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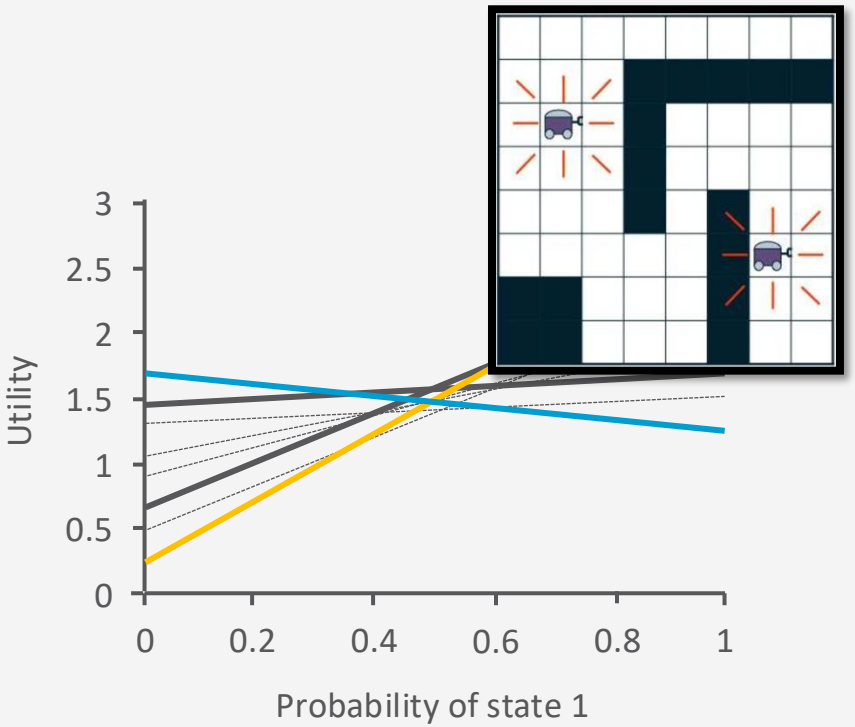

Automated Planning and Acting

Decision Making: Extensions

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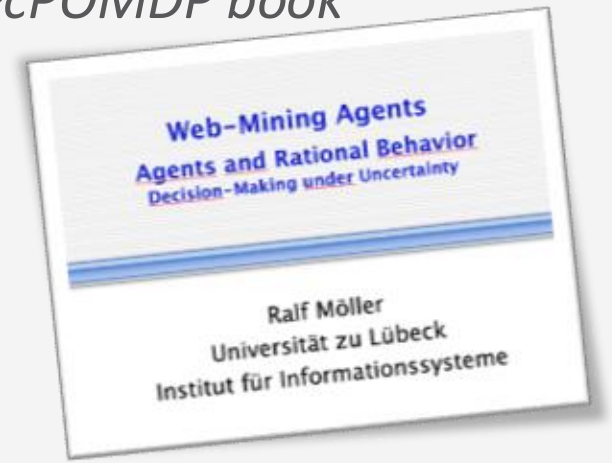
Content: Planning and Acting

1. With **Deterministic** Models
2. With **Temporal** Models
3. With **Nondeterministic** Models
4. With **Probabilistic** Models
5. By **Decision Making**
   A. Foundations
   B. Extensions
      - Partially observable MDPs (POMDPs)
      - Decentralised POMDPs (decPOMDPs)
   C. Structure
6. With **Human-awareness**

Acknowledgements

- Part 1 based on material from Lise Getoor, Jean-Claude Latombe, Daphne Koller, and Stuart Russell, Xiaoli Fern compiled by Ralf Möller
  - In part based on *AIMA Book, Chapter 17.4*
- Part 2 based on tutorial by Matthijs Spaan, Christopher Amato, Shlomo Zilberstein on *Decision Making in Multiagent Settings: Team Decision Making*
  - In part based on *DecPOMDP book*



http://rbr.cs.umass.edu/camato/decpomdp/overview.html

Outline: Decision Making – Extensions

Partially Observable Markov Decision Process (POMDP)

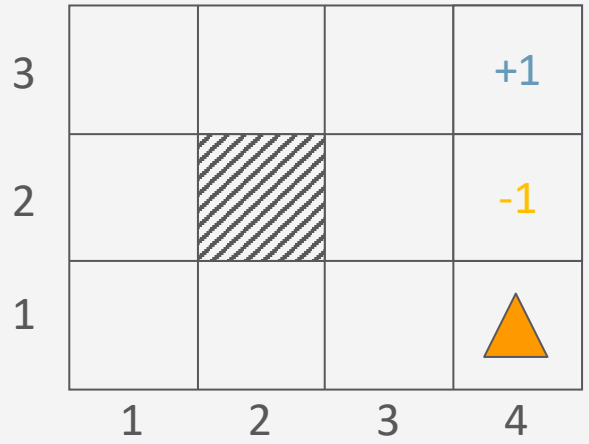
- POMDP agent, belief state, belief MDP
- Conditional plans, value iteration

Decentralised POMDP (Dec-POMDP)

- Dec-POMDP, local policy, joint policy, value function
- Communication, full observability, Dec-MDP
- Solutions for finite, infinite, indefinite horizon

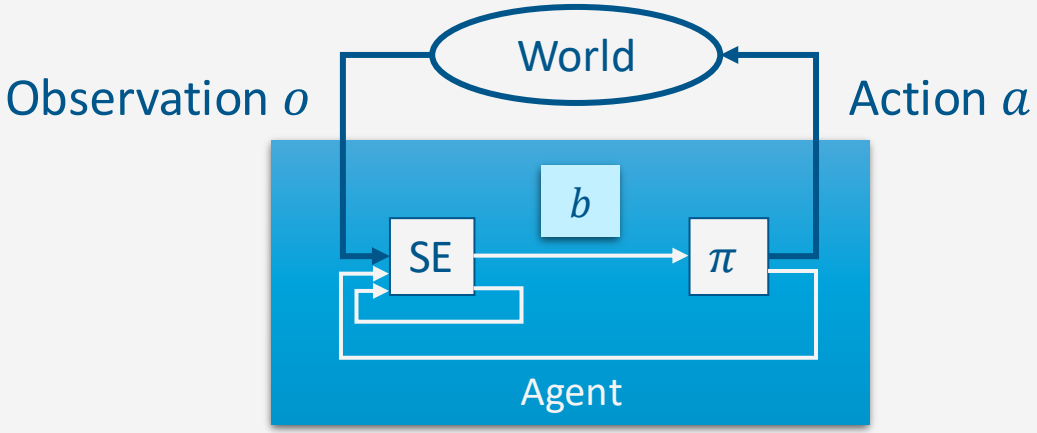
POMDP

- POMDP = Partially Observable MDP
  - Sensing operation returns multiple states, with a probability distribution
    - Sensor model Ω that encodes $P(o|s)$ (or $P(o|s,a)$)
      - Probability of observing o given state s (and action a)
    - Example:
      - Sensing number of adjacent walls (1 or 2)
      - Return correct value with probability 0.9
  - Formally, POMDP is a six-tuple (S, A, T, R, O, Ω)
    - MDP (S, A, T, R) extended with a set of observations O and a sensor model Ω
  - Choosing action that maximizes expected utility of state distribution assuming "state utilities" computed as before not good enough → Does not make sense (not rational)
- POMDP agent: Constructing a new MDP in which the current probability distribution over states plays the role of the state variable



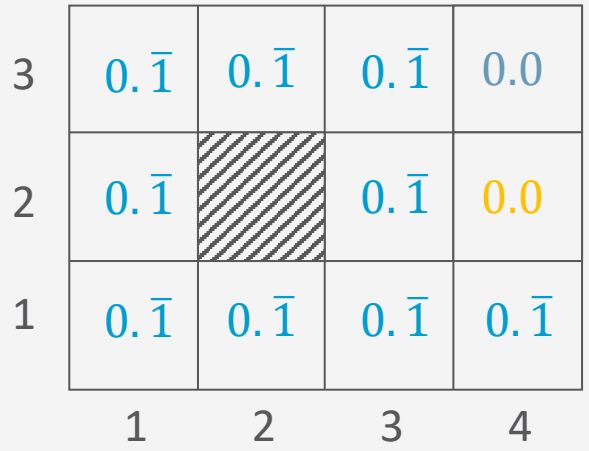
Decision cycle of a POMDP agent

- Given the current belief state b and a policy π, execute the action
$$a = \pi(b)$$
- Receive observation o
- Set the current belief state to $SE(b, a, o)$ and repeat
  - SE = State Estimation



Belief State & Update

- Belief state $b(s)$ is the probability assigned to the actual state s by belief state b
- Initial belief state
  - Probability of 0 for terminal states
  - Uniform distribution for rest
  - Robot navigation example:
    - $b = \left(\frac{1}{9}, \frac{1}{9}, \frac{1}{9}, \frac{1}{9}, \frac{1}{9}, \frac{1}{9}, \frac{1}{9}, \frac{1}{9}, \frac{1}{9}, 0{,}0\right)$

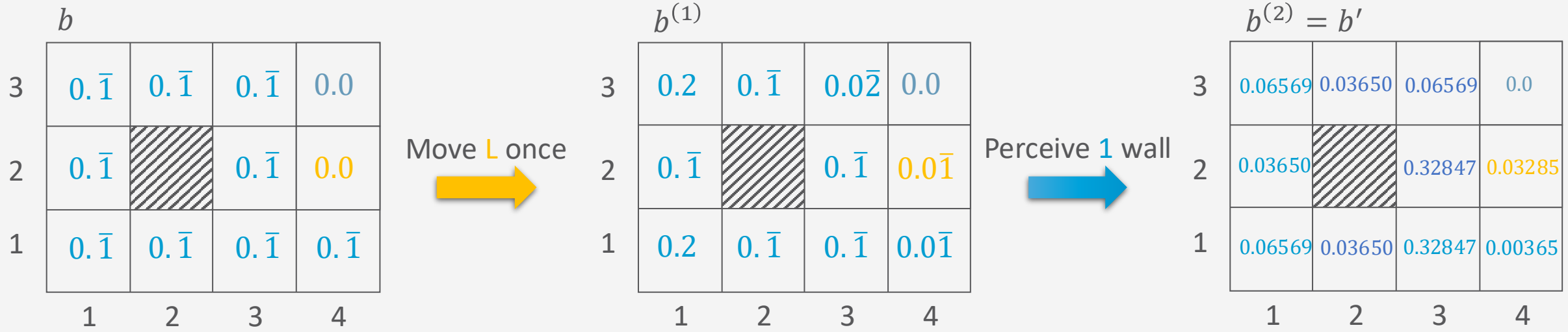


Belief State & Update

- Update $b' = SE(b, a, o)$

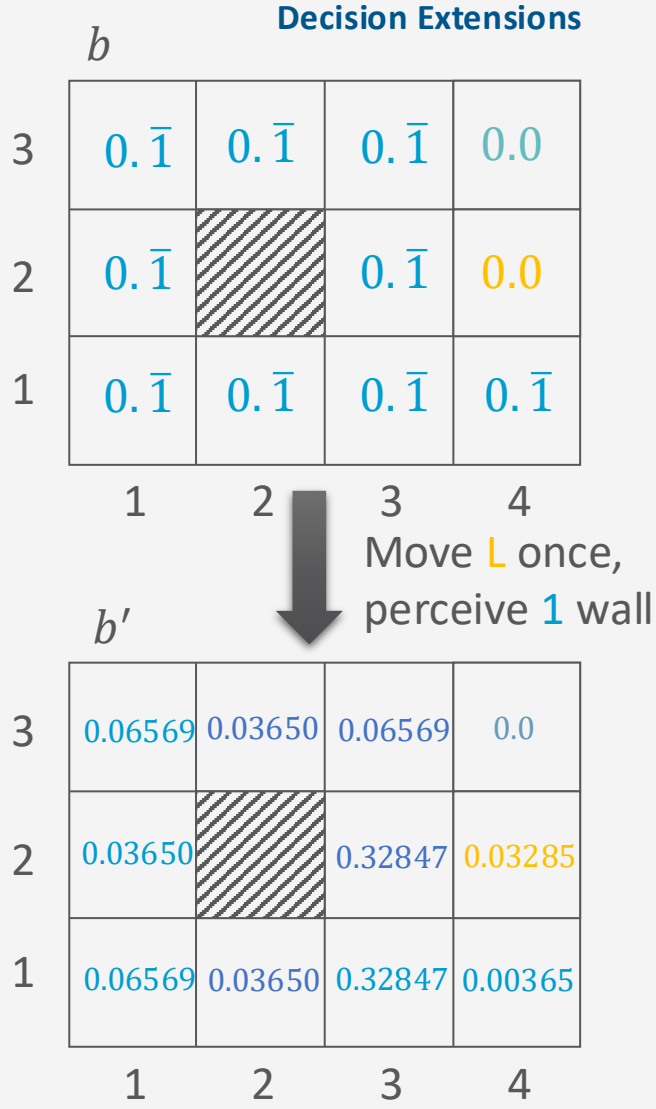
$$b'(s') = P(s'|o, a, b) = \frac{P(o|s', a) \sum_{s \in \mathrm{dom}(S)} P(s'|s, a) b(s)}{\sum_{s'' \in \mathrm{dom}(S)} P(o|s'', a) \sum_{s \in \mathrm{dom}(S)} P(s''|s, a) b(s)}$$

  - Consider as two-stage update: (1) Update for the action (2) Update for the observation



Belief MDP

- A belief MDP is a tuple $(B, A, \rho, T, O, \Omega)$
  - B = *infinite* set of belief states
    - Continuous!
  - A = finite set of actions
  - Reward function $\rho(b)$ (can also be defined with a)
    - Reward of belief state b
  - Transition function $T(b', b, a) = P(b'|b, a)$
    - Probability of new belief state b' given belief state b and action a
  - O = finite set of observations
  - Sensor model $\Omega(o, b) = P(o|b)$ (can also be defined with a)
    - Probability of observation o given belief state b (and action a)



Belief MDP: Express Functions using POMDP Functions

- Reward function: Sum over all actual states that the agent can be in

$$\rho(b) = \sum_s b(s)R(s)$$

- Transition function: Sum over all possible observations

$$P(b'|b,a) = \sum_o P(b'|o,a,b)P(o|a,b) = \sum_o P(b'|o,a,b) \sum_{s'} P(o|s') \sum_s P(s'|s,a)b(s)$$

  - where $P(b'|o,a,b) = 1$ if $b' = SE(b,a,o)$ and 0 otherwise

- Sensor model: Sum over all actual states that the agent might reach

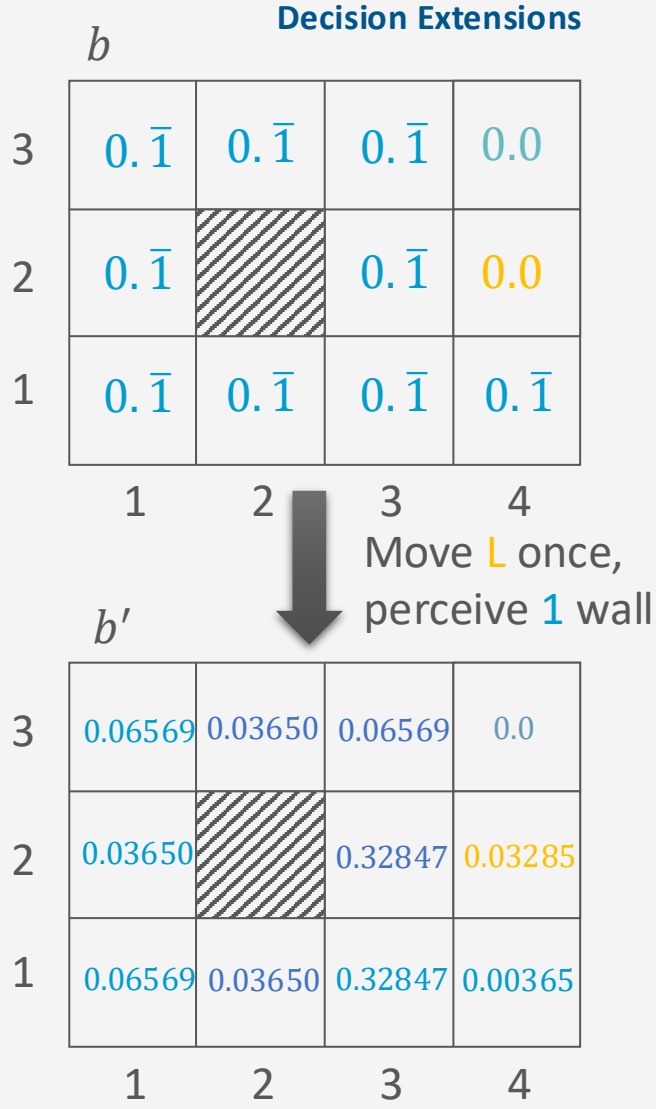
$$P(o|a,b) = \sum_{s'} P(o|a,s',b)P(s'|a,b) = \sum_{s'} P(o|s')P(s'|a,b) = \sum_{s'} P(o|s') \sum_s P(s'|s,a)b(s)$$

- $P(b'|b,a)$ and $\rho(b)$ define an observable MDP on the space of belief states

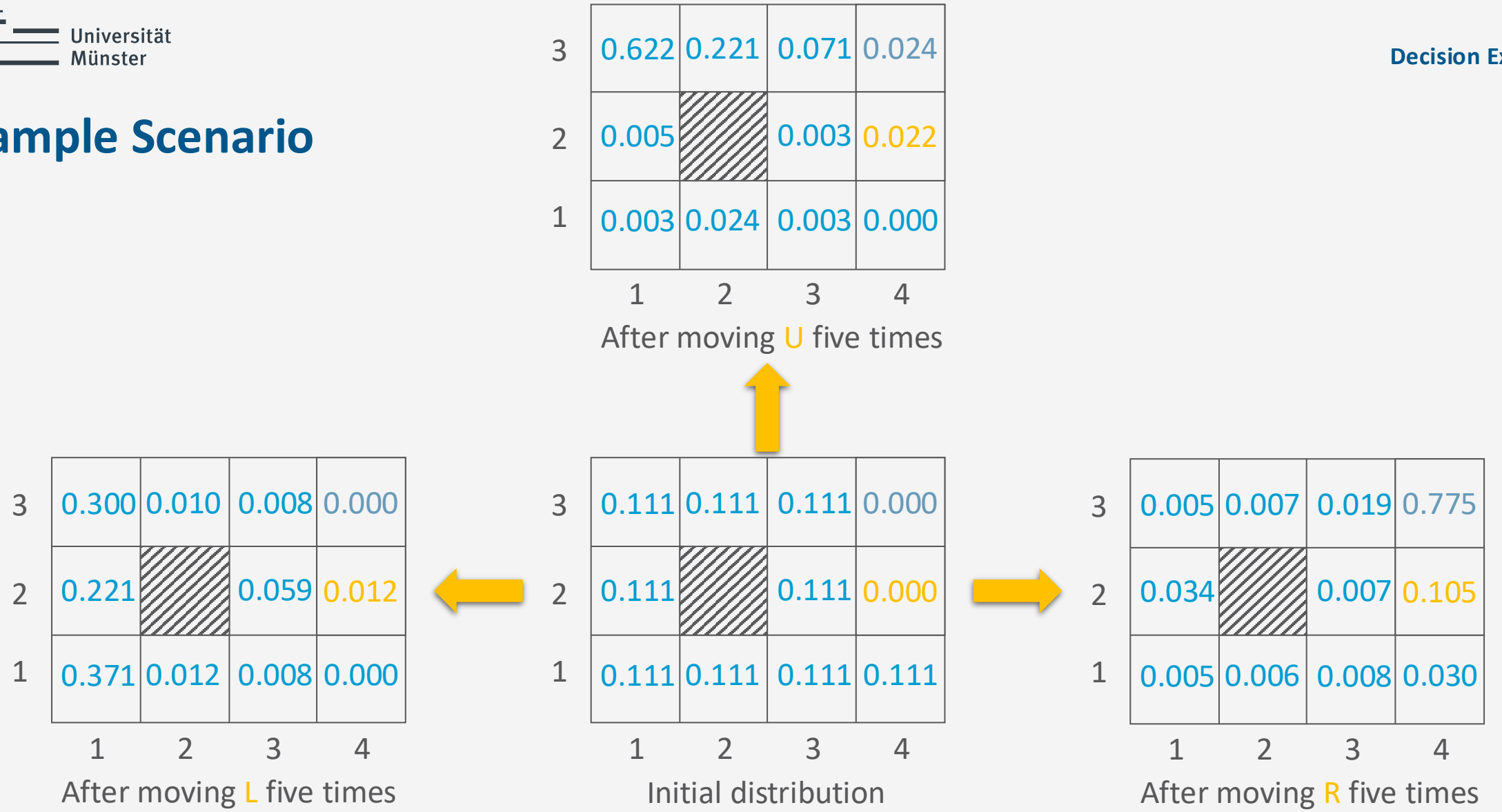
Belief MDP

- Optimal action depends only on agent's current belief state
  - Does not depend on actual state the agent is in

$\Rightarrow$ Solving a POMDP on a physical state space is reduced to solving an MDP on the corresponding belief-state space
  - Mapping $\pi^*(b)$ from belief states to actions



Example Scenario



Conditional Plans

- Example:
  - Two state world $0,1$
  - Two actions: $stay(P), go(P)$
    - Actions achieve intended effect with some probability P
  - One-step plan $[go], [stay]$
- Two-step plans are conditional
  - $[a1$, IF $percept = 0$ THEN $a2$ ELSE $a3]$
  - Shorthand notation: $[a1, a2/a3]$
- n-step plans are trees with
  - Nodes attached with actions and
  - Edges attached with percepts

Value Iteration for POMDPs

- Cannot compute a single utility value for each state of all belief states
- Consider an optimal policy π^* and its application in belief state b
- For this b, the policy is a conditional plan p
  - Let the utility of executing a fixed conditional plan p in s be $u_p(s)$
  - Expected utility $U_p(b) = \sum_s b(s)u_p(s)$
    - It varies linearly with b, a hyperplane in a belief space
  - At any b, the optimal policy will choose the conditional plan with the highest expected utility

$$U(b) = U^{\pi^*}(b) = \max_p \sum_s b(s)u_p(s)$$

$$\pi^* = \arg\max_p \sum_s b(s)u_p(s)$$

  - $U(b)$ is the maximum of a collection of hyperplanes and will be piecewise linear and convex

General Formula

- Let p be a depth-d conditional plan whose initial action is a and whose depth-$d-1$ subplan for percept e is $p.e$, then

$$u_p(s) = R(s) + \sum_{s'} P(s' \mid s, a) \sum_{e} P(e|s')\, u_{p.e}(s')$$

  - $d = 0$: $u_p(s) = R(s)$ for the empty plan $p = \perp$
  - $d = 1$: $p.e = \perp$ for all e, simplifying the last sum:

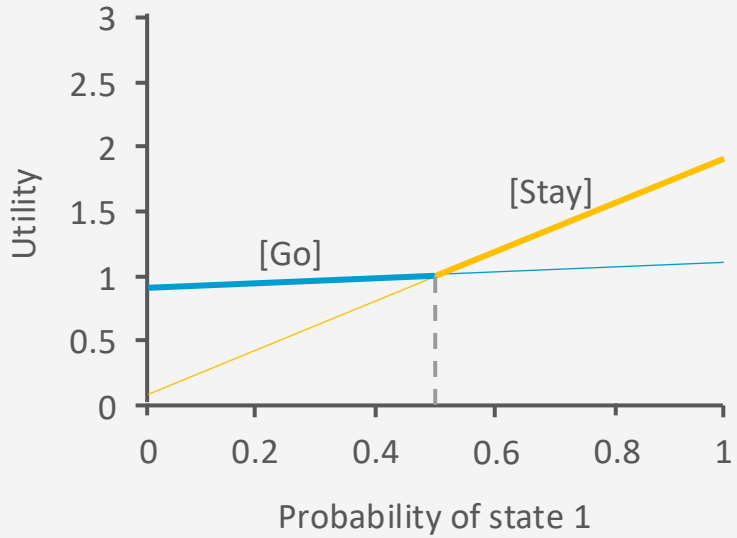
$$\sum_{e} P(e|s')\, u_{p.e}(s') = \sum_{e} P(e|s')\, u_{\perp}(s') = u_{\perp}(s') \sum_{e} P(e|s') = u_{\perp}(s') \cdot 1 = R(s')$$

- This gives us a *value iteration* algorithm
  - Elimination of dominated plans is essential for reducing doubly exponential growth:
    - Number of undominated plans with $d = 8$ is just 144, otherwise 2^{255} ($|A|^{O(|E|^{d-1})}$)
    - For large POMDPs this approach is highly inefficient

Example

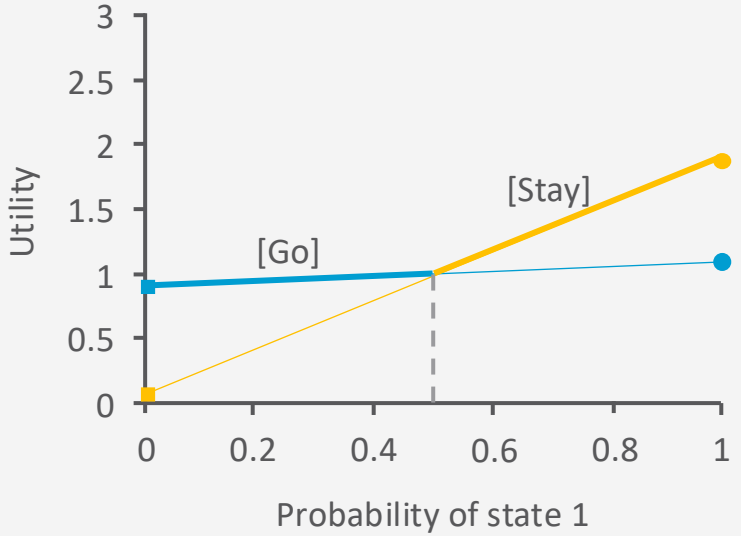
- Compute the utilities for conditional plans of depth 2 by
  - considering each possible first action
  - each possible subsequent percept
  - each way of choosing a depth-1 plan to execute for each percept

Utility of two one-step plans as a function of $b(1)$



Example

- Two state world $0,1$
- Rewards $R(0) = 0, R(1) = 1$
- Two actions: $stay(0.9), go(0.9)$
- Sensor reports correct state with probability of 0.6
- Consider the one-step plans $[stay]$ and $[go]$



state 0 state 1

- $u_{[stay]}(0) = R(0) + \overbrace{0.9R(0)}^{\text{state 0}} + \overbrace{0.1R(1)}^{\text{state 1}} = 0.1$
- $u_{[stay]}(1) = R(1) + 0.1R(0) + 0.9R(1) = 1.9$
- $u_{[go]}(0) = R(0) + 0.1R(0) + 0.9R(1) = 0.9$
- $u_{[go]}(1) = R(1) + 0.9R(0) + 0.1R(1) = 1.1$
- This is just the direct reward function (taking into account the probabilistic transitions)

8 distinct depth-2 plans for each state (16 plans)

Utility of depth-1 plan given state, outcome of first action, and percept

Choose action based on percept (0 : $\boldsymbol{stay}$); receive utility of actual state ($\mathbf{1}$): $u_{[\boldsymbol{stay}]}(\mathbf{1}) = 1.9$

Utilities of depth-1 plans

$u_{[stay]}(0) = 0.1 \quad u_{[go]}(0) = 0.9$

$u_{[stay]}(1) = 1.9 \quad u_{[go]}(1) = 1.1$

Reward of state

Probability of next state

Probability of percept

$$u_{[stay,stay/stay]}(0) = R(0) + \big(0.9 \cdot (0.6 \cdot 0.1 + 0.4 \cdot 0.1) + 0.1 \cdot (0.4 \cdot 1.9 + 0.6 \cdot 1.9)\big) = 0.28$$

$$u_{[stay,stay/stay]}(1) = R(1) + \big(0.1 \cdot (0.6 \cdot 0.1 + 0.4 \cdot 0.1) + 0.9 \cdot (0.4 \cdot 1.9 + 0.6 \cdot 1.9)\big) = 2.72$$

percept 0 percept 1 percept 0 percept 1

state 0 state 1

Sum over states reachable with first action

Sum over possible percepts

$u_{[stay,go/stay]}(0), u_{[stay,stay/go]}(0), u_{[stay,go/go]}(0)$

$u_{[stay,go/stay]}(1), u_{[stay,stay/go]}(1), u_{[stay,go/go]}(1)$

$$u_{[go,stay/stay]}(0) = R(0) + \big(0.1 \cdot (0.6 \cdot 0.1 + 0.4 \cdot 0.1) + 0.9 \cdot (0.4 \cdot 1.9 + 0.6 \cdot 1.9)\big) = 1.72$$

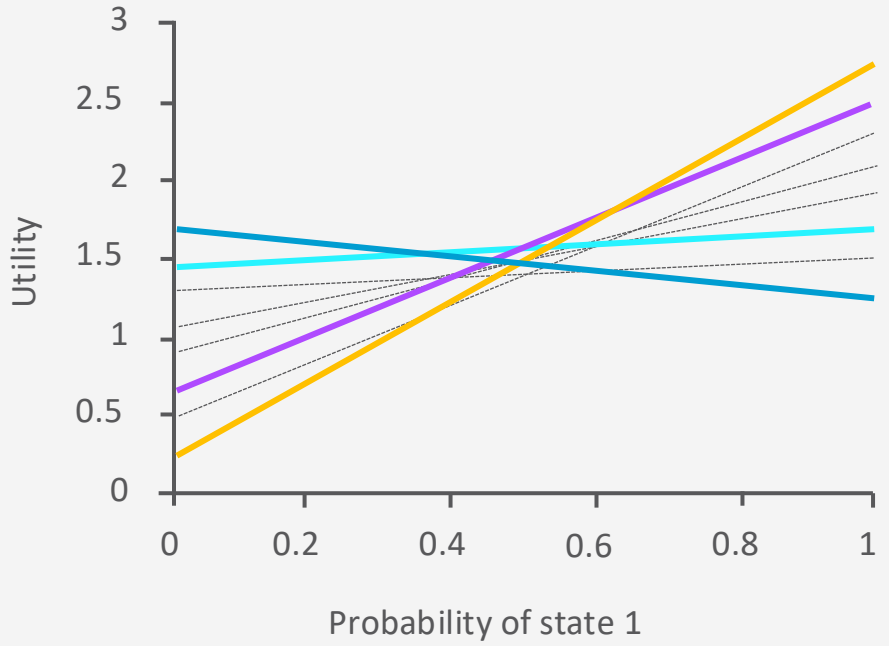
$$u_{[go,stay/stay]}(1) = R(1) + \big(0.9 \cdot (0.6 \cdot 0.1 + 0.4 \cdot 0.1) + 0.1 \cdot (0.4 \cdot 1.9 + 0.6 \cdot 1.9)\big) = 1.28$$

$u_{[go,go/stay]}(0), u_{[go,stay/go]}(0), u_{[go,go/go]}(0)$

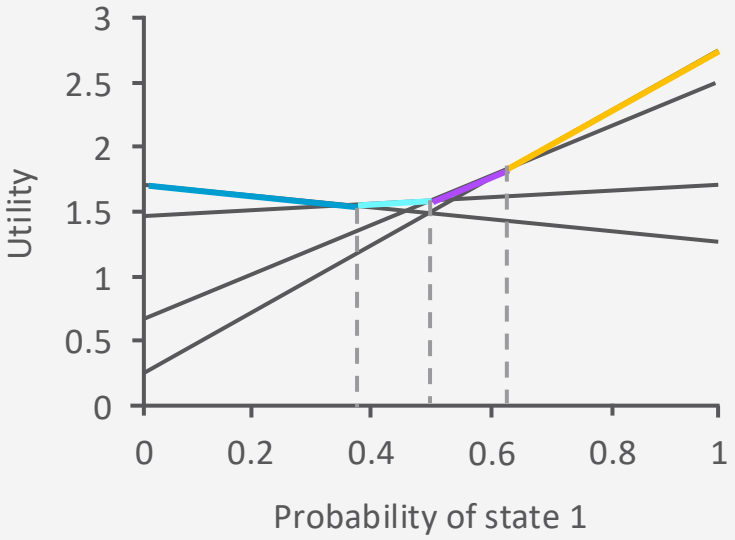
$u_{[go,go/stay]}(1), u_{[go,stay/go]}(1), u_{[go,go/go]}(1)$

Example

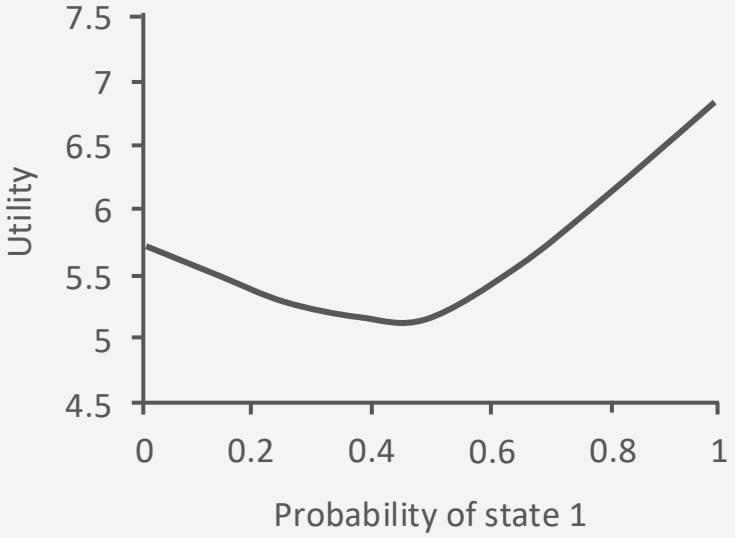
- 8 distinct depth-2 plans for state 1
  - 4 are suboptimal across the entire belief space (dashed lines)
  - With probability $b(1) = 0$
    - $u_{[stay,stay/stay]}(0) = 0.2$
    - $u_{[go,stay/stay]}(0) = 1.7$
  - With probability $b(1) = 1$:
    - $u_{[stay,stay/stay]}(1) = 2.72$
    - $u_{[go,stay/stay]}(1) = 1.28$



Example



Utility of four undominated two-step plans



Utility function for optimal eight step plans

Value Iteration: Algorithm

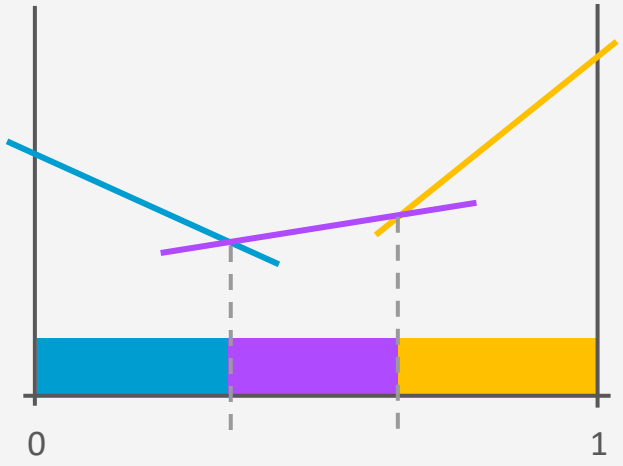
- Returns an optimal set of plans
- Inputs
  - POMDP $pomdp$
    - States $\mathrm{dom}(S)$
    - For all $s \in \mathrm{dom}(S)$,
      - Applicable actions $A(s)$
      - Transition model $P(s'|a,s)$
      - Sensor model $P(o|s)$
      - Rewards $\rho(s)$
    - Discount γ
  - Maximum error allowed ϵ

```
function value-iteration(pomdp,ϵ)
    U′ ← a set containing the empty plan [] with u_[](s)=R(s)
    repeat
        U ← U′
        U′ ← the set of all plans consisting of an action and,
                for each possible next percept, a plan in U with
                utility vectors computed as on previous slide
        U′ ← Remove-dominated-plans(U′)
    until Max-difference(U,U′) < ϵ(1-γ)/γ
    return U
```

- Local variables
  - U, U' sets of plans with associated utility vectors u_p

Solutions for POMDP

- Belief MDP has reduced POMDP to MDP
  - MDP obtained has a multidimensional continuous state space
- Extract a policy from utility function returned by value-iteration algorithm
  - Policy $\pi(b)$ can be represented as a set of regions of belief state space
  - Each region associated with a particular optimal action
  - Value function associates distinct linear function of b with each region
  - Each value or policy iteration step refines the boundaries of the regions and may introduce new regions



Intermediate Summary

- POMDP
  - Uncertainty about state → belief state
  - Solving a POMDP = Solving an MDP on space of belief states
  - Policy = conditional plans
  - Value iteration to find optimal policy
    - Very expensive, even with deletion of dominated plans

What to do alternatively? Find sub-optimal plans
- Sampling approaches
- In combination with deep learning methods

Outline: Decision Making – Extensions

Partially Observable Markov Decision Process (POMDP)

- POMDP agent, belief state, belief MDP
- Conditional plans, value iteration

Decentralised POMDP (Dec-POMDP)

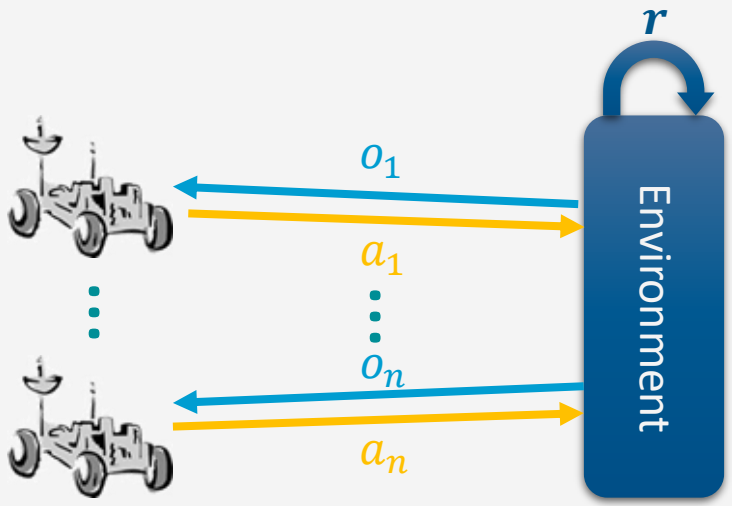
- Dec-POMDP, local policy, joint policy, value function
- Communication, full observability, Dec-MDP
- Solutions for finite, infinite, indefinite horizon

Multi-agent Scenarios

- Ambulance allocation
  - Multiple ambulance services
    - Business oriented operation
    - Competition for government funds and public opinion
  - Given several locations that require medical assistance, how many ambulances from which firm will go to which location?
- Firefighters
  - Maintain effort toward saving the building or draw back and minimise spread of fire?
  - Concentrate on a multitude of smaller fires or allow controlled unification and deal with only one location?
    - Will transportation routes be endangered?
    - Are there still civilians evacuating from the area / building?
  - Push through the fire to victims or save the fire crew and pull out?
    - If multiple crews are on site, which one goes? When?

Setting

- Single and repeated interactions with *joint rewards*: traditional game theory
- Interactions involving *joint state + reward* focus of decision-theory inspired approaches to game theory
  - Extensions of single-agent models to multi-agent settings
- Multi-agent setting
  - Co-operation of agents (team)
    - Vs. self-interested acting (all the way to hostile settings)
  - Problem: planning how to act
    - Joint payoff $\boldsymbol{r}$ but *decentralised* actions a_i and observations o_i
    - Joint state, influenced by actions, can influence rewards
    - Perfect vs. incomplete information about others



Decentralised POMDP (Dec-POMDP)

- Dec-POMDP: tuple $(I, S, \{A_i\}_{i \in I}, \{O_i\}_{i \in I}, T, R, \Omega)$
  - I = a finite set of agents indexed $1, \ldots, N$
  - $\mathrm{dom}(S)$ = a finite set of states
  - A_i = a finite set of actions available to agent $i \in I$
    - $\vec{A} = \bigotimes_{i \in I} A_i$ set of joint actions
  - O_i = a finite set of observations available to agent $i \in I$
    - $\vec{O} = \bigotimes_{i \in I} O_i$ set of joint observations
  - Transition function $T(s', s, \vec{a}) = P(s' | s, \vec{a})$
  - Reward function $R(s)$ or $R(\vec{a}, s)$
  - Sensor model (observation function) $\Omega(\vec{o}, \vec{a}, s) = P(\vec{o} | \vec{a}, s)$
- Co-operative, decision-theoretic setting: Joint reward function R, joint state space S

Generalising Dec-POMDPs

- Partially observable stochastic game (POSG)
  - Dec-POMDP $(I, S, \{A_i\}_{i \in I}, \{O_i\}_{i \in I}, T, \cancel{R}, \Omega)$ but with individual reward functions $\{R_i\}_{i \in I}$
  - Reward function R_i for each agent $i \in I$

- For self-interested or adversarial acting
  - Local optimum not guaranteed to be the global optimum
  - Dominant strategy equilibrium: best response (highest utility) given any state
    - Not guaranteed to exist
    - Prisoner's dilemma: Dominant strategy not always pareto-optimal strategy
  - Nash Equilibrium: No agent has incentive to change its strategy if no other agent changes its strategy
    - Always exists

Policies for Dec-POMDPs

- Local policy π_i for agent i
  - Representations: Mappings…
    - from local histories of observations $h_i = \left(o_i^{(1)}, \ldots, o_i^{(t)}\right)$ over O_i to actions in A_i
    - from local abstraction of joint state s in S to actions in A_i
    - from (generalised) belief states B_i to actions in A_i
      - Belief MDP
    - from internal memory states to actions
- Joint policy $\pi = (\pi_1, \ldots, \pi_N)$
  - Tuple of local policies, one for each agent in I

Value Functions for Dec-POMDPs

- Value functions work as before given a joint policy
  - Value of a joint policy π for a finite-horizon Dec-POMDP with initial state $s^{(0)}$

$$V^{\pi}\left(s^{(0)}\right) = E\left[\sum_{t=0}^{h-1} R\left(\vec{a}^{(t)}, s^{(t)}\right) \middle| s^{(0)}, \pi\right]$$

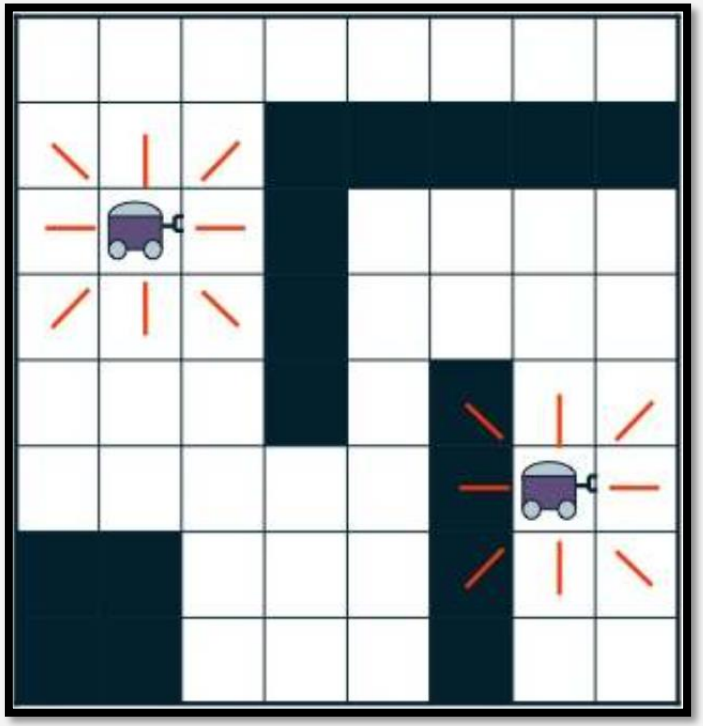
  - Value of a joint policy π for a infinite-horizon Dec-POMDP with initial state $s^{(0)}$ and discount factor $\gamma \in [0,1)$

$$V^{\pi}\left(s^{(0)}\right) = E\left[\sum_{t=0}^{\infty} \gamma^t R\left(\vec{a}^{(t)}, s^{(t)}\right) \middle| s^{(0)}, \pi\right]$$

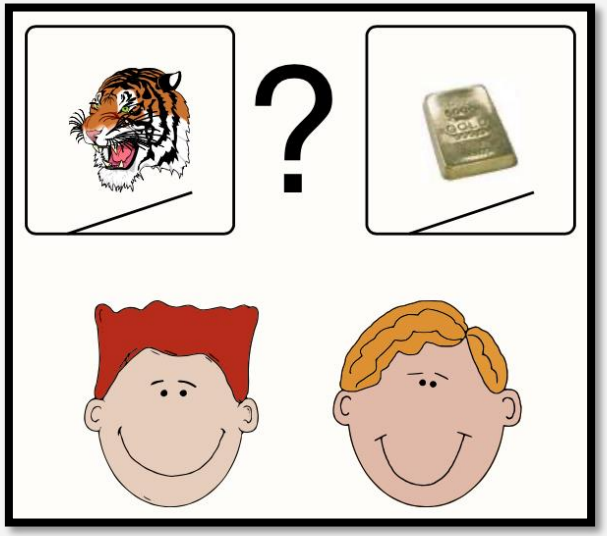
  - $\vec{a}_t$ joint action at time step t

Example: Two-agent Grid World

- Agents: two
- States: grid cell pairs
- Actions: move U, D, L, R, stay
- Transitions: noisy
- Observations: cell occupancy in the directions of the red lines
- Rewards: negative unless sharing the same square



Example: The Dec-Tiger Problem



- A toy problem: *decentralized tiger*
- State space:
  - Position of tiger behind one of two doors (left / right)
  - Treasure behind other door
- Reward:
  - Opening correct door: both receive treasure
  - Opening wrong door: both get attacked by a tiger
- Actions: Agents can open a door, or listen
  - After opening a door, game is reset with tiger behind a randomly chosen door
- Observations: Two noisy observations: hear tiger left or right
- Agents do not know the other's actions or observations

Worst-case Complexity of DecPOMDP

- Space complexity
  - Transition model: $\mathcal{O}(s \cdot s \cdot a^N)$
  - Sensor model: $\mathcal{O}(s \cdot o^N)$ or $\mathcal{O}(s \cdot o^N \cdot a^N)$
  - Reward function: $\mathcal{O}(s)$ or $\mathcal{O}(s \cdot a^N)$
- Runtime complexity of brute-force search
  - Evaluation cost of a joint policy: $\mathcal{O}\left(s \cdot o^{Nh}\right)$
  - Policy space: $\mathcal{O}\left(a^{\frac{N\left(o^h-1\right)}{o-1}}\right)$
- Notations
  - $s = |S|$
    - State space size
  - $a = \max_{i \in I} |A_i|$
    - Largest individual action space size
  - $o = \max_{i \in I} |O_i|$
    - Largest individual observation space size
  - h
    - Horizon
  - N
    - Number of agents

Communication?

- Can make working towards a common goal easier
  - Agents in grid world can communicate their intent (direction of travel)
- Definitely makes the formalism more complicated
  - Dec-POMDP with communication (Dec-POMDP-Com)
    - Dec-POMDP $(I, S, \{A_i\}_{i \in I}, \{O_i\}_{i \in I}, T, R, \Omega)$ defined as before extended with
      - Alphabet Σ for communication
      - $\sigma_i \in \Sigma$ an atomic message sent by agent i
      - $\vec{\sigma} = (\sigma_1, \ldots, \sigma_n)$ a joint message
      - $\varepsilon_\sigma \in \Sigma$ a null message, sent by an agent that does not want to transmit anything to the others (no cost of sending ε_σ)
      - Cost function C_Σ for transmitting atomic message
      - Reward function $R(\vec{a}, s', \vec{\sigma})$ incorporating joint message

New dimensions:
- Do agents always share information?
- Can they intentionally withhold information?
- Can they lie?

Dec-MDP

- Joint full observability
  - Collective observability
  - A DEC-POMDP is jointly fully observable if the N-tuple of observations made by all the agents uniquely determine the current global state
    - That is, if $P(\vec{o}|\vec{a}, s') > 0$, then $P(s'|\vec{o}) = 1$
- Dec-MDP ≙ Dec-POMDP with joint full observability
  - Same as before:
    MDP ≙ POMDP with full observability
  - Alternative name: multi-agent MDP

Solving Dec-POMDPs

- Problem: No joint belief available
  - Only partial information about state available to each agent

- Complexity: NEXP-complete
  - Optimal solutions using dynamic programming paradigm + exploiting structure if present
  - Reduction to NP when agents mostly independent + communication can be explicitly modelled and analysed
    - Requires that one can factorise the joint state space into a state space for each agent that is mostly independent of all others
    - The same goes for the observations and the reward function

Exhaustive Search

- Optimal solution approach for general models with a finite horizon h
- Procedure:
  - Do a search for each agent to find optimal local policies with a limited depth of h
  - Prune dominated search paths/strategies locally by considering the joint state and other agents' policies (globally)
    - Requires central oversight
    - Cannot be done locally without a huge amount of communication
- Even with pruning, still limited to small problems

Multi-agent Dynamic Programming

- Iteratively eliminate weakly-dominated policy trees using a dynamic programming
  - A policy $\pi_{i,j}^t$ of agent i is weakly dominated if there always exists another policy $\pi_{i,j'}^t$ with higher value over all possible states s and all possible policies of the other agents $\boldsymbol{\pi}_{-i}^t$:

$$\forall s \in S, \boldsymbol{\pi}_{-i}^t \in \boldsymbol{\Pi}_{-i}^t \; \exists \pi_{i,j'}^t \in \Pi_i^t : V_i\left(s, \pi_{i,j}^t, \boldsymbol{\pi}_{-i}^t\right) \leq V_i\left(s, \pi_{i,j'}^t, \boldsymbol{\pi}_{-i}^t\right)$$

    - Solvable by a linear programme

Dynamic Programming Operator:

Input: sets of policies and corresponding value vectors for each agent.

Process:

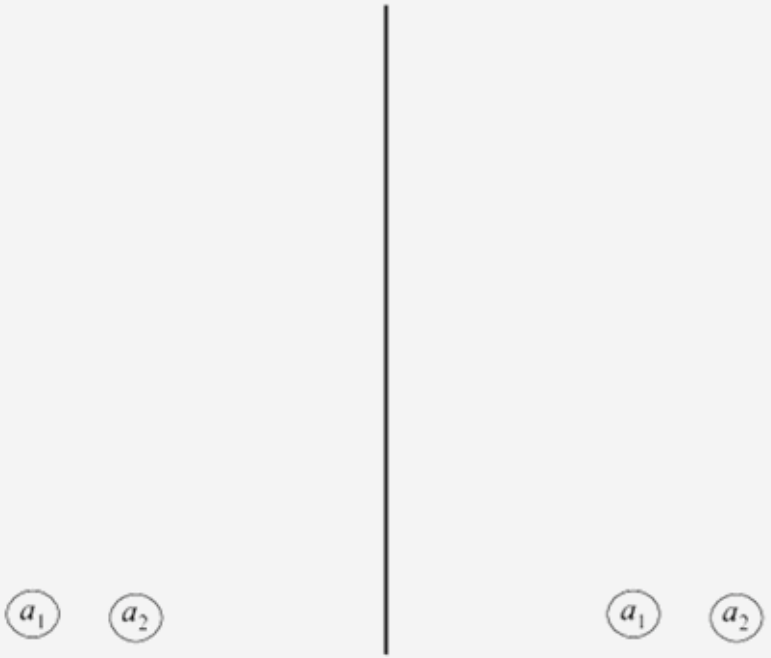
- Perform exhaustive backup using policies for each agent.
- Calculate new value vectors.
- Prune weakly dominated policies per agent.

Output: Updated sets of policies and value vectors for each agent.

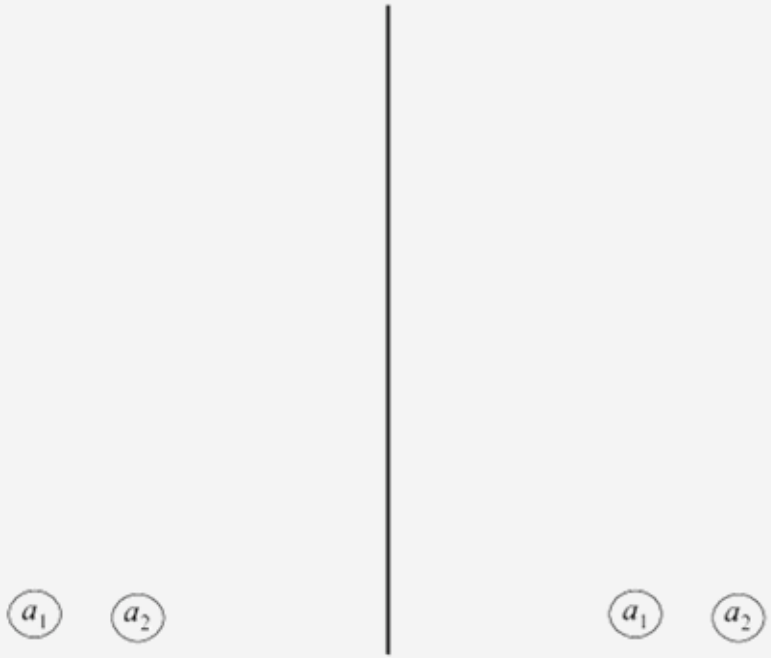
Hansen, E.A., Bernstein, D. S., & Zilberstein, S. 2004

Exhaustive Search and Pruning

With Pruning

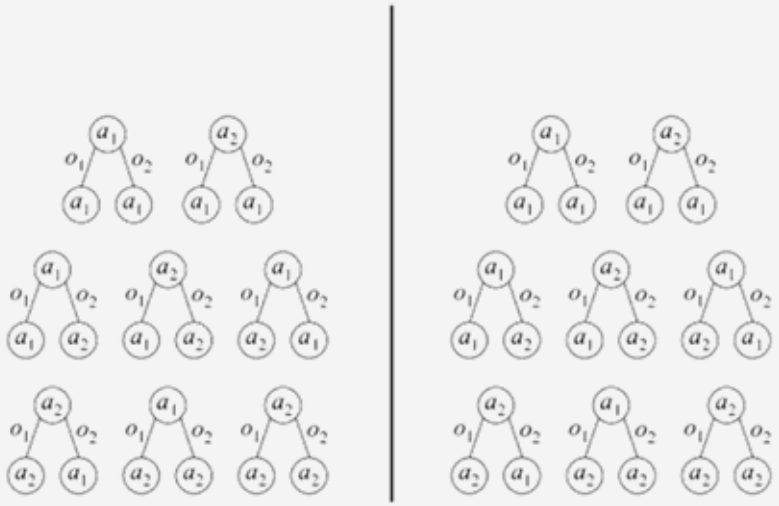


Without Pruning



Exhaustive Search and Pruning

With Pruning



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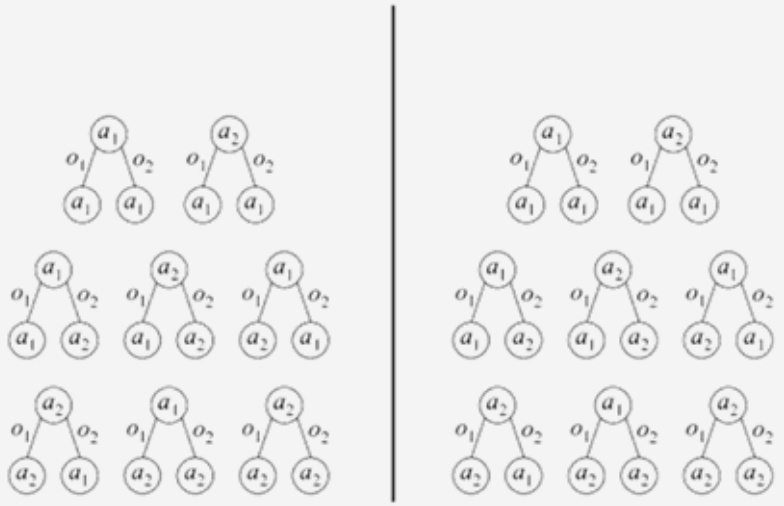
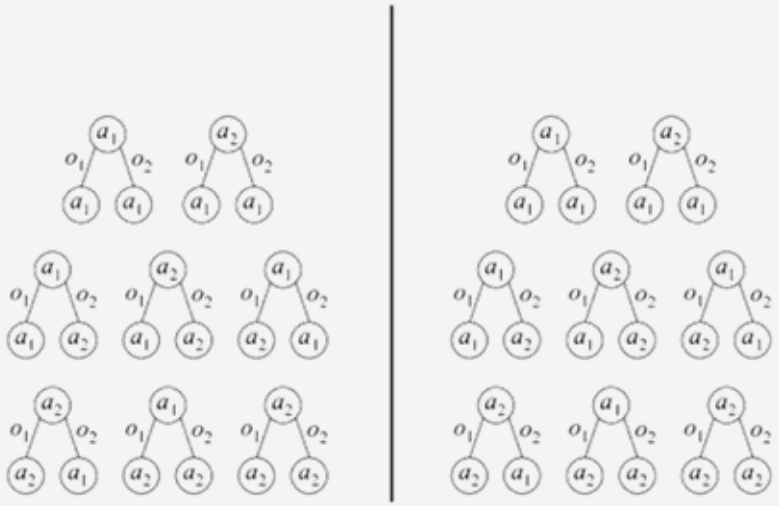


Exhaustive Search and Pruning

With Pruning



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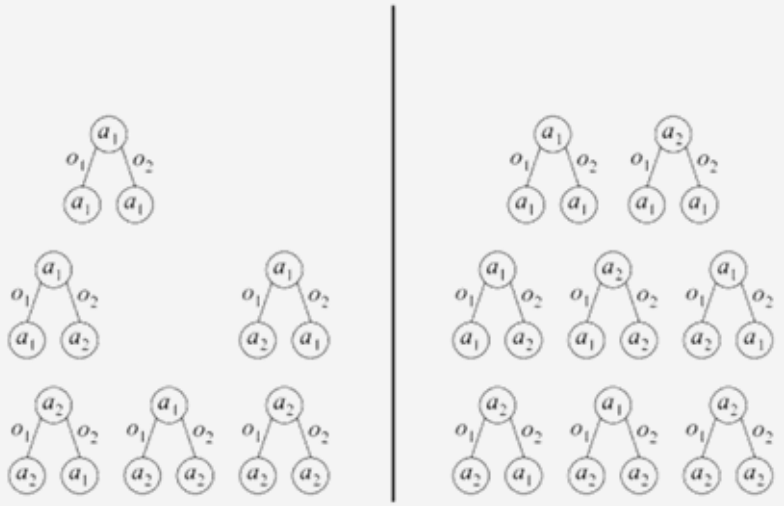
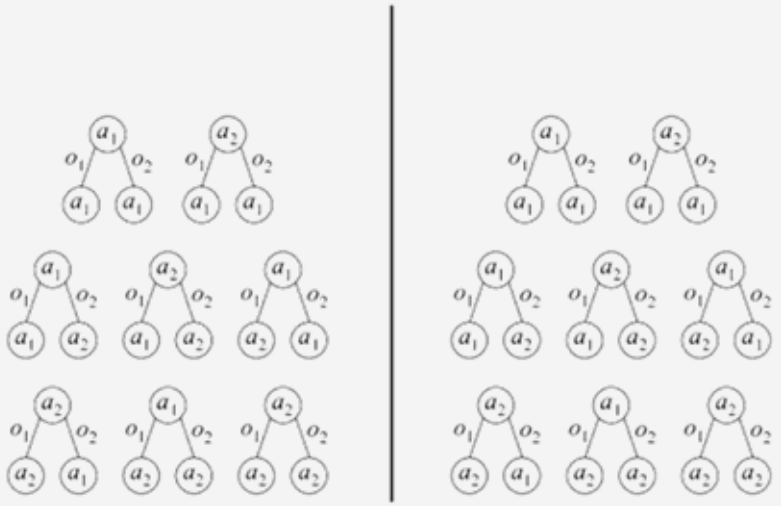


Exhaustive Search and Pruning

With Pruning



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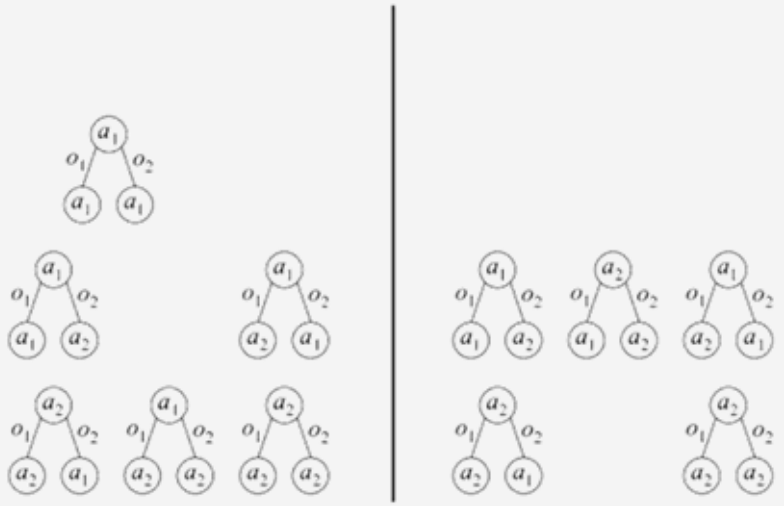
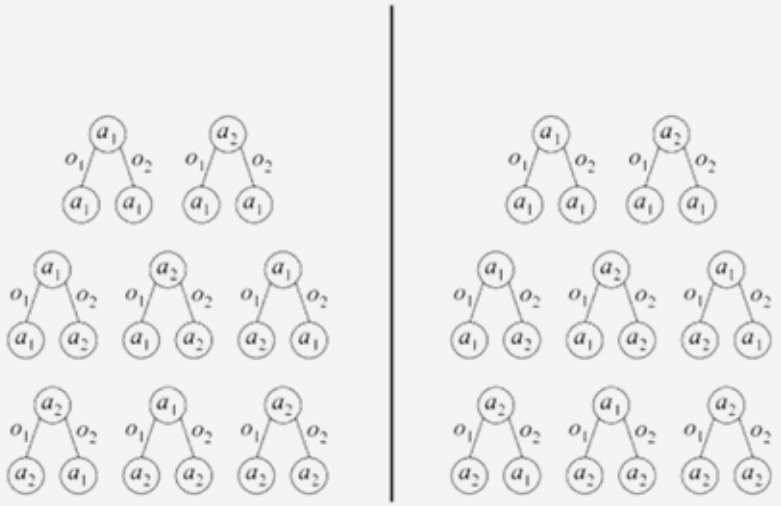


Exhaustive Search and Pruning

With Pruning



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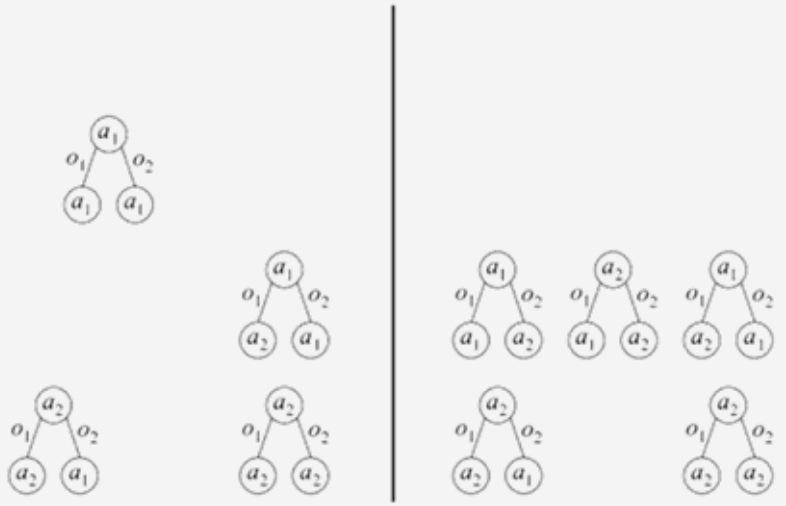
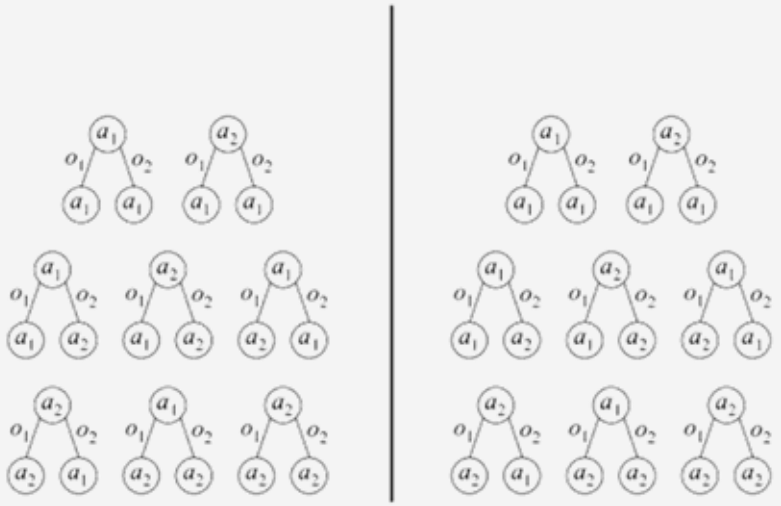


Exhaustive Search and Pruning

With Pruning



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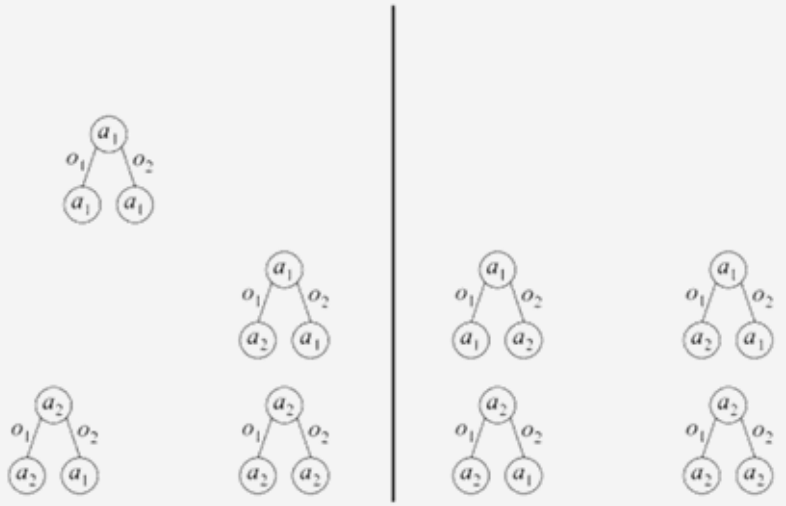
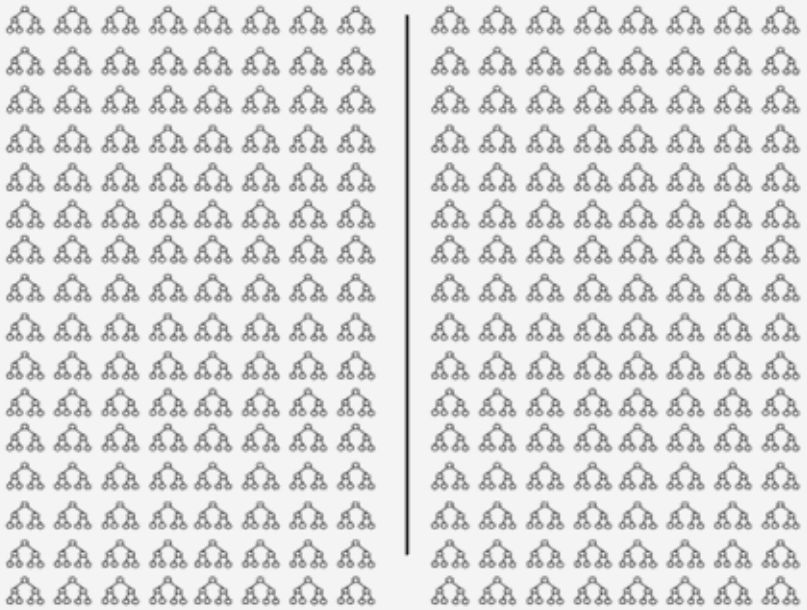


Exhaustive Search and Pruning

With Pruning



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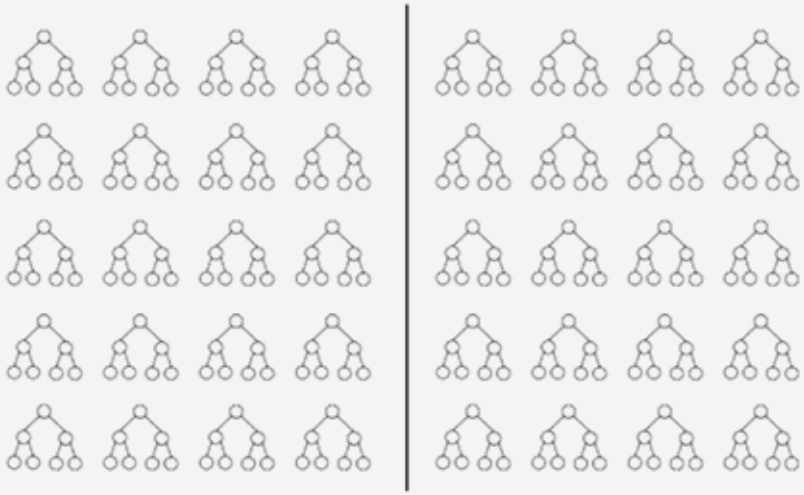


Joint Equilibrium Search for Policies

- Approximate solution approach for general models with a finite horizon h
  - Input:
    - DecPOMDP $(I, S, \{A_i\}_{i \in I}, \{O_i\}_{i \in I}, T, R, \Omega)$
    - Horizon h
    - Possibly error margin ε

```
JESP(dec-pomdp, h)
    while not converged do
        for i = 1 to n do
            Fix other agent policies
            Find a best response policy for agent i
```

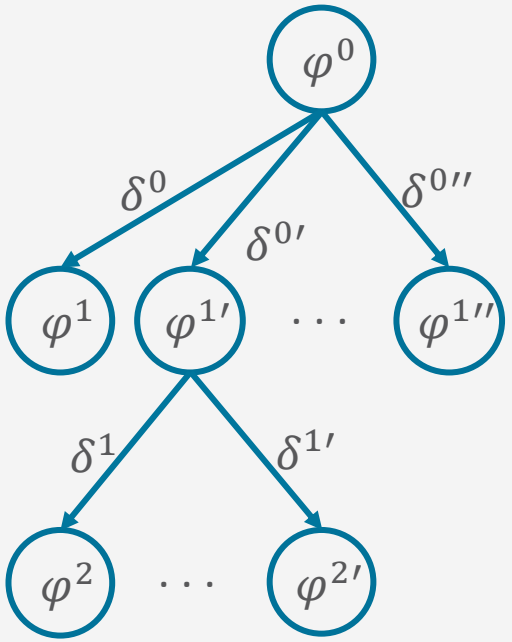
Turns DecPOMDP into a POMDP for i

- Instead of exhaustive search, find best response
  - Nash equilibrium: no agent has incentive to change its policy if no other agent changes its policy
  - Convergence criterion needed
    - E.g., no change (or only ε change) in any policy
  - Same worst-case complexity, but in practice much faster
    - Can include pruning, further heuristics when looking for best response policy

Multi-agent A* (MAA*)

- Optimal solution approach for general models with a finite horizon h
  - Inputs:
    - DecPOMDP $(I, S, \{A_i\}_{i\in I}, \{O_i\}_{i\in I}, P_{tr}, R, P_{obs})$
    - Horizon h
    - Heuristics $\hat{V}(\varphi^t)$
- A*-like search over partially specified joint policies
  - $\varphi^t = (\delta^0, \dots, \delta^{t-1})$
  - $\delta^t = (\delta_0^t, \dots, \delta_n^t)$
  - $\delta_i^t : \vec{O}_i^t \rightarrow A_i$
- Requires an admissible heuristic function $\hat{V}(\varphi^t)$

$$\underbrace{\hat{V}(\varphi^t)}_{F} = \underbrace{V^{0\dots t-1}(\varphi^t)}_{G} + \underbrace{\hat{V}^{t\dots h-1}(\varphi^t)}_{H}$$



How to Get a Heuristic Function?

- Solve simplified settings, e.g.,
  - Solve the underlying MDP (approximately or optimally) given assumptions:
    - Centralised observations
    - Full observability
      - Simulate / sample unobserved values
  - Solve a belief MDP given assumption
    - Centralised observations
- Domain-specific heuristics

Memory Bounded Search

- Approximate solution approach for general models with a finite horizon h
  - Inputs:
    - DecPOMDP $(I, S, \{A_i\}_{i \in I}, \{O_i\}_{i \in I}, T, R, \Omega)$
    - Horizon h
- Do not keep all policies at each step but a fixed number for each agent $maxTrees$
  - Select $maxTrees$ in a way that $maxTrees \cdot |I|$ trees fit into memory
    - Can be difficult to choose; often small in practice
  - Select trees by using heuristic (like A*)

```
MBDP(dec-pomdp, h)
    Start with a one-step policy for each agent
    for t = h downto 1 do
        Backup each agent's policy
        for k = 1 to maxTrees do
            Compute heuristic policy and resulting
                    belief state b
            Choose best set of trees starting at b
    Select best set of trees for initial state b0
```

MBDP =
Memory
Bounded
Dynamic
Programming

Infinite Horizon

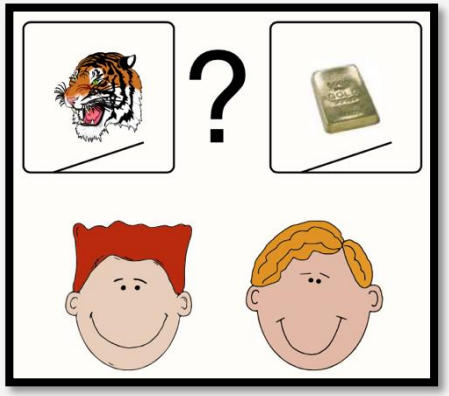
- Approximate using a large enough horizon h
  - Neither efficient, nor compact
- Selection of solution approaches based on solution approaches already seen for MDPs / POMDPs:
  - Policy iteration
    - Start with one-step plans, extend further
    - Automata-based approaches (Moore/Mealy automata to represent policy)
    - Intractable for all but the smallest problems
  - Best-first search
    - Finds optimal fixed-size solutions; use start state info
    - High search time → small sizes only
- Further solution approaches use non-linear programming

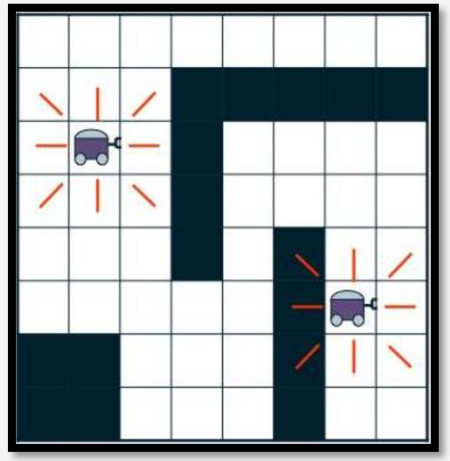
Indefinite Horizon

- Many natural problems terminate after a goal is reached
  - Meeting or catching a target
  - Cooperatively completing a task
- Unclear how many steps are needed until termination
- Under certain assumptions can produce an optimal solution
  - E.g., terminal actions and negative rewards
    - Such as the 4x3 grid:
      terminal states, negative rewards for all but one terminal state
- Otherwise, can bound the solution quality by sampling

Benchmark Problems

- *DEC-Tiger*
  - (Nair et al., 2003)
- BroadcastChannel
  - (Hansen et al., 2004)
- *Meeting on a grid*
  - (Bernstein et al., 2005)
- Cooperative Box Pushing
  - (Seuken and Zilberstein, 2007a)
- Recycling Robots
  - (Amato et al., 2007)
- FireFighting
  - (Oliehoek et al., 2008b)
- Sensor network problems
  - (Nair et al., 2005; Kumar and Zilberstein, 2009a,b)





Software for Dec-POMDPs

- The *MADP toolbox* aims to provide a software platform for research in decision-theoretic multiagent planning
  (Spaan and Oliehoek, 2008)
- Main features:
  - Uniform representation for several popular multiagent models
  - Parser for a file format for discrete Dec-POMDPs
  - Shared functionality for planning algorithms
  - Implementation of several Dec-POMDP planners
- Released as free software, with special attention to the extensibility of the toolbox
- Provides benchmark problems
  - Such as on the previous slide

```
agents: 2
discount: 1
values: reward
states: tiger-left tiger-right
start:
uniform
actions:
listen open-left open-right
listen open-left open-right
observations:
hear-left hear-right
hear-left hear-right
```

```
#include "ProblemDecTiger.h"
#include "JESPExhaustivePlanner.h"
int main()
{
    ProblemDecTiger dectiger;
    JESPExhaustivePlanner jesp(3,&dectiger);
    jesp.Plan();
    std::cout
         << jesp.GetExpectedReward()
                 << std::endl;
    std::cout
         << jesp.GetJointPolicy()->SoftPrint()
                 << std::endl;
    return(0);
}
```

```
# Transitions
T: * :
uniform
T: listen listen :
identity
# Observations
O: * :
uniform
O: listen listen : tiger-left : hear-left hear-left : 0.722
O: listen listen : tiger-left : hear-left hear-right : 0.1275
[...]
O: listen listen : tiger-right : hear-left hear-left : 0.0225
# Rewards
R: listen listen : * : * : * : -2
R: open-left open-left : tiger-left : * : * : -50
[...]
R: open-left listen: tiger-right : * : * : 9
```

Dec-Tiger Problem Specification and Program

Interim Summary

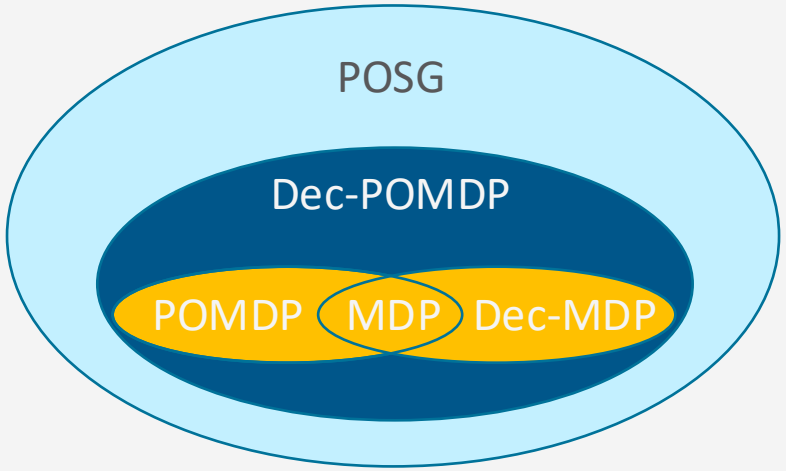
- Dec-POMDPs
  - Local policies, joint policy, value functions
  - Communication, full observability, Dec-MDP
- Solutions for
  - Finite horizon
  - Infinite horizon
  - Indefinite horizon
- MADP toolbox
  - Benchmark problems

Hierarchy of Formalisms

- Most general: *POSG*
  - Set of agents, individual reward functions
  - Environment only partially observable
- Specifications
  1. Decentralisation
     - Joint reward function

  2a. Observable environment

  2b. Multi to single agent
- Most specific: *MDP*
  - One agent, (therefore) one reward function
  - Observable environment



Modelling without any structure
in the agent set or state space
→ potential for efficiency wasted

Outline: Decision Making – Extensions

Partially Observable Markov Decision Process (POMDP)

- POMDP agent, belief state, belief MDP
- Conditional plans, value iteration

Decentralised POMDP (Dec-POMDP)

- Dec-POMDP, local policy, joint policy, value function
- Communication, full observability, Dec-MDP
- Solutions for finite, infinite, indefinite horizon

$\Longrightarrow$ Next: Decision Making – Structure