### Report: Sketch Map Representation and Alignment Methods

An analysis of qualitative spatial calculi suitable for representation of sketch maps for alignment with metric maps and other sketch maps.

Version 0.1

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#### 1 Graph Based Representations for Sketch Map Alignment

#### 1.1 Graphs

Graphs are convenient representational structures for many spatial representations. Graphs are composed of nodes and edges connecting those nodes. An edge may have a given direction in which case one of the nodes incident on the edge is the source node and the other the destination node. Such edges are called directed edges and graphs containing directed edges are called directed graphs. There are many classes and types of graphs that are distinguished by among other things, type of edges (directed or undirected), the connectivity of the graph (connected vs disconnected), the distribution and number of edges (complete vs incomplete), presence of cycles (trees, acyclic graphs vs cyclic graphs), presence of metadata for nodes and/or edges (weighted, labelled vs unweighted, unlabelled), etc. While in the general case many computations over graphs have exponential complexity in the number of graphs nodes, specialized notions of the graph restricted by any of the above distinctions or others have enabled researchers to develop efficient methods for performing some of the hard computations (graph-subgraph isomorphism, maximum match detection, clique enumeration etc.) in polynomial time.

#### 1.2 Application in Sketch Map Alignment

Graphs are used to represent constraint networks in qualitative spatial, temporal, and spatio-temporal reasoning. They have also been used to represent network structures such as street, road, or river networks. Adjacency networks represent the adjacency relations among spatial entities in 2D or higher dimension spaces – they are also graphs. Conceptual graphs can be constructed from the semantic information about spatial entities and their relations.

The discussion in this section will be focus on the computations of interest to the sketch alignment task and each of the representations above will be evaluated within the context one or more of these computations. The main computations on graphs with respect to sketch map alignment are graph matching (graph and subgraph isomorphism, inexact graph matching, and constrained graph matching) and graph based constraint satisfaction.

#### 1.2.1 Graph Matching

Graph Matching is the process by which the structure and semantics of two (or more) graphs are compared to determine whether the graphs are same. In the base case, an algorithm will try to determine the corresponding nodes and edges from the input graphs so that every node (viz-a-viz edge) in one graph has exactly one corresponding node (viz. edge) in the other – known as exact matching. Partial mappings are also considered where one graph is bigger than the other. However this strict definition of the task is not helpful in many applications including sketch map alignment so more generalized and restricted versions of the graph matching problem have been proposed. Graph and subgraph isomorphism correspond to the based case of exact graph matching. Another class of methods of interest here are those designed to find some similarity between the two graphs. In these methods, known as inexact graph matching, the restriction that there must be a 1-to-1 correspondence is lifted and only that there is substantial similarity between the graphs. This allows for errors to be taken into account.

Graph matching can be used in two separate scenarios. First, it can be used to match graphs that are directly derived from the positions of objects in the sketch map and the metric map, resulting in a structural alignment. Three examples of such graphs are street networks, adjacency graphs, and conceptual graphs. Second, it can be used to find similar spatial scenes based on the similarity of their qualitative constraint networks (or parts of them). This type of application of graph matching is more complex as it requires the detection of inconsistent matches in the context of the underlying qualitative calculus for each possible way of matching the two graphs.

#### 1.2.1.1 Graph matching for street network alignment

To represent street networks, there are three possible representations we can consider:

#### The street network graph

The street network can be represented by a graph G defined in the standard was as follows. Let  $G = (V, E, L_V, L_E, \mu, \nu)$  be a graph where V is a set of nodes,  $E \subseteq (V \times V)$  is a set of edges incident to the nodes,  $L_V$  is an alphabet of node labels,  $L_E$  is an alphabet of edge labels,  $\mu: V \to L_V$  is a mapping that assigns labels to nodes, and  $\nu: E \to L_E$  is a mapping that assigns labels to edges. Let R be the set of named geometric objects representing street segments of the street network and I the set of points representing end-points of the street segments. Then the graph I is a representation of the street network if there exists a mapping I is I is an alphabet of edges. We assume here that the edge labels are the same as the street names but this need not be the case since not all street names are known and will be assigned the null label.

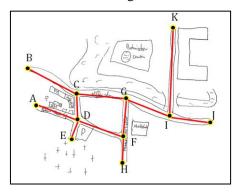


Figure 1: Graph representation of street network

```
V = \{A, B, C, D, E, F, G, H, I, J, K\}

E = \{(A, D), (B, C), (C, D), (C, G), (D, E), (D, F), (G, F), (F, H), (G, I), (I, J), (I, K)\}
```

Given two graph representations  $G_1$  and  $G_2$  of street networks from a pair of maps, the task of aligning them involves finding a map (bijective function) from one graph (typically the smaller one) to other such that edge and node adjacencies and (if possible) node cardinalities are preserved and label assignments are consistent. This means that the two networks have equivalent topologies. This is achievable using approaches for finding graph/subgraph isomorphism. For finding an isomorphism between the graphs we first can compute the graph dissimilarity based on the graphs structural properties. This enables us to eliminate very dissimilar graphs. Gärtner et al. (Gärtner, Flach, and Wrobel 2003) showed that dissimilarity computation based on the number of matching random walks in the two graphs can be performed using the product of the graphs without actually enumerating the walks. Other dissimilarity measures that attempt to detect graph or subgraph isomorphism without finding it can also be used. Once dissimilarity is estimated a choice can then be made on whether to proceed with actually finding the isomorphisms between the two graphs. Graph and subgraph isomorphism can be found using general or specialized methods. General methods can be applied on both directed and undirected graphs as well as on both labelled and unlabelled graphs. Specialized methods apply to graphs that are constrained in some way for example allowing only labelled edges or nodes. The problem sketched so far corresponds to exact graph matching. But knowing that sketch maps are often less detailed than cartographic maps or official spatial databases and that different sketch maps include and omit different street segments, a more useful way to formulate that matching problem is as an inexact graph matching problem. There are 3 scenarios that we are interested in:

- 1.  $G_1$  represents a street network from a sketch map and  $G_2$  represents a street network from a metric map,
- 2. Both  $G_1$  and  $G_2$  represent street networks from individual sketch maps, and
- 3.  $G_1$  represents a street network from a sketch map and  $G_2$  represents the results of aligning street networks from several sketch maps.

If  $G_2$  represents a street network from a metric map then we expect  $G_2$  to be more detailed than  $G_1$ . Consequently, there are edges (street segments) in  $G_2$  that are not present in  $G_1$ . Conversely there may possibly be paths (which will be interpreted as streets) that are not officially considered as streets and therefore not part of the metric map. These additional/missing street segments will have consequences on the structure of the resulting graph. Assuming that there is a partial match between graphs of two street network representations of the same street network, there are two main occurrences that may cause the incompleteness: an incoming street segment at a 4-way (4+) junction is displaced thereby creating two t-junctions, or a junction is omitted ( see Figure 2). From graph matching theory the consequence of an additional edge can be expressed in terms of the edit distance from the subgraph containing the additional edge and all edges directly connected to it, to the matching subgraph in the other graph.

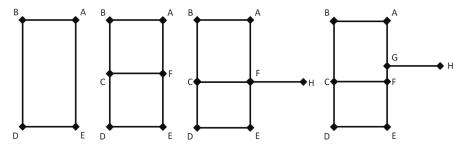


Figure 2. Effects of missing edges in one of the matched subgraphs. To match (a) to (b) requires introducing three edges and two nodes while matching (b) to (c) requires introducing only one edge and one node. Similarly to match (c) to (d) an edge

The **graph edit distance** of two graphs  $G_1$  and  $G_2$  is the minimum cost of transforming one graph into the other. There are 3 traditional operations that can be applied to nodes and edges during the transformation, namely insert, delete, and substitute. But other operations have also been suggested which include node merge (merge two nodes) and node split (split a node into two separate nodes). The latter two can also be expressed in terms of a sequence of the inserts, substitutes, and deletes. A sequence of edit operations is known as an **edit path** from  $G_1$  to  $G_2$  if it transforms  $G_1$  into  $G_2$ . The edit distance of  $G_1$  and  $G_2$  is thus given by the lowest cost edit path where the cost of an edit path is the sum of the costs of its individual operations. Figure 3 shows a transformation of graph (c) from Figure 2 to graph (d) of the same figure and the edit operations performed.

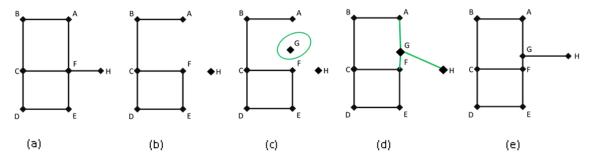


Figure 3. Edit path from graph (c) from Figure 2 to graph (d) of the same Figure 2. The operations on this edit path are del(AF), del(FH), ins(G), ins(AG), ins(GF), ins(GH).

In summary, inexact graph matching, although still computationally expensive, is more suitable for street graph alignment than exact graph matching. The main advantage is that errors can be tolerated and the best match out of several can be chosen based on other criteria e.g. matching adjacent landmarks.

#### Advantages

• Graph representations of the street network are easy to derive,

- There are already many matching algorithms for performing alignment within reasonable time bounds.
- The definition of nodes and edges is general and they also allow the incorporation of metadata as labels and weights making them highly customizable,
- They can represent networks at different complexities and with a large number of possible restrictions making them both versatile and hard to compute with. For example we can limit the cardinality of nodes to numbers other than 2 ensuring that we do not take into account corners where only two street segments meet. This may in turn help to better decide appropriate edit costs for computing graph edit distances.

#### Disadvantages

- Relative positions cannot are not directly preserved and limited only to the interpretation assumed. For example relative and even absolute orientations have to be maintained as metadata of some sort and are not explicitly expressed by the graph itself,
- As a result of the first disadvantage, detecting inconsistent scenes can be difficult since additional reasoning mechanisms must be employed. E.g. the maximum common subgraph (MCS) of two graphs represents a consistent spatial configuration since it is part of actual/original data but together with the other parts (not part of the MCS) the resulting configuration may in fact not be realizable in the plane,
- While inexact matching enables error tolerance, conceptual neighborhood reasoning is not possible. Conceptual neighborhood reasoning is allows for cognitively supported error mitigation strategies i.e. some errors are more severe than others from a cognitive standpoint.

#### 1.2.1.2 The conceptual graph of street network elements

Kopcynski and Sester employed the conceptual graph representation of the content of sketch maps. Because their human computer interface was a digital touch screen they were able to force users to provide compatible semantic information about the objects in the sketch maps. The conceptual graph introduced discussed at length in (Sowa 2008) allow them to represent both spatial entities and spatial relations with the same representational primitive — a graph node. A node in the conceptual graph represents a concept while an edge represents a relation between the concepts whose nodes are incident to it (Figure 5). Although only concepts relating to the street network representation are being considered here, the original representation of Kopczynski and Sester considers all sketch objects employing a large corpus of spatial and semantic concepts and relations.

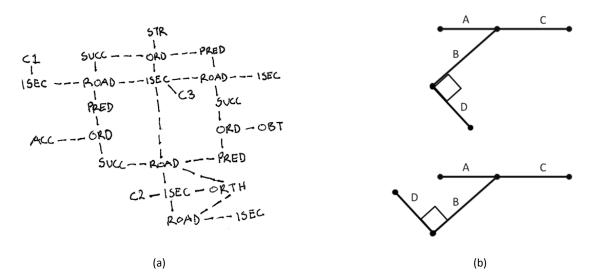


Figure 4. Conceptual graph (a) of at least two street network configurations. B and D are the road objects connected with an ORTH intersection, A and C, A and B, and B and C are connected with STR, ACC, and OBT intersections respectively.

Two primary concepts are ROAD for road segments and ISEC for their start and end points. An ISEC can have a relation with a class type representing its cardinality C1, C2, C3.... Subsequent road segments are can be related to a conceptual class describing their angular relation – STR for straight, ORTH for orthogonal, OBT for obtuse and, ACC for acute. For a many street junction (3+), a relation ORD is used together with sequential relations PRED for predecessor and SUCC for successor to describe the cyclic order of the road segments. With this representation, graph matching algorithms notably the VF-algorithm of Cordella et al. (Cordella et al. 1999) are used to find a subgraph isomorphism between the sketch map CG and the metric map CG. VF is an exact graph matching algorithm (see previous section above) but inexact graph matching methods can be employed as well. In the case of inexact graph matching, the resulting conceptual graph for the matched concepts must be verified for consistency with both the metric map and the sketch map.

#### Advantages

- Conceptual graphs are very expressive (formalized in first order logic) such that a great many assertions can be expressed using them,
- Matching of conceptual graphs proposed by the authors uses traditional graph matching methods so even inexact matching is possible.

#### Disadvantages

- High expressiveness implies hard computability so it is challenging in terms of both implementation and optimization,
- Conceptual neighbourhood reasoning can be employed but it is not clear how since a host of relation types are embedded in a single representation,
- It requires explicit knowledge of concepts referring to the objects in the sketch map or metric map,
- Some scenarios cannot be distinguished by a limited vocabulary and the result may be that one needs to keep adding new concepts and relations in order to distinguish unforeseen scenarios or develop a large theory in the first place. This may be because there is no specification of the granularity and scope (what entities to consider) of the representation.

#### 1.2.1.3 The Dipoles Relation Algebra (DRA)

The Dipole Relation Algebra (DRA) was presented in (Moratz, Renz, and Wolter 2000) to represent relative positions of oriented line segments. The oriented line segments are also known as dipoles. A dipole is an ordered pair of points in  $\mathbb{R}^2$  which can be written as  $\mathbf{a} = (a_s, a_e)$ , where  $a_s$  and  $a_e$  are the start- and end-point of  $\mathbf{a}$  respectively. A basic DRA relation between two dipoles A and B is represented by a 4-tuple of facts  $s_B e_B s_A e_A$  where  $s_B$  is the position of the start-point of dipole B with respect to dipole A. The other three elements of the relation  $e_B$ ,  $s_A$ , and  $e_A$  are defined analogously. The full DRA,  $DRA_{69}$ , with 69 relations embeds topology, ordering, and orientation information. The possible positions of the start-/end-point of one dipole with respect to another dipole are s (coincides with the start-point), s (coincides with the end-point), s (is on the straight line through the dipole and between the start-point and end-point), s (is on the straight line through the dipole and in front of the end-point), s (is on the left side of the dipole), and s (is on the right side of the dipole).

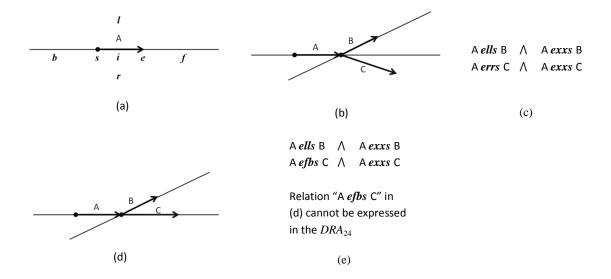


Figure 5. Relations of DRA at three different granularities.

 $DRA_{24}$  strips out the three dipole-point relations b, i, f and with them ordering information equivalent to Allen's Interval Algebra. However to is still possible to represent orientation information using the dipole algebra's l and r dipole-point relations. To capture the topology of the street network it is enough to use a further coarsened subset,  $DRA_7$ . For  $DRA_7$  the possible positions of the start-/end-point of one dipole with respect to another dipole are s (coincides with the start-point), e (coincides with the end-point), and e (coincides with neither start-point nor end-point). In the proposed representations, the start- and end-points of street segments are used to define the dipoles. Together the  $DRA_7$  basic relations capture the connectivity information of street segments and therefore the topology of the street network in sketch maps.

For example in Figure 1 the junctions B, C, D, E, and G define the dipoles BC, CB, GC, CG, DC, CD, DE, and ED. The relations for (BC, CG), (CG, BC), (CB, CG), and (BC, DE) are exxs, xsex, xsex, and xxxx respectively. Note that in this example the dipole relations can be derived directly from the labelling of junctions. The seven basic relations of  $DRA_7$  include the four listed above, sese, eses, and sexe.

As demonstrated above, DRA can capture many aspects of the street network at different levels of granularity. Course representations can be derived from more detailed representations. But the usefulness of additional more precise knowledge such as front, inside, behind is questionable since it is unlikely that subsequent street segments will be at exactly 180° to each other. But the inside relation is required when attempting to introduce a new dipole that starts/ends inside the referent and thus indicates that the referent should be split into two dipoles which both meet at the intersection point. This is important with respect to missing/additional street segments during alignment. To align the DRA representation of the sketch map with the metric map representation the constraint networks of the two representations are matched using graph matching methods (see section 2.1.2 below).

#### Advantages

- DRA can represent spatial configuration of the street network at different granularities at least 69, 24, 7 relations and coarse representations can be derived from more granular representations,
- The reasoning mechanisms are already implemented in general constraint reasoning systems such as SparQ,
- For the lower granularities, there is a limited number of distinctions to be made although  $DRA_{69}$  has many relations,
- For simple scenarios with few dipoles involved (e.g. less than 6) matching can be achieved very fast,

- Conceptual neighbourhood reasoning can be employed to allow error tolerant matching.
- It is possible to infer aggregated street segments by inspecting dipole-point relations.

#### Disadvantages

- Matching complexity is increased due to the additional inconsistency checking required for the matching,
- Specialized methods may be required to reason about junction information and possibly other things. For example, in Figure 6(d) the  $DRA_{69}$  relation A *efbs* C cannot be expressed in  $DRA_{24}$  although the latter maybe more suitable for reasoning about junctions relations.

#### 1.2.1.4 Graph matching for qualitative constraint networks

Matching two qualitative constraint networks (QCNs) involves matching the graph representations of the two networks. The standard way of doing this has been to construct an association graph of the two input graphs being matched. The association graph is constructed by inserting a node for every possible pair of nodes such that each node in the pair is from a different graph resulting in  $m \times n$ nodes where the two input graphs have m and n nodes respectively (See Figure 7). For each pair of nodes of the association graph, an edge is inserted between them if the constraints between the nodes of the input graphs match and none of the input graph nodes are identical. Matching the QCNs then reduces to finding maximal cliques (maximal complete subgraphs) of the association graph. The largest such clique if it exists is the maximum clique. A different way to do this matching is to use the interpretation tree approach from computer vision. The interpretation tree is a representation of the association graph in which each node at level i is an extension of the solution at level i-1. The full interpretation tree has a depth of n and at each level between 1 and n every node has m children. The width of the tree is therefore  $m \times n$  and has  $m^n + 1$  node. Each path from the root of the tree to a node at level i represents nodes of the association graph that are together a possible solution to the matching problem. In order to reduce the search space, for both clique enumeration and interpretation tree approaches, a node should be added to an accepted partial solution if including it does not lead to an inconsistent solution.

For identical spatial scenes, their QCNs will be identical. As such a complete solution will be obtained whenever clique enumeration or interpretation tree search is performed. However, this is not always the case and therefore relaxation mechanisms need to be applied. Relaxations can be applied to mitigate errors of position, incompatible object compositions, and additional/missing/spurious objects.

#### **Errors of position**

Errors of position correspond to extreme mis-positioning of a known object with respect to other known objects. They are common in sketch maps and have been associated with cognitive distortions and schematization. In terms of qualitative spatial representations these errors correspond to a change in the qualitative relations of the erroneously positioned object and the objects with respect to which it has been erroneously positioned. If only one object in one set is affected by a position error then during the matching only this object will be left out of the solution. On the other hand, if several objects are affected, then these objects will together be left out of one solution but may individually or in conjunction contribute to other wrong solutions (see Figure 8). If there are errors in the representation to which the matching is being performed then these errors may also give rise to wrong solutions.

Positional errors can be remedied during matching in part by considering the conceptual neighbourhood of the relations involved. The conceptual neighbourhood of a relation is that set of relations reachable through some spatial transformation without passing through any other relation in the process. However, the conceptual neighbourhood of a relation may consist of several relations. This means that whenever we are performing a relaxation we expand the search space by some factor of the size of the neighbourhood.

#### **Incompatible object compositions**

Errors of position are not always a result of cognitive distortions or schematization. They can also result from the operations at the conceptual level. The notion of object composition here refers to the process by which objects represented in the sketch map are constructed from parts that are other, smaller or otherwise less significant, objects. Some authors have categorized this notion as a particular form of abstraction. Timpf (Timpf 1998) has termed it aggregation and defines it in terms of a spatial information hierarchy where objects higher up the hierarchy are composed of objects directly below them (children). From this perspective, there are several questions we may wish to answer: what are the lowest level features of the world from which sketch objects can be composed? How many hierarchies can be constructed from a given spatial scene? What spatial constraints must be satisfied between elements of the hierarchy?

Another definition of abstraction is based on information reduction processes over sets of feature vectors. This type of abstraction operates on the spatial relations as opposed to spatial objects. An abstraction in this sense is a non-injective function  $K: S \to T$  that maps the feature space S into a smaller space T. We can attempt to model the problem of object composition as an aggregation with a family of constraint preserving functions  $K_{i,j}$  that map i-tuples of objects to j-tuples of objects (j < i) such that reduction from i to j objects is the result of substituting i-(j+1) objects with their sum. The functions  $K_{i,j}$  must be consistency preserving functions in the sense that the constraints of the sum must be consistent with the constraints of the summands.

#### Additional, missing, spurious objects

Additional, missing, and spurious objects may be the result of perceptual errors (seeing a mirage), errors in memory, or differences in object compositions, or indeed other causes. Differences in the two spatial scenes arising from additional, missing, or spurious objects can be quantified as that portion of the dissimilarity of the two scenes due to the presence of the object in question. If the dissimilarity of the spatial scenes cannot be accounted for by the approaches given above, then this measure of dissimilarity can be used to decide if ignoring this object is more reasonable than taking it into account.

#### 2 Orientation Information of Street Segment

Most approaches to qualitatively representing and reasoning about orientation information deals with points and line segments (generated from set of points as the basic entities) Orientation information of street segments in sketch map is used to identify the orientation of connected segments.

#### 2.1 Street Segment as Reference Object

In sketch maps most of the street networks are represented as set junctions and street segments. Where junctions are represented as vertices  $(V_i)$  and street segments as edges  $(S_i)$ .

Suggested calculi for the Orientation information of Street Segments:

#### 2.1.1 Dipole Relation Algebra (DRA)

The qualitative spatial calculi are dealing with two directed line segments, which are known as diploes. Each dipole is defined by two points, the start point sA and end point eA. The initial DRA calculus introduced by (Schlieder 1995) deals with the orientation of two dipoles as set or basic relations like lrrr, llll or rrrr etc. used to indicate basic orientation relations between given dipoles. Moratz et al. in (Moratz, Renz, and Wolter 2000) extended DRA by introducing six basic relations known as DRAc like eses, elrs, etc. These relations are used to identify orientation (either line segment is connected from left-side or right-side) as well as connectivity (segments have common junction) between given dipoles. In Sketch map, we are interested to identify the orientation as well as connectivity of the road segments. The most suitable version of Dipole relation algebra is DRA-24, If we want to find out the angle spanned by given dipole then we can use extended version known as DRAfp introduced by (Dylla and Moratz 2005). They introduced parallelism (P), anti-parallelism, mathematical angle (negative (-) or positive (+) to represent angel between dipoles with relation like llrrP, llrr+, seseP, rrrrA etc. At the moment SparQ support on DRAc which is known as DRA-24.

#### Advantages

- Coarser version of Dipole Relation Algebra (DRA<sub>7</sub>) is provides only connectivity of street segments exxs, xexe, and xxxx for disconnected street segments.
- The reasoning mechanisms can easily implemented in general constraint reasoning systems such as SparQ
- DRAc (DRA-24) contains set of relations that provides information about connectivity as well as orientation information of line segments like sese, slsr, and lere relations.
- For the lower granularities, there is a limited number of distinctions to be made although  $DRA_{69}$  has many relations.

#### Disadvantages

- DRA<sub>7</sub> does not provide orientation information of connected street segments.
- DRA<sub>c(24)</sub> does not specify the angle of connectivity between given dipoles. It provides basic orientation information that dipole is either connected from left-side or right-side.
- The reasoning mechanism for DRA<sub>cf(69)</sub> is not implemented in general constraint reasoning systems such as SparQ.
- All Dipole Relation Algebra (DRA) representations require precise geometric information to define orientation relations such as front and back. In freehand sketch maps, it is not possible to fine precise geometric information

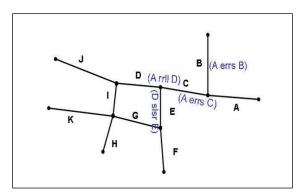


Figure 6. Orientation and connectivity information of street-segments using DRA24.

# **2.1.2 Single Cross Calculus (SCC)-Orientation information between Points-** (Freksa 1992).

The single cross calculus is a ternary calculus that de-scribes the direction of a *point C* (the referent) with respect to a *point B* (the relatum) as seen from a third *point A* (the origin). The plane is partitioned into regions by the line going through A and B and the perpendicular at B. This results in eight possible directions for C (see Figure 10)

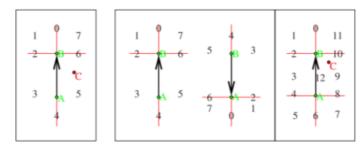


Figure 7. orientation information of junction using the SCC and DCC.

#### Advantages

- SCC provides the orientation information of points with respect to oriented line deduced by two ordered points
- Two order set of points can share one position without losing granularity. There for we can find orientation of "BC w.r.t AB" or vice versa
- Suitable for street-segment use as reference where we use oriented street segment as an edge between two junctions.
- The reasoning mechanism for SCC is not implemented in general constraint reasoning systems such as SparQ.
- SCC is ternary calculi and it need at least three spatial objects to capture qualitative knowledge

#### Disadvantages

- Representation has fixed granularity.
- SCC handles points as primitive entities
- Need precise geometric information to define front and back relations.
- Also need precise geometric information for the relation with respect to perpendicular line which partitions the plane in SCC.
- Not-suitable, if we consider reference street segment as an extended object.
- Due to fixed granularity minor distortions of angles at junctions leads to different relations.

Example: orientation information of Junction using DCC

Let suppose we have road junction A, B, C and D, the Single Cross calculus is used to extract

qualitative spatial relations between given objects using one junction as frame of reference. The order set of points A and B generate oriented line segment AB. By using line AB as frame of reference we can deduce possible qualitative relation of junction C. In this example C is in the relation (1) which indicates *right-front* portion of plane produced by using Single cross calculus. Similarly the position of junction A w.r.t BC is in relation (3) which is basically *right-back*.

# **2.1.3 Double Cross Calculus (DCC)-Orientation information between Points -**(Freksa 1992).

The double cross calculus can be seen as an extension of the single cross calculus adding another perpendicular. The relations are interpreted as the combination of two single cross relations. DCC first describes the position of C with respect to B as seen from A and the second with respect to A as seen from B.

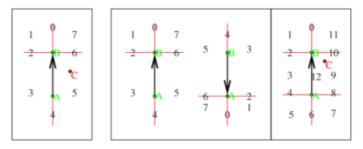


Figure 8. The orientation information of streets using the DCC.

#### Advantages

- Suitable for orientation information of street junctions based on local frame of reference.
- The double cross calculus takes the junctions of street segment as relatum and origin to capture orientation of junctions of other connected street segments.
- SCC is ternary calculi and it needs at least three spatial objects to capture qualitative knowledge. For the orientation of street segment we also need relatum and origin as together as reference object.
- The reasoning mechanism for DCC is implemented in general constraint reasoning systems such as SparQ (Wallgrun Oliver et al. 2007).
- DCC handles points as primitive entities. In street segment, junctions are also point entities

#### Disadvantages

- DCC has fixed granularity.
- Due to fixed granularity minor distortions of angles at junctions leads to different relations.
- Need precise geometric information to define front and back relations.
- Also need precise geometric information for the relation with respect to perpendicular lines which partitions the plane in DCC.
- Not-suitable, if we consider reference street segment as an extended object.

#### **2.1.4** FlipFlop (FFC)-Orientation information between Points – (Ligozat 1993).

The Flipflop calculus can be seen as an coarser representation of the SCC without perpendicular line. It describes the position of C with respect to B as seen from A.

#### Advantages

- FFC provides the orientation information of points with respect to oriented line deduced by two ordered points
- Two order set of points can share one position without losing granularity. There for we can find orientation of "BC w.r.t AB" or vice-versa

- Suitable for street-segment use as reference where we use oriented street segment as an edge between two junctions.
- The reasoning mechanism for FCC is not implemented in general constraint reasoning systems such as SparQ.
- FCC is ternary calculi and it need at least three spatial objects to capture qualitative knowledge

#### Disadvantages

- Representation has fixed granularity.
- FCC handles points as primitive entities
- Need precise geometric information to define front and back relations.
- Not-suitable, if we consider reference street segment as an extended object.
- Due to fixed granularity minor distortions of angles at junctions leads to different relations.

#### 2.1.5 Reasoning about ordering based on clockwise and counter-clockwise orientation of triple of points- (Schlieder 1995).

Schlieder, C. proposed ordering information of points in the plane. He introduced three basic relations negative (-), positive (+) and zero (0) to represent orientation of triple of points in the space. The line segment made by two points Pi and Pi is oriented positively iff the path from Pi, Pi follows the positive direction of line and from orientation information of points one can easily obtains ordering relation on the points because Pi<Pj.

#### # Advantage

#### Disadvantage

Useful for junctions and landmarks as point

ordering information w.r.t Does not clearly define connectivity of line segment.

If points are collinear the orientation of given It is not implemented in SparQ points will be indicated as zero (0)

Identify conceptual neighborhood relations.

Example: Ordering information and orientation of points

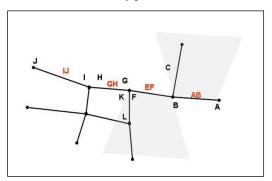


Figure 9. The orientation information of line segment using the Schlieder's calculus.

Let's suppose we have two line segments AB and CD with starting points and ending points.

The possible qualitative relations between these two like segments can be AB=+, BC=0, CD=+ It indicates that point A is before B, B and C are at the same position, point C is before D. The triangle made by these points can be used to identify orientation of line segment. For example Clockwise [ABC] = - as C lies on the right side of line AB and Counterclockwise [EFL] = + as point L lies on the left side of line segment EF.

#### **2.1.6** Cardinal Direction defined by four half-plane with natural zone - (Frank 1996).

Frank, A., 1992 introduced two methods to partition the space for orientation of objects w.r.t. each

other, which are called the projection based model and cone-based model of cardinal directions with neutral zone. The projection based cardinal direction is used to present object related to the earth. The neutral zone region is import in the situation where one cannot decide that the given point lie in which four regions.

#### # Advantages

1 Cardinal direction with neutral zone provides 9 possible cardinal direction of line segment

Neutral zone provide solution to choose appropriate cardinal direction of objects, it support to adjust reference object in neutral zone.

#### **Disadvantages**

It can't provides connectivity information of line segment together with orientation like DRA does

Size of neutral zone must be consistent, as it size effect the directional values assign to object.

Most of sketcher use local frame of reference for sketching. But projected base cardinal direct is used for reasoning on cardinal direction w.r.t global frame of reference. (Note: have doubt)

Example: Cardinal direction defined by half plane with neutral zone

To demonstrate cardinal direction on sketch data, I consider same sketch road network and applied cardinal direction projection based mode with neutral zero.

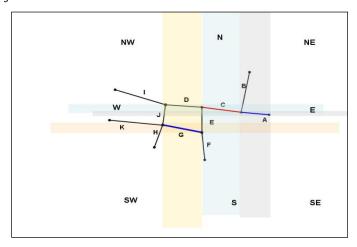


Figure 10. The cardinal directions of objects with respect to street segments using neutral zone.

Let suppose, we have line segment A in the neutral zone, the cardinal direction of remaining line segments with respect to A can be: (A ne B), (A nw C)(A nw, w, sw E)(A nw W) and so on.

Similarly: if we consider line segment (G) at neutral zone then the cardinal relations of other line segments with respect to G can be: (G ne A), (G ne B), (G ne C), (G ne E), (G n D), (G se F), (G w K), (G w H) and so on. The cardinal direction calculus provides possible cardinal information of line segments w.r.t each other.

#### **2.1.7** Egocentric reference frame with intrinsic front -(Mukerjee and Joe 1990).

Mukerjee et al., (1990) tried to focus on the concept of tangency, where the given objects are aligned at some face, line or at point. They considered one-dimensional relations between 1D point (A) and interval (B), between intervals and between Points and consider Allen time interval relations to identify possible ordering information.

In qualitative model of space they consider objects with arbitrary angle defined as intrinsic front. It

enables user to identify several qualitative different zones like front, left, right and back.

#### **Collision Parallelogram concept**

In linear case Mukerjee and Joe considered object as two end lines instead of intervals. When object is moving along travel of line the intersection at an angle will form parallegorm which is known as collision parallelogram (CP). The defined object has three qualitative relations with respect to CP. That can be before (-), inside (i), back (b), front (f), after (+). To find out possible orientation information of given object needs to find out quadrant information w.r.t reference object. For position information we need to consider CP.

For example, Pos (A/B) by considering CP between these two object. Dir (A/B) by considering direction of first object w.r.t quadrant generated by second object with the help of its intrinsic front.

#### # Advantage

# 1 Orientation information, for multidimensional objects with defined intrinsic front

It uses intrinsic front of the object to generate possible quadrant around the object to find out possible orientation information.

It is good to preserve the information of contact and tangency or not contact of object

The CP is use to identify positional information of object w.r.t CP of second object as set of relation like (b, i, f, -, +).

Can be suitable where quadratic information and position information required

#### Disadvantages

Does not provide clear connectivity information of segments

Not implemented in SparQ

Provides limited set of qualitative relations.

Requires intrinsic front of street-segment.

Example: Orientation and position of line segments using intrinsic front

Lets suppose, we have sketch map of road network. By considering road segment as 2D with defined intrinsic front we can infer the orientation quadrants (I, II, III, IV) of line-segments with respect to other line-segment. For example, we have line segment A with intrinsic front. The orientation of line segment B with respect to A can be in quadrant III or IV and the orientation of A can be in I, II quadrant with respect to B. Similarly the orientation of G w.r.t A is in quadrant IV.

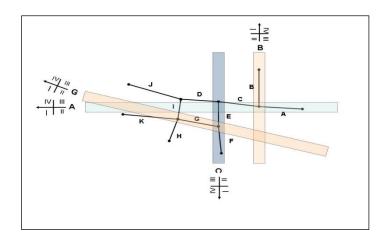


Figure 11. The orientation information of line segments as set of quadrants I, II, III, IV.

#### 2.2 Junctions as Reference object

Junctions (vertices) in the street network are used to represent starting and ending point of street-segments. Junctions are represented as either isolated point or as extended 2D object.

#### 2.2.1 Junctions as isolated Points

The orientation information of junction is based on the used frame of reference and the interpretation of junction. In sketch map most of the sketcher use local frame of reference to represent the objects in the space.

# **2.2.1.1 Direction information of Junction based on Cone- based Cardinal directions** (Frank 1996).

A cone-shaped (or triangular) area of acceptance is related with angular direction between a position and a destination to some direction fixed in space. The set of directional symbols for this system is V9 = {N, NE, E, SE, S, SW, W, NW, 0}. These directional symbols can interpret as orientation symbols like front, back, left, right, front-right, front-left, back-right, back-left.

#### # Advantages

## Most general system for representing direction of the object in the space

# The relations based on cone-based model of direction can directly translated in to relative orientation relation by introducing left, right, front and back

Suitable for orientation of point like objects

#### **Disadvantages**

Uncertainty in direction related with object increase w.r.t distance

Area of acceptance increase with respect to distance between observer and destination.

More Euclidean approximate and identity results then projected-based cardinal direction

Difficult to decide orientation of object close to reference object

Not suitable for extended object as division of space with w.r.t shape of object is difficult.

Example: orientation information of Junctions using cone-based cardinal direction relations

Let suppose, we have junctions and we are interested in orientation information of sketch objects w.r.t. junction as reference object in local frame of reference. For example, we have junction A as reference junction and consider A at identical (I) position. The possible orientation information can be B is *west\_of* A, and C is on *NW* of A. Similarly, if we consider E as reference junction then the cardinal directions w.r.t C can be: A is E of C and B is on NE of C, and D is N of C.

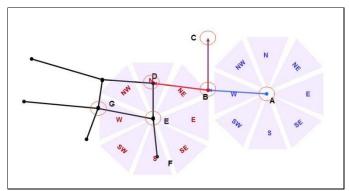


Figure 12. The orientation information of junction based on the cone-based cardinal directions.

#### **2.2.1.2 Direction Information of Junctions based on Star calculus-** (Renz and Mitra 2004).

Star Calculus is used for representing and reasoning about qualitative direction between points (junctions) in a 2D space w.r.t given reference direction. For each point, the star calculus divides the plane into several zones based on the defined granularity, which form the possible qualitative directions between given points. For each point (p) the lines (m) partition space in two 4m+1 disjoint zone represented as even and odd number like {1,3,5} and {0,2,4.....}. The set of 4m+1 basic relation of star calculus are considered to be regular if all zones have equal size. The star calculus provides high-level of granularity as compare to other proposed cardinal direction calculi.

#### # Advantages

# 1 Star calculus provides high-level of granularity as compare to other cardinal direction calculi like cardinal direction by frank, which has fixed granularity with limited defined relations.

- It provides qualitative relations between points by dividing the plane into several equal size zone with angular orientation from range (0 -360).
- It is powerful to express geometrical statement .e.g if we have star calculus with granularity (M3) then we can easily find out consistency of point. As well as can infer possible relations for all instantiation.

#### **Disadvantages**

Selection of granularity-level for specific purposes is difficult.

Star calculus is not implemented in SparQ.

It provides qualitative relations between points by dividing the plane into several equal size zones

Example: Orientation information of Junctions using the Star-calculus

The star calculi with  $m\ge3$  is so expressive that the quantitative aspect of a coordinate system can be emulated by the star calculus. The sub-calculus of star calculus which is known as revised star calculus. It provides possible relation as set of equal size regions to deal with qualitative spatial representation. It provides set of JEPD relations. Supposed we have same street segment with junction A, B, C ...etc. We apply revised star calculus with granularity (m4), which produce 10 equal size zones in 2D space.

For example, we have junction A used as a reference point. Orientation information of junctions with respect to A can be: A r8 B, and A r4 C means that orientation of junction B w.r.t A is "r8" similarly orientation of C w.r.t A is "r7". Based on these constraints between A, B and C, we can also infer possible orientation information of A w.r.t B and C, which can be "r4" and "r3".

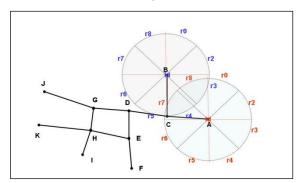


Figure 13. The orientation of junction with respect to the reference junction A and B using the Star calculus.

#### 2.2.2 Junctions as 2D objects

To find out the orientation information as set of qualitative relations needs to apply qualitative calculi dealing with 2D objects. The following calculi are considered to capture the orientation information of street junctions. The suggested calculi used to find out qualitative orientation relations between set of objects using local frame of reference.

#### **2.2.2.1 Egocentric reference frame with intrinsic front-** (Mukerjee and Joe 1990).

The model proposed in (Mukerjee and Joe 1990) considers 2D objects in the space as representational primitives. By applying calculi, we can find both orientation information between objects (junctions) as well as ordering information. In qualitative model of space they considered object with arbitrary angle defined as intrinsic front. It enables user to identify several qualitative different zones like *front*, *left*, *right* and *back*. By consider junction as 2D object with intrinsic front we can able to identify qualitative relations between given objects. It has following advantages and disadvantages.

#### # Advantages

# 1 If junctions are consider 2D object with intrinsic front, calculus provides set of orientation relations as quadrants.

Suitable when both orientation and ordering information of object required.

#### **Disadvantages**

Difficult to decide intrinsic front of junction.

It doesn't provide connectivity information of junction with street-segment.

It does not provides precise orientation information

Example: Orientation information of Junctions using intrinsic front with egocentric frame of reference suggested.

Let suppose, we have 2D junctions with intrinsic front. The model divides the space into four quadrants, named as I, II, III, IV. For example, we have 2D junction A with intrinsic front. The orientation information of junction B w.r.t A can be in *quadrant-IV* or *quadrant-III* and the orientation of junction G w.r.t A can be *quadrant-IV* as the intrinsic front of G is in the direction of *quadrant-IV* of the junction A. Similarly if we consider junction G as a reference object, then the orientation of A w.r.t G in the *quadrant-I* of G and orientation of C w.r.t G is either *quadrant-I* or *quadrant-II* of G.

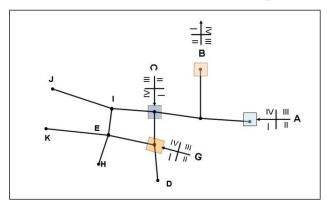


Figure 14. The orientation information of junctions with intrinsic front based on the qualitative model of space.

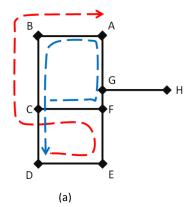
#### **3** Ordering information

It appears that attempting to capture order relations among street segments is an unproductive task since street networks are planar graphs. An order relation must be established not only on the segments of the streets but first on their endpoints. Since the end points of the streets are embedded in the plane, any ordering must be based on some sort of planar reference system (a frame of reference endowed with some operations). Choosing the form and orientation of this reference system

constitutes a main challenge since global frames of reference are already out the question from our standpoint. As such we may have to rely on the order street segments along a given path. But this is equivalent to determining the connectivity of the street network for the given path (see the applicable interval ordering relations in the next section), which we have already dealt with in the first chapter on Graph Matching. Some of the issues with problem are outlined below.

#### **3.1** Allen's Interval Algebra in the graph –(Allen 1983).

Interpreting junctions as isolated points on directed street segments we can order a set of street segments along a given direction provided that there is a path from the first to the last street segment in the order (sequence). The problem of this type of representation is that street segments are required to be directed and if the direction of any one segment on the path is reversed the path breaks. Further given a small street network of say five directed segments there are potentially many possible paths between any two junctions or endpoints and a street segment may participate in any number of paths. Consequently the representation is weakly constrained and there can be so many identical paths in every street network representation that matching street networks based only on such ordering requires comparing all possible paths (n! paths for n nodes in the case of complete graphs). This increases the size of the search space for the reasoning tasks unreasonably especially if errors or deviations in the representations are to be taken into account. But more importantly, IA is far more expressive than the situations encountered here require. For any two street segments only one of {meet, meet-inverse, after, before} can be true. Perhaps it is better to use a less expressive language in this case. Figure 10 shows an example representation using this approach and shows some disadvantages. On the other hand, Allen's algebra has been studied extensively and implementations for it exist.



O1: DE {m EF, b (FC, CB, BA)}
O2: CF {m FG, b (GA, AB, BC, *CD*)}

The two orderings O1 and O2 have the same pattern although O2 is longer than O1. They share some of the same edges and are indistinguishable from a purely syntactic perspective. On the other hand if the segments can be labelled in a sensible way then sequences can be compared since each "interval" will be uniquely identifiable. But even in this case, finding the sequences to be compared will be unnecessarily costly.

(b)

Figure 15. (a) Problems of representing the ordering information in a graph, and (b) ordering information using the Allen's Interval Algebra.

#### 3.1.1 Using Allen's Interval Algebra by projecting street segments onto a line

Projecting streets onto lines or planes to order them by some relation does not seem practical. There is no room for projections if there is no fixed perspective and no global orientation for all streets. The problems described in 2.2 and 2.2.1 above would nonetheless still persist.

# 3.2 Order of city blocks (in the case of countries like Japan, which do not have street names but city block names

Spatial relationships between city blocks can be captured using their adjacency relations. These relations form a graph structure in which a node represents a city block and an edge represents an adjacency between the city blocks corresponding to the nodes incident to it. For ordering we therefore fall back to the same trap as above.

# 3.3 Change in orientation of street segments/Straightness of street segment intersections

Continuity of street segments refers to the change of orientation from one street segment to another. This can be viewed as a categorisation of turn angles between two street segments at their intersection. In (Klippel and Montello 2007) Klippel et al. investigate the categorisation of turn direction in linguistic and non-linguistic conceptualizations. They found that the angles 90°, 180°, and 270° degrees act as prototypical angles for the categories right, straight, and left resp. in both linguistic and non-linguistic conceptualizations. However in the non-linguistic formalization they also act as boundaries of the turn direction categories. They found eight directional clusters altogether with straight having the narrowest angular range. We believe that these directional clusters or categories can be useful for formalizing the relative change in orientation between adjacent street segments but they must be organized in such a way as to have at least the algebraic operations conversion, identity, and weak composition on them (i.e. to form a non-associative algebra). The easiest way may be to map them to existing qualitative calculi. There are three classes of spatial configurations that require consideration – intersections of only two street segments, sequences of two street segment intersections, and intersections of more than two street segments (Figure 17 a, b, and c resp.).

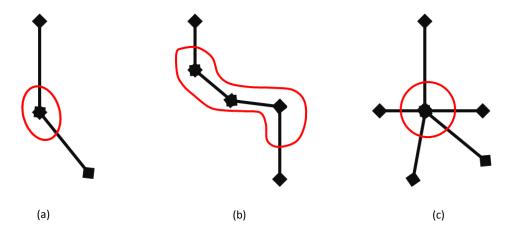


Figure 16. Three types of intersections at which change in the orientation is considered.

*Intersections of only two street segments* 

The change in direction at intersections of only two street segments can be represented using the multi granular OPRA calculi (Moratz, Dylla, and Frommberger 2005) restricted in a certain way. Klippel and Montello's results could provide guidance on how to create angular partitions that are cognitively representative. But there must be a way of mapping the cognitive turn direction categories to the OPRA relations that is closed under the relational operations (composition, conversion, union, etc). Figure 18 shows an example situation where Klippel's categories are used to determine turn directions and the result of overlaying Klippel's turn directions categories with OPRA4 partitions.

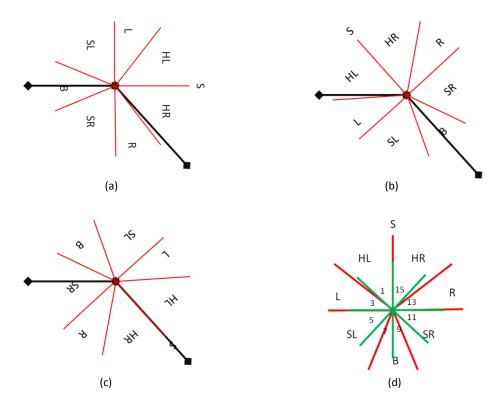


Figure 17. Possible model for determining change in orientation between adjacent street segments using Klippel's model. In (a) the change in orientation longer (slanted) line segment from the perspective of the shorter (horizontal) line segment is g.

#### Advantages

Based on a cognitively motivated model

#### Disadvantages

• Not designed as a formal qualitative calculus therefore consistency, composition, and conceptual neighborhood reasoning not investigated.

#### Sequences of two street segment intersections

A sequence of two street segment intersections reflects the curvature of the aggregate or combination of the street segments. The problem is how to combine the change in orientation between each pair of street segments into the overall change in orientation for all the street segments. One approach is to always apply simplification algorithms from cartographic generalization to replace sequences of street segments of a certain minimum length with a straight line segment that connects the first and last endpoints of sequence. The constraint for this is that no topological relations may be violated (must be topologically invariant). Alternatively this information can be added as attributes to the nodes.

#### *Intersections of more than two street segments*

Intersections of more than two street segments represent junctions. The orientation information for street segments (Section 2.1) will be used here.

#### 4 Qualitative Representation of City Block

According to Wikipedia, a city block is the smallest element of urban planning and urban design. It is the smallest area that is surrounded by streets. The extent of a city block is defined by the road centerlines of its bounding streets. This definition is substantially restrictive to use in sketch map representations. People do not always sketch complete city blocks. They may sketch a network of

streets without any loops (the network is a tree) because they have omitted other streets. At the same time a city block plays the role of a container for other landmarks. A useful definition must therefore take these considerations into account allowing city blocks to be nested and combined systematically by adding or removing parts of their boundary.

In "The image of the City" Kevin Lynch describes how residents see and think about the city in terms of elements of the physical structure and how those elements relate to the residents' experiences in the city.

#### 4.1 City blocks in sketch maps

The following recursive definition of a (representation of a) city block in a sketch map is proposed: A city block in a sketch map is

- i. The smallest region of the map that is delimited by lineal representations of street segments or other path-like objects such as rivers, or
- ii. Any combination of such regions provided that they form a topologically connected surface created by deleting certain region boundaries.

To account for omitted street segments, a looser characterization of a city block (from now on simply CB) is first given here. A CB is a region whose boundary has segments of path like sketched objects and/or possibly parts of the border of the sketching surface (page) – see **Error! Reference source not found.**. So CBs form part of the basic spatial partition of the urban sketch map and may be littered with landmarks. In particular, every landmark in a sketch map is contained in some CB.

From the foregoing at least two types of CBs can be distinguished. Interior CBs are bounded only by sketched objects. They conform, somewhat, to the urban planning definition cited above and, strictly, to the recursively stated definition proposed above. Border CBs contain at least some part of the page border in their boundary and therefore their boundary is partly or completely artificial. In reality a city block may have a vague boundary (e.g. edge of a forest) but such boundary is expected to be sketched explicitly for interior CBs. Vagueness is however present in border CBs because their boundary is not well defined. To overcome this it is assumed that each border CB is a union of parts of interior CBs. So if BCB is a border CB there are regions ICB<sub>1</sub>, ..., ICB<sub>N</sub>, P<sub>1</sub>ICB<sub>1</sub>, ..., P<sub>1</sub>ICB<sub>N</sub>, P<sub>2</sub>ICB<sub>1</sub>,..., P<sub>2</sub>ICB<sub>N</sub> such that BCB =  $U_{i=1 \text{ to } N}(P_1ICB_i)$ , where, for each i in  $\{1,...,N\}$ ,  $ICB_i = P_1ICB_i \cup P_2ICB_i$  is an interior CB (see Figure 12) with  $(P_1ICB_i \cap P_2ICB_i)^\circ = \emptyset$ . Here  $X^\circ$  is the topological interior of X. Because interior CBs are bounded by path like sketched objects, this assumption says that given BCB there exists a number of real world geographic features which if sketched into the sketch map would cause new interior city blocks (the ICB<sub>i</sub>s) to emerge whose sum completely covers BCB. The formulation given above allows us to talk about city blocks in a sketch map without requiring that each CB be an interior CB. It also, however, demands mechanisms for determining the regions ICB<sub>i</sub> and their parts when required. The challenge is now to formalize this representation (the above is a sketch) using familiar geometric structures within the existing qualitative spatial reasoning theories. The simple to obtain geometric representations are polygons and the qualitative representations of interest are sets of topological relations.

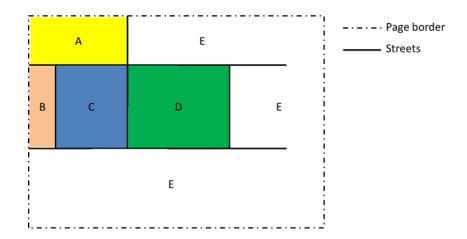


Figure 18. Example of how city blocks emerge in a sketch map. The streets and the border of the sketching surface define the boundaries of the city blocks C and D have well defined boundaries while city blocks A, B and E do not.

#### 4.2 City blocks as polygons

In this section we consider each city block as a polygon in  $\mathbf{R}^2$ . Topological relations among city blocks in a sketch map can therefore be determined using point-set topology or other topologies over regular closed regions. The topology can be represented using the 9 Intersection model, the 4 Intersection model, RCC8, RCC5, the Dimension Extended Method (DEM) or the Calculus Based Method (CBM).

In addition an adjacency matrix can be used to encode the adjacency relations among polygons where two polygons are adjacent if and only if their interiors are disjoint and they share a set of common boundary points  $B_{shared}$  that is continuous in space with  $|B_{shared}| > 1$  (or  $|B_{shared}| > n()$  where n() can be a real number or a function of the boundaries of the two polygons). Defining adjacency in this way reduces the set of topological relations to two relations:

- i. ADJ (for adjacent) is a subset of the EC relation obtained by eliminating 0-meets, and
- ii. NADJ (non-adjacent) is the set union of all the other seven RCC8 relations and the subset EC\ADJ.

The topological model is essential since it expresses the notion of continuity in space – there are no empty spaces between CBs. The adjacency model serves two purposes. First it constrains the CBs so that interiors of CBs at the same level of granularity do not overlap since in that case they are NADJ. Second it ensures that if two CBs are considered adjacent then the intersection of their boundaries is a 1-dimensional object i.e. a line. This is important because CBs should only be aggregated if the surface covering exactly the resulting aggregate (or composite) region has a continuous interior.

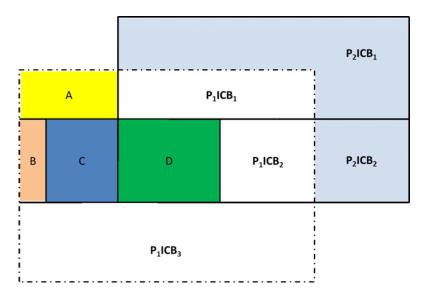


Figure 19. Border CBs are the union of parts of interior CBs. The border CB labeled E in the previous figure formed by combining parts of the interior CBs ICB1, ICB2, and ICB3.

#### **4.2.1 9- Intersection model** (Egenhofer and Herring 1991) **and RCC8**(Cohn et al. 1997)

9-Intersection relations among polygons can be computed from the component intersections in the standard way - i.e. by performing tests of the intersections of their boundary components and determining the nature of the intersection.

#### **4.2.2 RCC11-**(Jan et al. 2015)

The 9-Intersection model and the RCC8 do not distinguish between 0-dimension and 1-dimension contact of regions. To remedy this we introduced RCC11 which can make this distinction. This enables the extraction of the adjacency relations mentioned above during matching.

#### 5 Landmarks and Street Segments

In sketch map landmarks are generally defined as common anchors, representing significant places in the environment that can easily recognized by human beings. Landmarks are represented as isolated point or as extended object in 2D space. To align landmarks from sketch maps with metric map requires qualitative relations of landmarks w.r.t. other surrounding objects like street-segments, junctions or other dominant landmarks in the space.

#### 5.1 Distance Relations between Landmarks and Street-segments

To align landmarks from sketch map with metric map, we have both quantitative and qualitative distance measurement methods between landmarks and street-segments.

#### **5.1.1 Quantitative methods**

How we are going to define buffer around street-segment in sketch map?

In quantitative method, we are considering landmarks along the street-segment. We need to define buffer around street-segment and consider it as reference object. Based on buffer distance we will measure the distance between landmark and street-segment. One of the challenging tasks is to consider the shape and size of the buffer use for quantitative distance measurement.

#### 5.1.2 Qualitative Methods

In qualitative method of alignment of landmarks from sketch-map with metric map we are consider three different approaches.

**5.1.2.1** Landmarks (centroids) and street-segments as directed lines- (Zimmermann 1993). In (Zimmermann 1993) Zimmermann et al. introduced qualitative spatial reasoning for orientation of

point w.r.t oriented line(starting and ending point) which is basically single cross calculus Where we find orientation of point (1D or 2D) w.r.t two other order point that form oriented line. To infer qualitative distance between given object, they integrate distance knowledge based on del-calculus. To exploit distance they mapped generated vectors (AB, BC and AC) in to un-oriented edges and introduce orthogonal distance between point C and line ab, dx and distance DvA and DvB between point C and the two orthogonal lines. The distance between given edges are represented as a, b and c.

The possible distance relations between given object can be represented as A<B, A=C or B>C.

#### **# Advantages**

# distance together

#### Distance relations restrict orientation of object Does not provide clear qualitative distance in the space and vice versa.

#### **Disadvantages**

Suitable if we need mapping orientation and In sketch-map how we are going to calculate distance along x-axis and y-axis from reference point to landmark

relations.

Flexible granularity e.g A>B, but how much it is greater can represent as  $A \times B$ 

Example: distance relation between Landmark (Point and street-segment)

Let supposed, we have landmark (centroid) represented as C and street-segment (AB) is oriented line generated by two junctions A and B. The distance of point C to line-segment is represented by orthogonal lines are DyA, DyB and DX, which are represented as a, b and c. By using length of edges a, b and c, we can infer the orientation of the landmark w.r.t given street-segment.

If a < b it indicates that the landmark is closure to end junction of street-segment (B). If the length of a=c then the distance of landmark is around the junction B. similarly if length of edge b=c then it indicates that the landmark is around starting junction (A) of street-segment.

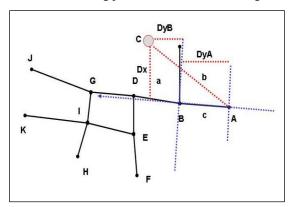


Figure 20. The distance between landmark (C) and street-segment AB using the representation proposed in (Zimmermann 1993).

#### 5.1.2.2 Landmarks (Extended objects) and street-segments as projected axis-(Jungert 1992; Chang and Jungert 1991)

In (Jungert 1992; Chang and Jungert 1991), Jungert and Chang introduced the concept of symbolic projection used for spatial reasoning on both 1D object and extended objects. Then qualitative distance technique is much similar to that introduced by Frank, where he split the space into three set of distance relations like [c, m and f]. For extended objects, the distance relations can infer based on the projection of the landmark boundaries on either x-axis or y-axis.

For qualitative distance between objects in the space it is important to consider direction relations between given objects and reference object as the objects at almost the same distance from the observer (reference object), but situated in different directions from the observer, are of importance to differentiate from each other.

#### 5.1.2.3 Landmarks (centroids) and space as circular regions- (Hernandez 1991).

Orientation relation of objects where object are placed relative to one another object, and can be defined in the terms of three basic concepts, the primary object, the reference object and the frame of reference. In (Hernandez 1991), Hernandez proposed set of qualitative relations between objects in the space by considering two relevant dimensions. One is projection which represents the connectivity relations and second one is orientation relation that represents position of object w.r.t reference object. For orientation information he divide space into following qualitative relations like left, right, front, back, left-front, right-front, left-back, right-back etc. For the representation of object in large scale environment the integration of distance relation with orientation information provides clear information.

In (Clementini et al. 1997), Hernandez et al. integrated the orientation and distance information of object to provide basis step for spatial reasoning. They consider same object as frame of reference for both orientation and distance information. For example positions of object B w.r.t. object A represented by the pair  $(d_{AB}, \theta_{AB})$ .

#### **# Advantages**

1 Suitable when we considering centroid of extended object as point

Suitable for distance relation between landmark and junction,

It provide both orientation and distance information together

#### **Disadvantages**

To define acceptance area of distance is challenging

In case of street-segment how we will decide if starting-point of street-segment is close to landmark but ending-point is far from landmark. In that situation how we will decide that street is near or far to the landmark?

Example: the distance relation between Landmark (centroid) and street-segment

Suppose we have landmark X and street-segment AB, to infer qualitative distance relation between given objects we applied Hernandez's and clementini's qualitative approach for orientation and distance information.

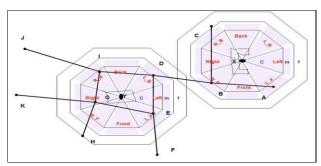


Figure 21. The qualitative distance relation between landmark and street-segment or junction using the angular orientation and distance relations.

According Hernandez's approach of orientation the street-segment is in "front, L-F, and R-F" of landmark (X). The distance relation of street-segment (AB) w.r.t. landmark is "near and middle". As the starting-point of street-segment AB is in middle range of distance and ending-point is also in middle range of distance, but the segment itself is in near range of distance.

 $d_{X,AB}$  (near, middle) and  $\theta_{x,AB}$  (R-F, Front, L-F) Similarly the distance and orientation relation of landmark(Y) w.r.t. landmark (X) can be  $d_{Y,X}$  (far) and  $\theta_{Y,X}$  (right, B-F). Based on given orientation relation we can infer the distance relation of object Y w.r.t. street-segment AB or vise-versa. If the direction relations between objects are opposite then distance relation will be close otherwise it will

#### 5.2 Orientation information of Landmarks w.r.t. Junctions / Street-segments

#### 5.2.1 Junction as isolated point

#### 5.2.1.1 Orientation relations between triples of Point -(Freksa 1992).

The single-cross calculus(SCC) and double-cross calculus(DCC) introduced in (Freksa 1992) divides the space into number qualitative relation likes left, right, front and back based on local frame of reference.

If we consider sketch objects particularly landmarks as extended objects and junctions/street-segment as reference object then the suggested orientation calculus has following advantages and disadvantages.

#### # Advantages

Suitable for orientation information based on local frame of reference

Single or double cross calculi required orientation information based on reference object and the relatum, that provides orientation of reference object

It is not possible to use single junction as reference object as it need orientation of junction and if we consider junction as point the how we will decide its orientation?

suitable calculi is ternary calculi like DCC, or FFC

Disadvantages

Limited granularity

If we consider street segment as oriented line produced by two junctions, then it will help use to infer orientation information of landmark with respect to this oriented line

Distance between reference object and relatum object also effect the orientation of landmark

If we use junction as isolated point as reference object then how we will consider its orientation?

Example: Orientation information of landmark (extended) w.r.t. junction as isolated point

Suppose we have landmark as extended object and street junction as reference object. The orientation information of landmark (X) as extended w.r.t. junction (B) is on *right-side* when we are looking from junction (A). Similarly orientation of landmark(Y) with respect to Junction D is on the *right-side*.

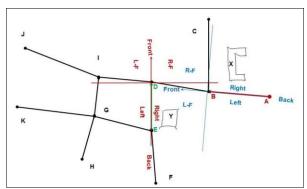


Figure 22. The orientation information of landmark with respect to junction .

# **5.2.1.2** Orientation of landmarks (extended) w.r.t. junction using cardinal directions-(Frank 1996).

The cone-based model of cardinal direction introduced (Frank 1996) provides angular direction between a position and destination to some direction fixed in the space as set of relations like north, south, east and west with identity relation at the center. The model has property that the area of acceptance for any given direction increase distance. There are following advantages and disadvantages of the calculus when we are considering it to infer orientation information between landmark (extended object) and junction as isolated point.

#### # Advantages

#### Cone-based model of orientation calculus provides angular direction between object and reference object particularly point as reference object

If we consider landmark as extended object and junction (isolated point) as reference object it provides orientation relation with high granularity.

For extended objects projected-based model of cardinal direction is suitable, as we can represent extended object in the natural zone and used as reference-object

#### **Disadvantages**

The area of acceptance increase w.r.t. increase distance

Calculus is not suitable for street-segment as reference object to infer orientation information of landmark as representation of street-segment in the model is not possible

Calculus is not suitable to use on extended object as reference object. As it's representation is difficult.

It is difficult to decide the orientation of object close to reference object

#### Example: Orientation information of landmark w.r.t. junction

Suppose we have sketched landmark (X) and junction (A) and (E) as point. When we apply cone-based cardinal direction calculus the possible orientation information of landmark (X) is *right-front* (R-F) w.r.t junction (A). Similarly if the orientation of landmark (Y) w.r.t. junction (E) is *left and left-front* (L-F) as the landmark is close to junction, there for the area of acceptance is decreased.

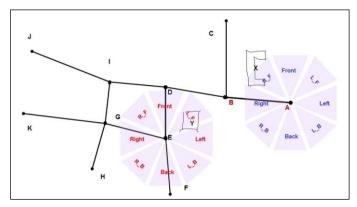


Figure 23. The orientation information of landmark w.r.t junction using cone-based cardinal directions.

# **5.2.1.3** *Orientation of landmarks (extended) w.r.t. junction using the angular orientation* -(Renz and Mitra 2004).

In angular-based orientation calculi are used for representing and reasoning about qualitative direction between objects in a two-dimensional space w.r.t. given reference object. Star-calculus introduced by (Renz and Mitra 2004) a is a generalization of several existing calculi such as the

different kind of calculi which Frank distinguished. It divides the plane into several cones which form the different qualitative relations. Star-calculus(2) is the simplest special case with five basic relations that could be semantically described as {Equality, Front(0), Above/Left(1), Back(2), Below/Right(3)}, depend upon used granularity.

On the other hand the angular orientation calculus proposed in (Hernandez 1991) divides the space into several cones and provide set of qualitative orientation relations like left, front, left-front, right-front, right, back, left-back, right-back and identity (Id). The orientation information of landmark (2D) w.r.t junction (point)/street-segment based on angular orientation calculi has following advantages and disadvantages to consider the star-calculus and angular orientation calculus by Hernandez.

#### # Advantages

The star-calculus divides space into several equal size cones depend upon required granularity level.

In star-calculus reference-object doesn't required direction information like intrinsic front.

In case of junction as reference object it is suitable because, junction doesn't provide an direction information

Star-calculus is not suitable for extended object and street-segment whole as reference object. Therefore we will need to consider centroid of object use as reference object

#### **Disadvantages**

Hernandez's angular orientation calculus provides set of relation with limited granularity.

Hernandez's calculus required intrinsic front of reference object. Therefor it is suitable for object with natural intrinsic front.

In case of junction as reference object, it is not suitable as junction is point without any orientation information.

Hernandez's calculus is suitable if we are considering junction as ending point of particular street-segment, as it will provide intrinsic front of junction.

It is not suitable for street-segment whole as reference-object

Example: Orientation information of landmark (2D-object) w.r.t. junction

Suppose we have landmark X representing building and reference object junction which is isolated point. The orientation of landmark (X) w.r.t. junction (A) is in qualitative regions ( $R_B$ ). If the landmark occupies more than a qualitative region can be managed in these systems by introducing a disjunction of relations. Similarly, orientation of landmark (Y) w.r.t. junction (E) is qualitative region ( $L_B$ ).

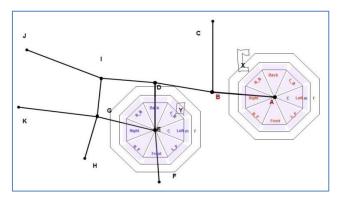


Figure 24. The orientation information of landmark (2D-object) w.r.t. junction as isolated point using the starcalculus and Hernandez's angular orientation

# 5.2.1.4 Clockwise and counterclockwise Orientation of landmarks w.r.t. junction as triple of points based on orientation calculus in (Schlieder 1995).

In (Schlieder 1995), Schlieder introduced the clockwise and counterclockwise concept to deal with the orientation of triple of point in the space. According to him order and orientation are close to each other as order of point effect the orientation information. If we have line segment with starting point  $P_i$  and ending point  $P_j$ . The orientation of line segment is considered to be positive if the path from  $P_i$  to  $P_j$  follows positive direction. From this oriented segment we can find order information of point which is  $[P_i < P_j]$ .

The orientation of points  $[Pi, P_j, P_k] = +$  if the path follow counterclockwise positive orientation, or if  $P_k$  lies on left side when the path follow for  $P_i$  to  $P_j$ . The point  $[P_i, P_j, P_k] = 0$  if point are at the same position and  $[P_i, P_j, P_k] = -$  if path follows negative orientation, or  $P_k$  lies on the right side of oriented line segment  $P_i, P_j$ .

#### # Advantages

#### Suitable for objects represented as 1D-point

Useful for orientation information of landmark (point/2D) and line-segment (starting and ending points)

Suitable for line-segment as reference object in the space.

#### **Disadvantages**

In case of extended object orientation w.r.t. point doesn't provide clear information, as it provides generalized orientation information.

Generalized set of relation like [-, 0, +].

Example: Orientation information of landmark (point) w.r.t. junction as isolated point

Suppose that we have landmark X and reference object represented as isolated points. The orientation relation of landmark with respect to oriented line-segment produced from ordered set of junction A and B is clockwise (negative) = "-". Similarly the orientation of landmark Y with respect to order set of junctions DE is counterclockwise (positive) = "+".

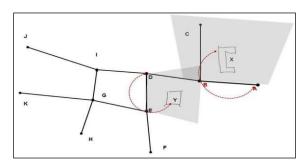


Figure 25. The orientation information of landmark represented as point and the junction as isolated point.

#### 5.2.2 Junction/Street-segment as 2D object

# 5.2.2.1 Orientation of landmark (extended) w.r.t junction/street-segment with intrinsic front -(Mukerjee and Joe 1990).

In (Mukerjee and Joe 1990), Mukerjee et al. introduced qualitative orientation model to represent the orientation information of 2D object in the space. They considered egocentric reference frame with intrinsic front of 2D objects. By applying calculi we can find both orientation information between objects (junctions) as well as ordering information. In qualitative model of space they considered the objects with arbitrary angle defined as intrinsic front. It enables user to identify several qualitative different zones like front, left, right and back.

#### # Advantages

It is suitable when we used 2D reference object particular street segment where the order of street junctions provide street-segment intrinsic front, but adjustment of street-segment in the model is still challenging

This calculus is appropriate for orientation information of other sketch objects w.r.t extended object particularly landmark with clear intrinsic front. Orientation of other object like junctions and street-segment w.r.t. landmark

#### **Disadvantages**

It is not suitable for single Junction as 2D object, as single junction does not provides their intrinsic front information

It generalize and divides space into four quadrants like I, II, III, IV to provide orientation information

Limited set of qualitative relations

Example: Orientation information of landmark (2D/1D-point) w.r.t. junction/street-segment

Suppose that we have landmark X and junction A represented as extended object. The orientation information of landmark w.r.t. junction A is in *quadrant-IV*. Similarly the orientation of landmark Y w.r.t junction D is in *quadrant-III*.

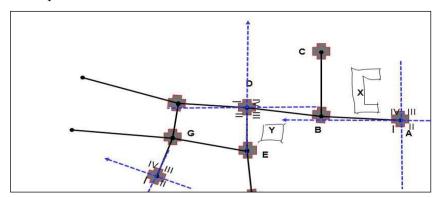


Figure 26. The orientation information of landmark (2D/1D-point) w.r.t. junction/street-segment.

## 5.2.2.2 Orientation of landmark (extended) w.r.t junction/street-segment with intrinsic front and frame of reference using the triangular model -(Hernandez 1991).

The triangular model determines the directional relations between extended object particularly the directional relations between two simple polygon of arbitrary shape and size in the two dimensional space. There are following general properties of the relations between extended object.

- It is binary relation
- Each direction is coupled with a semantic inverse

E.g. south  $(A, B) \Leftrightarrow north (B, A)$ 

- The area of acceptance increase w.r.t. increase in distance
- The area of acceptance increase with the dimension in the facing direction of the reference object in relation to the second object

The triangular model is modified by Peuquet and Zhan to overcome the uniform shape and size restrictions. Where the vertex of triangular area (90-degree) is moved backward s or forward of the object centroid until the boundary of the triangular area touch the vertex of the object.

#### **# Advantages**

Suitable for extended object with uniform and non-uniform shape and size object particular landmarks and 2D street segments.

#### **Disadvantages**

Calculus is not suitable for junctions represented as point.

It just provides simple set of qualitative relations. The area of acceptance increase with respect to increase in distance

It is not implemented in SparQ

Example: Orientation information of landmark (2D) w.r.t. junction/street-segment

Suppose we have landmark(X) as extended object and reference object is junction (2D). The orientation information of landmark w.r.t junction (2D) object is front. Similarly the orientation of landmark(Y) w.r.t junction (E) is front. If the extended object w.r.t to reference object belong to two orientation region, then we can using second rule of triangular mode of orientation. According to it, if the orientation of object belongs to two regions then use second object as reference object to find out orientation of first object and apply inverse operation.

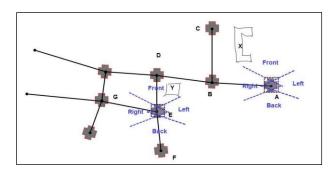


Figure 27. The orientation information of landmark (2D) w.r.t. junction/street-segment as extended objects.

### 5.2.2.3 Orientation information of Junction/Street-segment w.r.t. landmark (point/extended) object.

Qualitative orientation information of junction / street-segment w.r.t landmark as point or extended object is depend upon the used frame of reference. In general user sketched object w.r.t local frame of reference.

#### 5.2.3 Landmark as isolated Point

Suggested calculi for Orientation information of junctions/street-segment w.r.t landmark (isolated point)

### 5.2.3.1 Orientation information between triples of point- (Freksa 1992).

If we consider landmark as point, the orientation of junction (point)/ street-segment (starting and ending point) w.r.t landmark using the orientation calculi in (Freksa 1992) has following advantages and disadvantages

### # Advantages

## Suitable if we consider junctions and landmark as point

#### **Disadvantages**

Most of the landmark sketch maps are represented as polygon, the suggested calculi is not suitable for extended object as reference object

Orientation of street-segment w.r.t landmark Need to consider centroid of the landmark

is difficult as street-segment can contain multiple orientation regions

Calculi provides easy understandable set of How to decide intrinsic front of point. qualitative relations

Orientation calculi by Freksa,1992 is ternary calculi that required 3 object for reasoning. It required ordered line segment made to infer knowledge by two set of points.

Orientation calculi by Freksa,1992 requires orientated reference like made by reference object and other object.

Example: Orientation information junction/street-segment w.r.t. landmark (point)

Suppose we have landmarks represented as isolated point X and Y. The orientation information of street/segment and junction using orientation calculi in (Freksa 1992)can be: Junction A is in left-back region w.r.t landmark X and the junction B is in right-back region with respect to the landmark X.

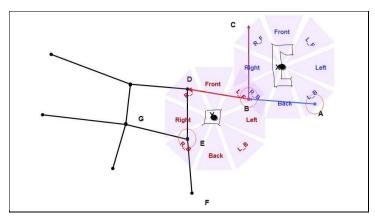


Figure 28. The orientation information junction/street-segment w.r.t. landmark (point) by Freksa, 1992.

### 5.2.3.2 Orientation information between triples of point using Cone-Based orientation calculus -(Frank 1996).

#### # Advantages

### Cone-based orientation calculus is suitable when we use landmark (point ) as reference object

#### Provides easy understandable qualitative set of relations

#### **Disadvantages**

Most of the landmarks are represented as polygon in sketch maps.

Provides limited set of qualitative relation with high level of granularity

Orientation information of object close to reference object is vague

Area of acceptance increase with increase in distance between reference object and target object.

Example: Cone-based orientation information junction/street-segment w.r.t. landmark (point)

For example we have landmarks represented as isolated point X and Y. The orientation information of street/segment and junction using the orientation relations in (Frank 1996) can be: Junction A is in *left-back* region with respect to the landmark X and the junction B is in *right-back* region with respect to the landmark X.

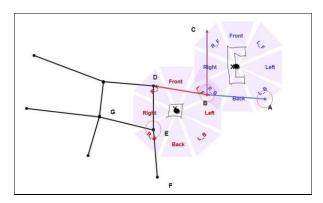


Figure 29. The Cone-Based orientation information between the junction/street-segment and landmarks.

## **5.2.3.3 Orientation information between triples of point using the angular orientation** (Hernandez 1991).

If we consider sketched objects as isolated points. The orientation information of junctions and street-segments using the angular orientation in (Hernandez 1991) has following advantages and disadvantages.

#### **# Advantages**

# Hernandez's angular orientation calculus divides space into angular orientation zone with equal size.

#### It requires intrinsic front of reference object

Suitable for landmark that contains natural intrinsic front.

#### **Disadvantages**

It supports point type entities but most of the landmarks are represented as polygon in sketch maps.

Area of acceptance increase with increase in distance between reference and distention object

Orientation of object close to reference is vague.

Requires centroid of extended object

Example: Angular orientation of junction/street-segment w.r.t. landmark (point).

Let suppose we have extended object X representing build with intrinsic front and street junctions A and B. The orientation information of junction with respect to the landmark X is *left-front* and the orientation of junction B w.r.t landmark X is *right-front*. Similarly if we consider landmark Y, the orientation of junction C w.r.t landmark is *right-front*. The orientation of street-segment AB w.r.t landmark X is *left-front*, *front*, and *right-front*.

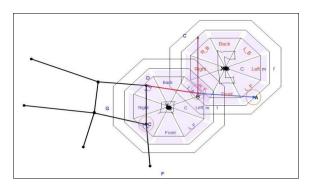


Figure 30. The angular orientation of junction/street-segment with respect to landmark.

## **5.2.3.4** Orientation information between triples of point using clockwise and counterclockwise orientations -(Schlieder 1995).

If we consider landmark object as point, the orientation information of junction (point) and street-segment w.r.t landmark using clockwise and counter-clockwise orientation calculi in the (Schlieder 1995) has following advantages and disadvantages.

#### **# Advantages**

Calculus is suitable for reasoning on point like objects. Where the order information of object effects orientations

Not suitable for extended object

#### **Disadvantages**

Provides limited set of orientation information like left, and right and same(that represents that third object is on same orientation like reference object

Fixed granularity

Always requires centroid of extended object

Example: Orientation information of junction/street-segment w.r.t. landmark.

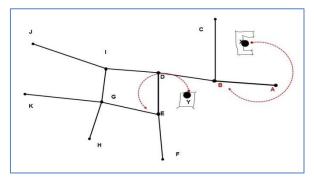


Figure 31. The clockwise and counter-clockwise orientation information of street-segment and junction with respect to landmarks.

#### 5.2.4 Landmark as Extended Object

## 5.2.4.1 Orientation of Junctions/ Street-segment w.r.t landmark (extended) with intrinsic front and egocentric frame of reference - (Mukerjee and Joe 1990).

Suppose we have landmark as extended object with intrinsic front, the orientation of Junctions / street-segment w.r.t landmark using qualitative orientation approach in (Mukerjee and Joe 1990) as follow advantages and disadvantages.

#### **# Advantages**

**Disadvantages** 

Naturally Landmark contains arbitrary intrinsic front, and most of the user sketched extended object as polygon. This approach is suitable for extended object with arbitrary intrinsic front

This approach provides limited orientation information as set of 4 quadrants that divides space in four equal planes

Suitable for extended objects

Fixed granularity

Suitable for Orientation information of junction w.r.t to landmarks as reference object

Not implemented in SparQ

Distance doesn't affect the orientation information.

No suitable for street-segment orientation information .w.r.t landmarks as street-segment may locate in multiple orientation region

Example: Orientation information of junction/street-segment w.r.t. landmark (2D).

For example, we have landmark represented as X, the orientation information of junction A and B with respect to landmark X is junction B belong to  $3^{rd}$  quadrant w.r.t landmark X, and junction A belong to  $4^{th}$  quadrant w.r.t landmark A. Similarly if we consider landmark A as reference object, the orientation of junction A belong to quadrant-A iv of A as it intrinsic front is toward North in global frame of reference. If we consider street segments, in this case some street segment are belongs to two quadrants like street-segment A in this case it is difficult to find out appropriate orientation information using above mention calculus.

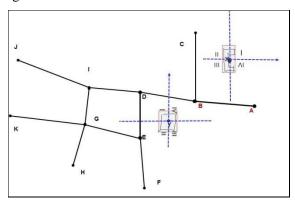


Figure 32. The orientation information of junction/street-segment with respect to landmark.

## 5.2.4.2 Orientation of Junctions/ Street-segment w.r.t landmark as extended object with intrinsic front and egocentric frame of reference using the Triangular model -

(Hernandez 1991).

Using the triangular model in (Hernandez 1991), the orientation information of junctions/street-segment with respect to the landmark as extended object with intrinsic front has the following advantages and disadvantages

### # Advantages

#### **Disadvantages**

Naturally Landmark contains arbitrary intrinsic front, and most of the user sketched extended object as polygon. This approach is suitable for extended object with arbitrary intrinsic front.

It requires egocentric frame of reference

Closeness of reference object does not affect Fixed granularity

orientation

Suitable for Orientation information of junction w.r.t to landmarks as reference object

Not implemented in SparQ

Provides limited set of qualitative relations

Not suitable for street-segment orientation information .w.r.t landmarks as street-segment may locate in multiple orientation region

Area of acceptance depend upon dimension of the face of the reference object

Example: Orientation information of junction/street-segment w.r.t. landmark (2D) using the triangular model.

For example, we have a landmark X used as a reference object. The orientation information of Junction (point/extend object) and street-segment (segment with starting and ending junction) using triangular model can be: Junction B and A is on the right-side with respect to the landmark X, and junction C is on *left-side* of landmark X. Similarly, if we consider landmark Y as reference object the orientation of junction A and B is on right-side and junction C is in front of landmark Y. if the object belong to two orientation region then it is possible to infer orientation of object by changing reference object.

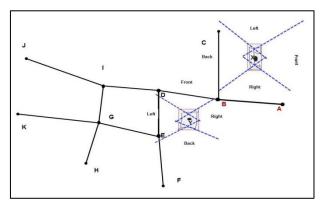


Figure 33. The orientation information of junction/street-segment with respect to landmark using the triangular model.

## 5.2.4.3 Orientation of Junctions/ Street-segment w.r.t landmark (extended) with intrinsic front and egocentric frame of reference using the projected-based cardinal directions-(Frank 1996).

If we consider landmark as an extended object with intrinsic front, the orientation of other objects like junction /street segment using the cardinal direction calculus with neutral y zone equal to the size of landmark provides orientation information of object around it. There are following advantages and disadvantages of the above mentioned calculi for orientation information.

### **# Advantages**

#### **Disadvantages**

suitable for extended object to extract orientation relations direction information

Projected base cardinal direction by frank is Need to modify its relations in the terms of

Fixed granularity

Implemented in sparQ

Not suitable for street-segment orientation information .w.r.t landmarks as street-segment may locate in multiple orientation region

Example: Orientation information of junction/street-segment w.r.t. landmark (2D)

For example, we have landmark as an extended object with intrinsic front at a neutral zone. The cardinal direction N, S, E and W can be represented as front, back, left, and right. If the landmark X is at neutral zone the orientation of junction C with respect to X is left and junction A is on right-back (RB). Similarly the orientation of street/segment AB w.r.t landmark Y is on left-front (LF) by rotating orientation w.r.t intrinsic front of the landmark Y.

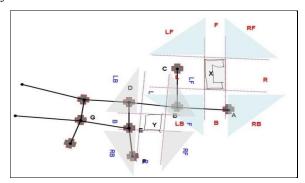


Figure 34. The orientation information of junction/street-segment w.r.t. landmark using the projection-based cardinal directions.

### 6 Orientation Information of Landmark and Landmark

In sketch map most of the landmarks are represented as extended objects. The orientation information of landmark with respect to reference object depends upon the dimension of the reference object. There are follow suggested calculi to formulize the orientation information of landmarks.

#### 6.1 Landmark as isolated Point

If we consider landmark as an isolated point represented as a centroid, there are following suggested calculi:

## **6.1.1 Orientation of landmark w.r.t landmark (point) using the orientation -**(Freksa 1992).

To capture the relative orientation of points, we used the representation the single cross calculus (SCC) and the double cross calculus (DCC) (Freksa 1992). Both the SCC and DCC calculi are ternary calculi. They require three objects, the relatum, reference and the target object itself. In case of landmark-landmark qualitative we can't use this calculus.

## 6.1.2 Orientation of landmark w.r.t landmark using the Cone-Based cardinal direction by Frank, (19969) (Frank 1996).

The cone-based cardinal directions capture the orientation information between objects based on the global reference system such as *north* and *south*. There are following advantages and disadvantages, when we consider these calculi to capture the orientation information of landmark with respect to landmark as points.

#### # Advantages

#### **Disadvantages**

Most general system for representing Uncertainty in direction related with object

direction of the object in the space

The relations based on cone-based model of direction can directly translated in to relative orientation relation by introducing left, right, front and back increase w.r.t distance

Area of acceptance increase with respect to distance between observer and destination.

Suitable for orientation of point like objects

More Euclidean approximate and identity results then projected-based cardinal directions

Easy to implement in the applications

Difficult to decide orientation of object close to reference object

Not suitable for extended object as division of space with w.r.t shape of object is difficult.

### Example with the cone-based cardinal directions

For example, we have landmarks represented as points (centroids), the orientation information of landmark Y with respect to landmark X using cone-based cardinal direction calculi can represented as set of relations such as *left*, *right*, *front*, and *back*. Figure 34 illustrates the orientation relations between two landmarks. The orientation of landmark Y is " $R_B$  (right-back)" with respect to landmark X and similarly the orientation of landmark X is in the " $L_F$  (left-front)" cone with respect to Y.

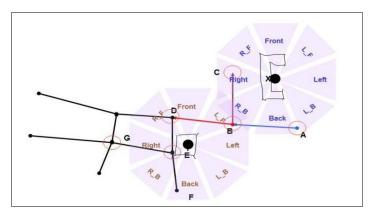


Figure 35. The orientation information of landmark Y with respect to landmark X using the cone-based cardinal direction relations.

## **6.1.3** Orientation of landmark w.r.t landmark using the Angular Orientation System (Clementini et al. 1997).

The angular orientation of landmark w.r.t landmark as isolated points has following advantages and disadvantages.

# ,	Aď	van	tag	es

## Clementini's angular orientation calculus divides space into angular orientation zone

divides space into angular orientation zone with equal size.

### It requires intrinsic front of reference object

Suitable for landmark that contains natural

#### **Disadvantages**

object

It supports point type entities but most of the landmarks are represented as polygon in sketch maps.

Area of acceptance increase with increase in distance between reference and distention

Orientation of object close to reference is

intrinsic front. vague.

Requires centroid of extended object

#### Example with the Angular Orientation System

Let suppose, we have extended object X representing build with intrinsic front and another landmark Y. The orientation information of Y with respect to landmark X is "right-front ( $R_F$ )". Similarly the orientation of landmark X with respect to landmark Y is left-back ( $L_B$ ).

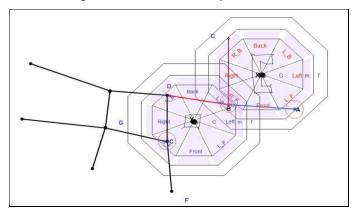


Figure 36. The angular orientation of landmarks with respect to landmarks using the angular orientation.

#### 6.2 Landmark as Extended Object

#### **6.2.1 Orientation of landmarks with intrinsic front -**(Mukerjee and Joe 1990).

In (Mukerjee and Joe 1990) Mukerjee et al. proposed a system to capture the orientation relations between extended objects. These spatial objects must have intrinsic front and geocentric frame of reference. However, in freehand sketches the depicted objects do not contains information about their intrinsic front. The proposed system has following advantages and disadvantages.

#### # Advantages

Naturally Landmark contains arbitrary intrinsic front, and most of the user sketched extended object as polygon. This approach is suitable for extended object with arbitrary intrinsic front

Suitable for extended objects

Suitable for Orientation information of point like entities w.r.t landmark as extended object

Area of acceptance increase w.r.t to increase in distance from reference object.

#### **Disadvantages**

This approach provides limited orientation information as set of 4 quadrants that divides space in four equal planes

Fixed granularity

Not implemented in SparQ

It divides the space based on centroid of landmark.

#### Example of Orientation information

For example, we have two landmarks X and Y with their intrinsic front. The landmarks Y is in the 3<sup>rd</sup> quadrant with respect to landmark X and landmark X is in the 4<sup>th</sup> quadrant Y. The calculus requires centroid of the landmark to divide the space in to 4 quadrants.

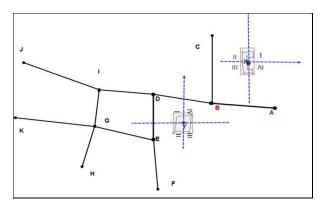


Figure 37. The orientation information of landmarks using the Mukerjee's representational system

#### 6.2.2 Orientation of landmarks with intrinsic front using the Triangular Model-(Hernandez 1991).

In (Hernandez 1991) Hernandez et al. proposed a triangular model to capture the relative orientation information. In the model, the vertex of the triangular area is position at the objects' centroids. The Triangular model requires the intrinsic front of the reference object. The model has following advantages and disadvantages.

#### **# Advantages**

#### Naturally Landmark contains arbitrary intrinsic front, and most of the user sketched extended object as polygon. This approach is suitable for extended object with arbitrary intrinsic front.

Closeness of reference object does not affect orientation

Suitable for Orientation information of Not implemented in SparQ extended object used as a reference object

Provides limited set of qualitative relations

Does not provides composition table as calculus vocabulary

#### **Disadvantages**

It requires egocentric frame of reference

Fixed granularity

Area of acceptance depend upon dimension of the face of the reference object

Example: Orientation information using the Triangular model

For example, we have landmark X used as reference object. Using the model the landmarks Y is right-side to the Landmark X. Similarly, if we consider landmark Y as reference object the landmark X is in front-side of Y. if the object belong to two orientation region then it is possible to infer orientation of object by changing reference object.

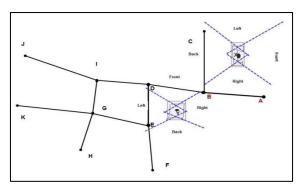


Figure 38. The orientation information of landmarks using the triangular model.

#### 6.2.3 Orientation of landmarks (with intrinsic front) using Projection-Based Cardinal **Directions** -(Frank 1996)

Projection-based cardinal direction calculus is used for direction information of objects in the space. The representational system uses the global frame of reference and captures orientation relations such as north, south, and west etc. If we modified the given set of relations and interpret them as orientation relations like font, back, left, right. The representational system has following advantages and disadvantages.

#### **# Advantages**

## suitable for extended object to extract direction information

#### **Disadvantages**

Projection-base cardinal direction by frank is Need to modify its relations in the terms of orientation relations

Fixed granularity In sketch maps, we do have single global reference frame, the drawn objects act as

reference objects.

Implemented in sparQ Not suitable for street-segment orientation

information .w.r.t landmarks as street-segment may locate in multiple orientation region

Example: Orientation information using the Projection-Based Cardinal Directions

For example, wee have two drawn landmarks X and Y. these spatial objects are approximated by polygons. The cardinal direction north, south, east, and west can be represented as front, back, left and right based on intrinsic front of the reference object. If the landmark X is at neutral zone the orientation of landmark Y w.r.t X is left\_back (L\_B). Similarly, the orientation of landmark X w.r.t landmark Y is *left-front* (L F) by rotating orientation sectors with respect to the intrinsic front of the landmark Y.

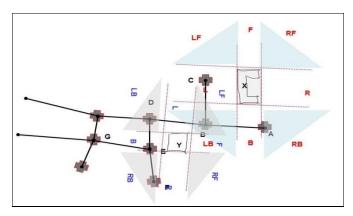


Figure 39. The orientation information of landmarks with intrinsic front using projection-based cardinal directions.

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