screening spatial solitons in photorefractive materials.[12]

To describe multicomponent vector solitons in the framework of model (1), first we look for stationary solutions in the form $E_j(x, y, z) = u_j(x, y)\exp(-i\beta_j z)$, where β_j is the propagation constant and $u_j(x, y)$ is the envelope of the jth component. Then, introducing the dimensionless parameter $\lambda_j = (1 - \beta_j)/(1 - \beta_1)$ and normalizing the field amplitudes, $u_j \to \sqrt{1 - \beta_1}\, u_j$, and the coordinates, $(x, y) \to (x, y)/\sqrt{1 - \beta_1}$, we obtain

$$\Delta_\perp u_j - \lambda_j u_j + F(I) u_j = 0\,, \tag{2}$$

where $I = \Sigma |u_j|^2$ is the normalized total intensity and $F(I) = I(1 + sI)^{-1}$, with the effective saturation parameter $s = 1 - \beta_1$.

First, following Musslimani *et al.*,[8] we seek multicomponent radially symmetric solutions of Eq. (2) for which the main component, $u_1(x, y) = U_1(r)$, has no nodes but each of the components u_k ($k > 1$) carries a different topological charge, $u_k(x, y) = U_k(r)\exp(im_k\theta)$. We denote such states as $(0, \ldots, m_k, \ldots)$, and an example for $N = 3$ is presented in Fig. 1(a), in which the same intensity distribution corresponds to two different states, (0, +1, +1) and (0, +1, −1).

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To study the stability of these composite solitons, we propagate them numerically and find that, provided that the total angular momentum is nonzero, all these multicomponent solitons undergo symmetry-breaking instability and fragment into a number of fundamental solitons, as shown in Fig. 1(b) for the case of (0, +1, +1). This instability is similar to the instability of the vortex-mode solitons described earlier for the two-component model. The resulting incoherent superposition of two parallel dipole components, u_2 and u_3, can be regarded as a generalization of a two-component dipole-mode soliton[13] $\{u_1, V\}$ to a three-component solution $\{u_1, u_2, u_3\}$ at $\lambda_2 = \lambda_3$ with the help of a transformation of the dipole components, $V \rightarrow \{u_2, u_3\}$, where $u_2 = V\cos\psi$ and $u_3 = V\sin\psi$ (ψ is a transformation parameter). Such a straightforward generalization is indeed possible for N-component system (2).

The most important property of the (0, +1, −1) solution is that its total angular momentum is zero, and this makes the solution stable. In our calculations, this vector soliton was observed to be unchanged for distances of the order of 10^3 diffraction lengths. However, as it is launched with additional noise, this soliton displays slowly growing modulations, as shown in Fig. 1(c). The total intensity of the modulated rings in Fig. 1(c) preserves the initial ring profile, resembling an incoherent superposition of two perpendicular dipole components.[11] Although the vector soliton, consisting of two crossed dipoles, has been shown to be unstable without the third main component,[11] we found that the three-component dipole-mode soliton is stable in our numerical simulations. Stabilization of the vector ring in the presence of the third component can be explained by the physics of the soliton-induced waveguides. Indeed, two crossed dipoles, u_2 and u_3, represent a vectorial guided mode of the induced waveguide. A nontrivial rotational transformation of such a solution (see Ref. 11 for details) allows us to find a whole family of possible superpositions of these modes, including, as a particular case, the vortex components shown in Fig. 1(a) and the N-component dipole-mode soliton.

To find the multicomponent solitary waves with a nontrivial geometry, we integrate system (2) numerically by means of a relaxation technique and find a novel class of the dipole-mode soliton that consists of perpendicularly oriented dipoles with different powers: The simplest possible solution of this type has $N = 3$ components, and it is described by two independent parameters, (λ_2, λ_3), as shown in Fig. 2(a). The family of these solitons ranges from solutions in which the fundamental mode dominates the entire structure to solutions in which one of the dipoles dominates, as can be seen in Fig. 2(b), in which, for fixed $\lambda_2 = 0.5$, the power of the components $P_j = \int |u_j|^2 d\mathbf{r}$ is shown as a function of λ_3.

Numerical propagation of these solitons has shown that from the lower cutoff value for λ_3, where the intensity of the u_3 component vanishes, up to a value of $\sim\lambda_3 = 0.7$ these vector solitons are stable, whereas for higher λ_3 they decompose to form new, different structures. As can be seen from Fig. 3, an unstable soliton breaks the symmetry along both symmetry axes of the initial distribution. The products of this instability (see the bottom row in Fig. 3) are a fundamental vector soliton and a rotating dipole-mode soliton, recently introduced in Ref. 14 as a propeller soliton. Those two simpler solitons fly away from each other after the breakup.

Near the instability threshold, for $0.7 < \lambda_3 < 0.8$, we observe intriguing dynamics, associated with weak oscillatory instabilities. Figure 4 shows a characteristic example of this dynamics, when the instability breaks the symmetry only along one of the symmetry axes (parallel to the orientation of the stronger dipole). The product of this instability is a structure consisting of a tripole, a dipole, and a nodeless beam. This structure is remarkably long lived and it has, as the snapshots show, swinging behavior resembling a swinging mode of a three-atom molecule. We could observe almost three periods of such oscillations, until a strong energy exchange between the two dipole beamlets sets in and destroys this structure. We expect that the vibrational degrees of freedom, which are likely associated with long-lived soliton internal modes, should manifest

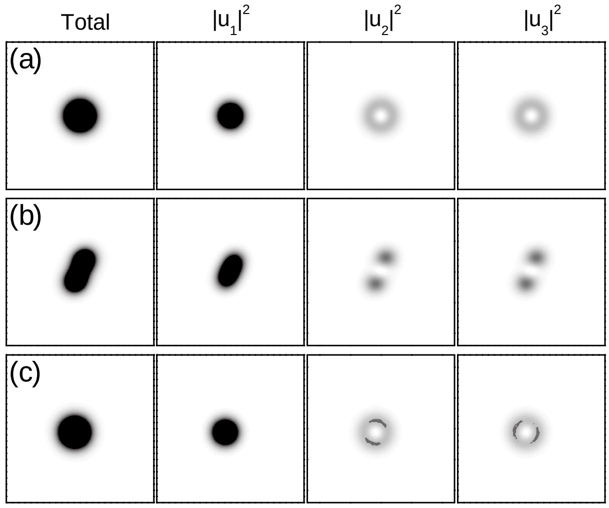

Fig. 1. Evolution of the three-component soliton: (a) stationary solution at $z = 0$, (b) symmetry-breaking instability of the (0, +1, +1) solution at $z = 80$, (c) long-lived quasi-stable propagation of the (0, +1, −1) state at $z = 500$.

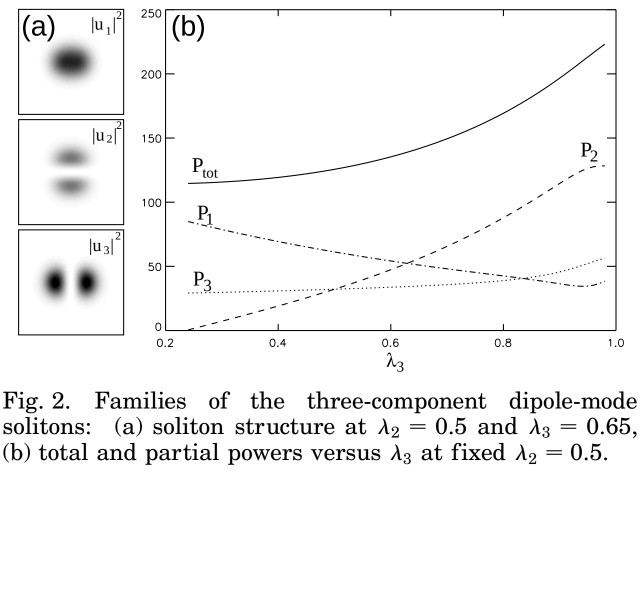

Fig. 2. Families of the three-component dipole-mode solitons: (a) soliton structure at $\lambda_2 = 0.5$ and $\lambda_3 = 0.65$, (b) total and partial powers versus λ_3 at fixed $\lambda_2 = 0.5$.

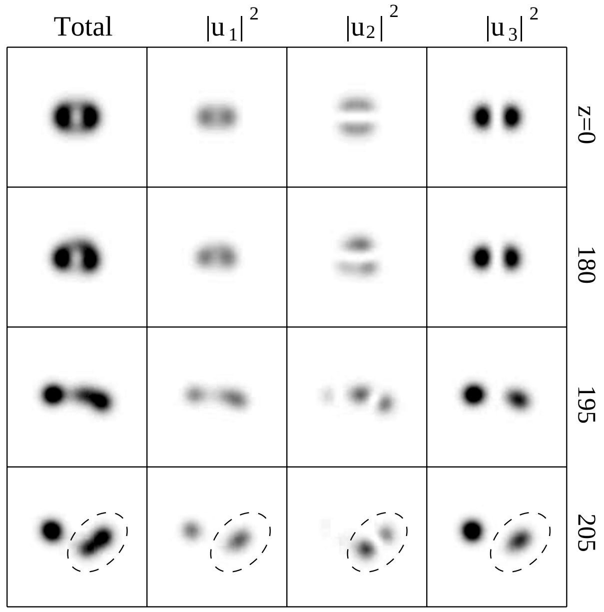

Fig. 3. Unstable propagation of a three-component dipole-mode soliton at $\lambda_2 = 0.5$ and $\lambda_3 = 0.8$ and its decay into a fundamental vector soliton and a vector propeller soliton (shown by the dashed circles).

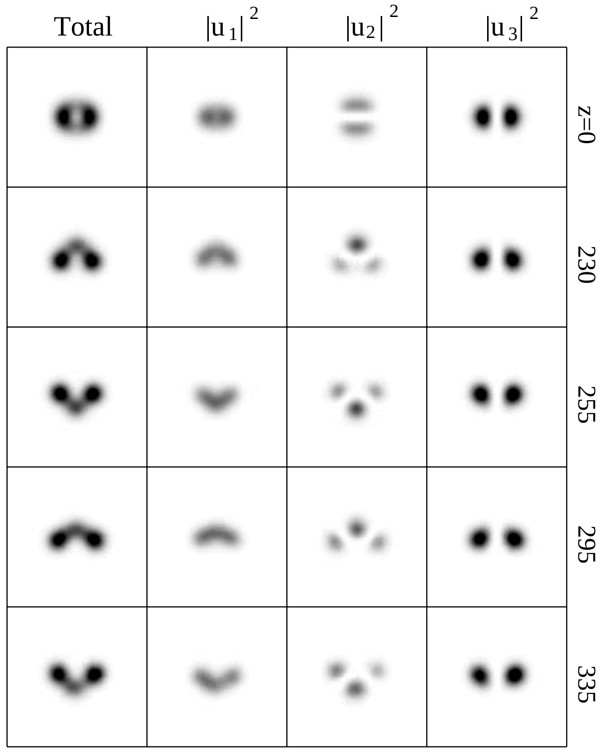

Fig. 4. Swinging dynamics of the vector dipole-mode soliton at $\lambda_2 = 0.5$ and $\lambda_3 = 0.75$.

themselves in the rich dynamics of soliton collisions, as is known from the study of a two-component model.[5]

Having found these novel composite solitons for the isotropic model, we wonder if similar multicomponent solitons can exist in an anisotropic nonlocal model that is more consistent with the experimentally studied photorefractive nonlinearities.[15,16] To verify that this is so, we have used the N-component generalization of the Zozulya–Anderson model, which takes into account the most important properties of photorefractive nonlinearities[17,18] and found similar classes of multicomponent localized solutions with perpendicularly oriented dipole components. Since the anisotropy allows stable stationary dipole modes that are oriented in two fixed directions only,[15,16] these solutions are found to be stable even in anisotropic media with a nonlocal nonlinear response. This leads us to expect the subsequent experimental observation of the novel type of vector solitons and swinging dynamics described above.

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