Stabilization and breakup of coupled dipole-mode beams in an anisotropic nonlinear medium

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We study the mutual trapping of two dipole-mode light beams in a medium with a nonlocal anisotropic saturable nonlinearity. It is shown that the incoherent attraction of perpendicularly aligned dipole-mode beams leads to the stabilization of their composite structure. Our numerical analysis gives a stationary solution that is stable only with respect to small amplitude modulations. The coupled solitary structure disintegrates in a well-defined way when the numerical perturbations or the experimental propagation length exceeds a certain limit. In this case a new and more robust, multicomponent solitary transverse light structure consisting of two dipole-mode vector solitons will be generated. © 2002 Optical Society of America

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1. INTRODUCTION

Spatial optical solitons have attracted much research interest during the past decade and were extensively studied in planar $(1 + 1)$D geometry as well in bulk medium in two transverse dimensions.[1] Stable self-trapped optical beams in both transverse dimensions will form only in materials that possess a saturable nonlinearity that prevents the beam from undergoing a catastrophic collapse, as it would in a material with a Kerr nonlinearity.[2] Recently, investigations of self-focused transverse light structures concentrated on the prediction and generation of various combinations of composite optical beams with complex internal structures in saturable nonlinear media.[3–9] The basic concept of these combined structures, also denoted vector solitons, is that they are a promising tool for stabilizing light beams, such as multihumped beams and optical vortex beams, that do not propagate in a self-consistent and stable way by themselves. The copropagation of such a beam that exhibits a complex internal structure with a mutually incoherent bell-shaped fundamental beam induces a refractive-index change in the nonlinear material that supports the stable and self-confined propagation of both contributing beams. The two beams induce an effective multimode waveguide in which they propagate as eigenmodes. Among various possible configurations is that of the dipole-mode vector soliton, which consists of a double-humped beam in one component and a bell-shaped Gaussian beam in the other and displays a surprising robustness.[7] For example, it has been shown that a vector soliton consisting of a fundamental beam and an optical vortex undergoes a transition into a dipole-mode vector soliton with nonvanishing angular momentum.[8,10,11] Further, the existence of various multipole vector solitons with higher order-components that exhibit quadrupole and even dodecagonal structures has been demonstrated.[12]

Recently the stabilization of multihump optical beams by means of incoherent interaction was shown for so-called necklace-ring vector solitons, which consist of azimuthally modulated components.[13,14] The simplest solution of this class of necklace ring-vector solitons is an incoherent superposition of two dipole components that complement each other in an isotropic medium to form a perfect ring structure for total intensity distribution. Here we study such a structure in an anisotropic model for a medium with photorefractive nonlinearity. We investigate the generation and the propagation behavior of a multicomponent solitary wave consisting of two dipole-mode beams without a fundamental stabilizing component. First, we analyze the vectorial attraction of diverging dipole modes that do not form a localized and self-trapped light structure when they propagate separately in an anisotropic saturable medium. We then apply the iterative technique of Petviashvili[15,16] and demonstrate that the incoherent attraction of the two modes compensates for the repulsion of their out-of-phase lobes, which supports the generation of a self-trapped ring-shaped solitary structure. To analyze the stability of the structure we start from this numerically exact solution and, using the beam propagation method described in Ref. 17, finally reveal the unstable propagation behavior that leads to a splitting of the composite structure. Because of numerical noise, the initially ring-shaped optical beam disintegrates into three composite quasisolitons that repel one another and separate in the radial direction. The arbitrary breakup behavior changes when we apply an initial perturbation to one of the components. Because of the anisotropic and nonlocal characteristics of the nonlinear material, the composite light structure spreads only in the horizontal direction, while one of the constituent dipole beams undergoes a transition into two dipole-mode structures that serve as the higher-order mode of two dipole-mode vector solitons. Our theoretical investigations are supported by experimental data derived from a setup with a photorefractive strontium barium niobate crystal as a nonlinear material. The controlled and determined disintegration of two-dimensional solitary light structures as a result of light-induced splitting is a new and unique property of composite optical spatial solitons.

2. THEORETICAL APPROACH

Choosing the direction of propagation along the ξ axis and neglecting absorption, we can describe the propagation of two incoherently coupled beams in a photorefractive nonlinear material in the paraxial approximation[18]:

$$i\frac{\partial U}{\partial \xi} + \frac{1}{2}\nabla_\perp^2 U = -\gamma \frac{\partial \phi}{\partial x} U,$$

$$i\frac{\partial V}{\partial \xi} + \frac{1}{2}\nabla_\perp^2 V = -\gamma \frac{\partial \phi}{\partial x} V, \tag{1}$$

where U and V are the slowly varying amplitudes of two optical beams, $\nabla_\perp^2$ is the transverse Laplacian ($\nabla_\perp^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$), $\gamma = 1/2k^2 w^2 n_e^4 r_{\text{eff}}$ is the coupling constant, k is the wave number of light, n_e is the ordinary unperturbed refractive index of the medium, w is the initial beam waist, and r_{eff} represents the effective electro-optic coefficient. The transverse coordinates x and y are scaled by beam waist w, and propagation coordinate ξ is scaled by diffraction length $L_D = knw^2$. With typical beam waist parameters of $w = 10–12\,\mu\text{m}$, a wavelength of $\lambda = 532\,\text{nm}$, and an ordinary refractive index of strontium barium niobate of $n = 2.3$, a single diffraction length is equivalent to 3–4 mm in a real optical system. ϕ describes the electrostatic potential induced by photoexcitation of charge carriers and satisfies the differential equation[19]

$$\nabla^2\phi + \nabla\phi\nabla \ln(1 + I) = E_0 \frac{\partial}{\partial x}\ln(1 + I). \tag{2}$$

Here, normalized intensity $I = |U|^2 + |V|^2$ is given in units of saturation intensity I_d, and E_0 describes an electric field that is applied in the transverse x direction. Diffusion effects of the charge carriers are dominated by drift in the external electric field and are therefore neglected here. We choose the parameters that correspond to our experimental conditions to be $E_0 = 3.6\,\text{kV/cm}$ and $r_{\text{eff}} = 180\,\text{pm/V}$. The system of Eqs. (1) and (2) is not integrable, and an analytical solution of the problem does not exist. Therefore we use a numerically exact procedure based on the work of Petviashvili.[15,16] As we are interested in solitary solutions that propagate along ξ without changing their transversal shape, we make the ansatz

$$U(x, y, \xi) = u(x, y)\exp(i\beta_1\xi),$$

$$V(x, y, \xi) = v(x, y)\exp(i\beta_2\xi), \tag{3}$$

where $\beta_{1/2}$ are independent propagation constants. Inserting Eqs. (3) into Eqs. (1), we obtain a new set of equations:

$$(\beta_1 - 1/2\nabla_\perp^2)u(x, y) = \gamma\partial_x\phi(x, y)u(x, y),$$

$$(\beta_2 - 1/2\nabla_\perp^2)v(x, y) = \gamma\partial_x\phi(x, y)v(x, y), \tag{4}$$

which represents an eigenvalue problem with eigenvalues β_1 and β_2 and eigenfunctions u, v, and $\partial_x\phi(x, y) = \partial\phi(x, y)/\partial x$. In what follows, we Fourier transform Eqs. (4) to express the problem by a pair of fixed-point equations:

$$u(\mathbf{k}) = \frac{\hat{F}[\gamma\partial_x\phi u(x, y)](\mathbf{k})}{\beta_1 + \mathbf{k}^2/2},$$

$$v(\mathbf{k}) = \frac{\hat{F}[\gamma\partial_x\phi v(x, y)](\mathbf{k})}{\beta_2 + \mathbf{k}^2/2}, \tag{5}$$

where $\hat{F}(\mathbf{k})$ is the two-dimensional Fourier transform in the transverse plane with transverse wave vector $\mathbf{k}$. The simple iterations of Eqs. (5) do not converge in general. Here the procedure proposed by Petviashvili defines a functional,

$$M_j = \frac{\int d\mathbf{k}\hat{F}[\gamma\partial_x\phi s_j(x, y)](\mathbf{k})s_j^*(\mathbf{k})}{\int d\mathbf{k}(\beta_j + \mathbf{k}^2/2)|s_j(\mathbf{k})|^2}, \tag{6}$$

with $s_1(x, y) = u(x, y)$ and $s_2(x, y) = v(x, y)$, that enables us to seek solutions to Eqs. (5). Suppose that $s_j(x, y)$ is a solution to Eqs. (4); then $M_j[s_j(x, y)] = 1$, and multiplying the right-hand sides of Eqs. (5) by $|M_j|^{-3/2}$ leaves the fixed points unaltered and finally leads to a convergence for the iteration of Eqs. (5). Our numerical procedure is as follows: Choosing the initial functions u_0 and v_0 similar to the expected solutions for u and v, we iterate

$$u_{n+1}(\mathbf{k}) = |M_1|^{-3/2}u_n(\mathbf{k}),$$

$$v_{n+1}(\mathbf{k}) = |M_1|^{-3/2}v_n(\mathbf{k}), \tag{7}$$

after each step calculating the new potential ϕ from Eq. (2), with $u_n(\mathbf{k})$ and $v_n(\mathbf{k})$ given by Eqs. (5). The iteration stops when the relative error becomes smaller than 10^{-5} in both components.

3. NUMERICAL RESULTS

The initial dipole-mode beams u and v are chosen as a pair of Gaussian beams with a relative phase shift of π [Fig. 1(a)]. The u component is aligned along the y axis, and the v component is oriented along the x axis and is therefore parallel to the external electric field. The two single lobes of the two components have the same initial beam profile and peak intensity, $I = 1$, and are slightly squeezed in the horizontal direction to match the elliptical shape of a photorefractive soliton.[20] When we apply the iterative process described above to the intensity distribution illustrated in Fig. 1(a), the routine converges and yields a transversal intensity distribution, which is depicted in Fig. 1(b) with the u component in the upper row and the v component below. The two components maintain their symmetry axis, which is horizontal for the

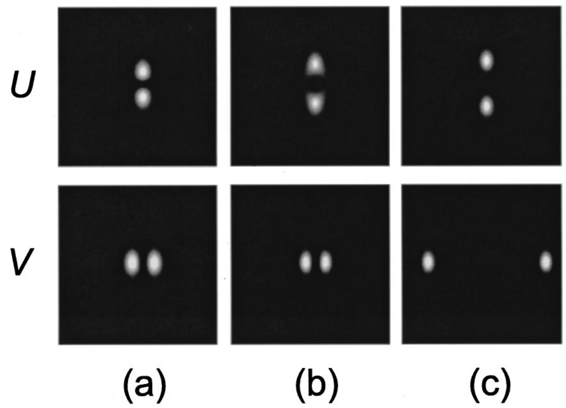

Fig. 1. (a) Initial intensity distribution and (b) the numerical exact solution of two perpendicularly aligned dipole mode beams given by the Petviashvili routine. (c) The same components after they have propagated separately in the nonlinear medium ($\xi = 4.5$).

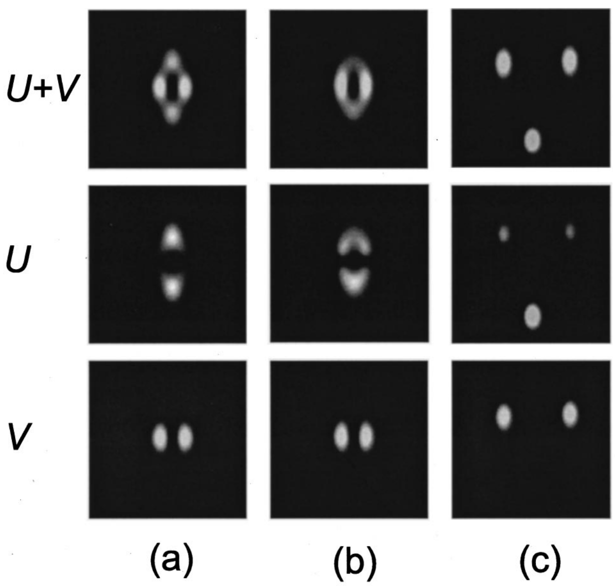

Fig. 2. Numerical simulation at several stages of propagation for simultaneous propagation of both components u and v. Top, $|u|^2 + |v|^2$; middle, $|u|^2$; bottom, $|v|^2$. (a) $\xi = 0$, (b) $\xi = 9.5$, (c) $\xi = 16$.

u and vertical for the v component; in particular, the u component displays a slightly different intensity profile, and the distance between its two lobes increases. This behavior is not surprising because our theoretical model is inherently anisotropic and does not support any solutions $|u|^2 + |v|^2$ with cylindrical symmetry. The anisotropic nature of the model becomes especially apparent when each component propagates separately in the medium. To investigate this behavior in detail we make use of the beam propagation method.[17] Figure 1(c) displays the intensity profiles after a propagation of $\xi = 4.5$ diffraction lengths. The single lobes remain self-trapped, whereas their separation is much more obvious for the v component. The light-induced refractive-index change in between the two lobes and at the horizontal margins of the beamlets is negative.[19] Because light tends to propagate toward the region of elevated index of refraction, the appearance of a defocusing effect leads to an effective repulsion of the two lobes in the horizontal direction. Perpendicularly to the applied electric field, i.e., along the vertical direction, the refractive-index change is always positive, and, because of the nonlocal nature of our model, it can even be strong enough to overcome the repulsion of the two out-of-phase lobes, leading to the formation of a bound state: a dipole.[21] This vertically aligned bound dipole solitary structure propagates in a nondiverging way for several propagation lengths and could therefore serve as a stabilizing component to prevent the other, horizontally aligned, dipole lobes from separating. Figure 2 illustrates the propagation behavior when both mutually incoherent dipole-mode beams propagate simultaneously in the nonlinear medium for various distances at [Fig. 2(a)] $\xi = 0$, [Fig. 2(b)] $\xi = 9.5$, and [Fig. 2(c)] $\xi = 16$. Here, the total intensity of the two components $|u|^2 + |v|^2$ is shown in the top row and the intensity distributions of the single constituents $|u|^2$ and $|v|^2$ are illustrated the middle and bottom rows, respectively. We start from the same initial conditions as depicted in Fig. 1(b) that were calculated by the Petviashvili procedure. Figure 2(b) represent the two beam profiles after they have propagated nine diffraction lengths. Now the repulsion of the horizontally aligned dipole [Fig. 2(b), bottom] is prevented by the other, vertically orientated component [Fig. 2(b), middle], which is affected as well, and its lobes display a kidneylike shape. The total intensity distribution, depicted in the top row of Fig. 2(b), becomes smoother and resembles a horizontally squeezed ring. On further propagation to $\xi = 16$, the two-component-structure displays a symmetry-breaking instability and splits into three beamlets that separate from one another in the radial direction; i.e., they fly apart. This behavior demonstrates that the numerically calculated solution given by the Petviashvili routine indeed forms a bound composite solitary state in which the two constituents attract and stabilize each other, at least for a certain propagation distance. The discretization to a numerical grid of 128×128 transversal data points for the beam propagation method includes a small amount of numerical noise, which is responsible for the final breakup of the light structure at larger propagation distances. Consequently, the composite light structure does not remain stable and splits into three composite quasi solitons that repel one another.

4. EXPERIMENT

The experimental setup for generating two coupled dipole-mode optical beams is similar to the one that was described earlier in Ref. 8. Here we derive four beams from a frequency-doubled Nd:YAG laser ($\lambda = 532\,\text{nm}$) with the help of an extended Mach–Zehnder-like configuration. Two of the beams are reflected by a mirror mounted upon a piezoelectric device and propagate transversely shifted to a second, mutually coherent beam. With the help of the piezoelectric device we shift the relative phase between a pair of beams by π to obtain two dipole structures. Subsequently, one of the dipole beams is reflected by a mirror mounted upon a piezoelectric device that oscillates at $\sim$1 kHz before it is aligned perpendicularly to the other dipole beam. The fast-oscillating mir-

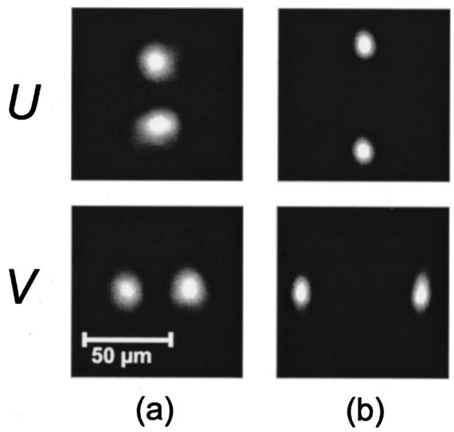

Fig. 3. Experimental results: (a) input intensity distribution, (b) beam profiles after 13.5-mm separate propagation.

ror effectively destroys the coherence between the two dipole modes as a result of the crystal's noninstantaneous response to variation in the intensity of light. This effect also permits the observation of the single constituents of composite solitary structures by blocking one component and recording the intensity of the remaining light from the other constituent within a short time ($\Delta t \approx 0.1\,\mathrm{s}$). Both dipole modes are then focused onto the front face of a strontium barium niobate crystal (i.e., SBN:60, doped with 0.002% cerium by weight). The nonlinear medium is biased by a 3.6-kV/cm dc electric field along its c axis, and the beams, which have a focal spot size of 12 μm (FWHM), propagate 5 or 13.5 mm inside the material.

The experimental results are depicted in Figs. 3 and 4. The input intensity distribution and the repulsive propagation behavior when each component propagates separately 13.5 mm in the crystal are depicted in Fig. 3. The single frames are arranged in the same way as for the numerical simulation shown in Fig. 1. The general agreement between the simulations and the experimental results is evident. The experiment reveals the anisotropic interaction of the single lobes of each dipole mode. The repulsion between the beamlets that are out of phase is slightly stronger along the horizontal than along the vertical direction. The relative distance between the two lobes of the u component increases from 37 to 58 μm, whereas the two lobes of the v components spread from 33 to 66 μm during propagation. The difference from the numerical simulations depicted in Fig. 1 can be explained by the fact that the initial separation and the transverse shape of the dipole lobes do not correspond exactly to the numerical initial conditions. Figure 4 depicts the two components copropagating simultaneously in the nonlinear material at different propagation steps. Here the single frames are arranged in the same way as in Fig. 2. The two components have almost equal total power of 2.3 μW, and the crystal is biased by a voltage of 3.6 kV/cm. Figure 4(a) illustrates the input intensity, and Fig. 4(b) shows the total intensity $|u|^2 + |v|^2$ and the single components u and v after a propagation of 5 mm. The total intensity distribution resembles a squeezed ring, and the structure does not break up yet. The two horizontally aligned lobes of the v component remain in their typical dipole shape [Fig. 4(b), bottom] whereas the perpendicularly aligned lobes of the u component are strongly affected, stretch in the vertical direction, and become kidney shaped [Fig. 4(b), middle]. When both beams propagate further to 13.5 mm, the breakup of the coupled dipole-mode solitary structure becomes apparent. The u component splits into a complex structure consisting of four peaks [Fig. 4(c), middle], whereas the v component becomes strongly vertically stretched [Fig. 4(c), bottom]. Even though the propagation distance of 13.5 mm, which corresponds to 4.5 diffraction lengths, is much shorter than in the numerical simulation depicted in Fig. 2(b), vertically aligned dipole mode u already displays a symmetry-breaking instability and no longer exists as a dipole mode. It is indeed surprising that the experiment displays decay of the u component with a subsequent formation of a four-peak structure, in contradiction to the simulations depicted in Fig. 2. Inasmuch as the v component is elongated along the vertical direction, the structures in Fig. 4(c) resemble two robust dipole-mode vector solitons. The elongated lobes of the v component act as a fundamental mode that traps two vertically aligned dipole modes. The stabilizing influence of the u component that was mentioned above was therefore not strong enough to prevent the breakup of the composite structure.

Comparing these experimental results with the theoretical ones, we have found a significant difference in the breakup behavior. However, the experimental generation of a certain initial profile is often not perfect because of the existence of optical impurities and aberrations. To account for these deviations of the ideal beam profile in our simulations, we simulate the propagation behavior, inducing a supplementary initial perturbation by changing the intensity of one component relative to the other. Starting from the exact results depicted in Fig. 1(b) and increasing the amplitude of the v component by 10% result in the evolution of the beams' intensities at different propagation stages as shown in Fig. 5, with [Fig. 5(a)] $\xi = 2.5$, [Fig. 5(b)] $\xi = 4.5$, and [Fig. 5(c)] $\xi = 10$. The single frames are arranged in the same way as in Fig. 4, and now the good agreement between numerical calcula-

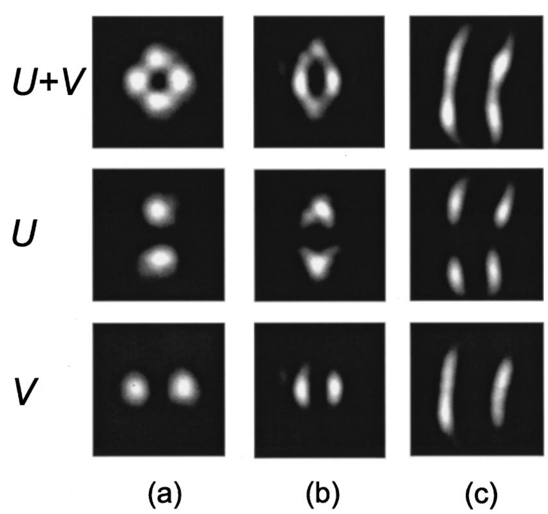

Fig. 4. Experimental results: (a) input intensity distribution; (b), (c) beam evolution after 5- and 13.5-mm propagation, respectively. The three rows depict the total intensity $|u|^2 + |v|^2$ and the single constituents $|u|^2$ and $|v|^2$.

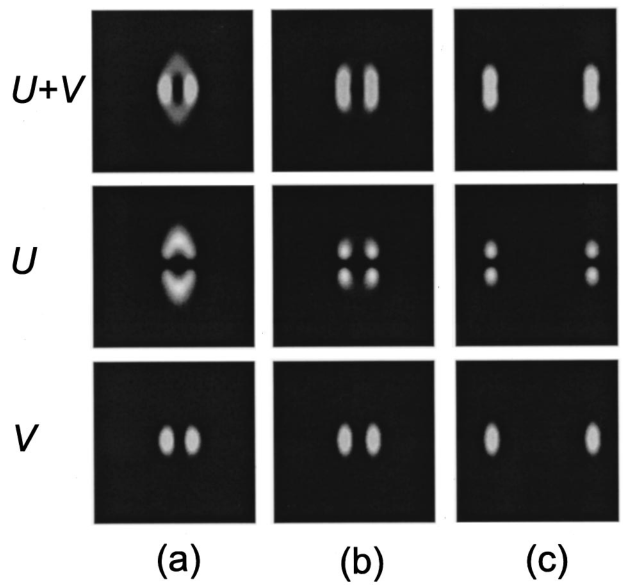

Fig. 5. The same sequence of frames as in Fig. 2 but with 10% initial perturbation for (a) $\xi = 2.5$, (b) $\xi = 4.5$, (c) $\xi = 10$.

tions and experimental results is obvious. Figure 5(a) corresponds to the experimental results depicted in Fig. 4(b). The squeezed-ring shape of the total intensity as well as the kidney shape of the u component is almost identical in both cases. At $\xi = 4.5$, which corresponds to a 13.5-mm propagation distance, the u component's break up into four beamlets is also evident. At larger propagation distances the destabilizing effect of the v component becomes apparent as the coupled structure splits into two equal parts that spread in the horizontal direction. This behavior, although it was expected for our experiments, could not be realized because of the limited length of our crystal sample. However, the vertical separation of the u component's lobes in Fig. 5(c) is much smaller than in the separate propagation depicted in Fig. 1(c), which is an indication that the lobes effectively get trapped by the copropagating lobes of the v component that serve as the fundamental constituents of two dipole-mode vector solitons.

5. CONCLUSIONS

We have demonstrated the effect of mutual stabilization of two perpendicularly aligned dipole-mode optical beams in an anisotropic saturable nonlinear medium. We have shown the existence of a numerically exact solution in the form of a composite solitary structure with a ring-shaped total intensity profile. The numerical beam propagation method has revealed symmetry-breaking instability at propagation distances that are typically larger than 10 diffraction lengths. The structure's splitting is strongly dependent on numerical noise and initial perturbations and therefore shows two different situations in which breakup can occur. Simulations with a remarkable initial perturbation display a spatial development than that is identical to experimental results. It is the inherent anisotropy of the medium in particular that is responsible for the well-defined splitting of one dipole into four parts that become the higher-order components of two mutually repelling dipole-mode vector solitons. To summarize, we have demonstrated the conversion of two mutually incoherent dipole-mode beams into a pair of dipole-mode vector solitons that survive for large propagation distances. These results underline the results of previous investigations[7–11] that have shown that the dipole-mode vector soliton is probably the most robust of the various configurations of multicomponent self-focused optical beams in saturable bulk nonlinear media.

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