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# Coloured rings produced on transparent plates

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**Abstract**
Beautiful coloured interference rings can be produced by using transparent plates such as window glass. A simple model explains this effect, which was described by Newton but has almost been forgotten.

## Interference colours where they are not expected

The formation of interference colours is usually associated with careful experimentation, specialized illumination and small dimensions. Students do not normally expect to see this phenomenon outside the laboratory, as it is not commonly known that interference colours can be seen on transparent material such as window panes which are several millimetres thick. However, if they look at a window in the usual way they will not see any interference colours—a specific perspective is required to see the coloured rings.

Interference colours similar to those produced by diffraction gratings can only be seen using thick glass within a small solid angle whose centreline is perpendicular to the plate. If the centreline combines a suitable light source with its reflection on the plate it is possible to see coloured rings concentric around the reflection.

## Some historical remarks

In 1704, Isaac Newton published the first observations of these kinds of interference colours in his book 'Opticks' [1]. Towards the end of the 19th century, distinguished scientists such as Thomas Young, John Herschel, George Stokes and Adolphe Quételet (see [2]) investigated this more thoroughly, and explained it on the basis of the wave theory of light. Newton's very detailed description of his experiments is an invitation for them to be reproduced (see [1], p 289).

He used the Sun's rays as a light source shining into a dark room through a small hole in a shutter. He let the rays fall onto a concave spherical mirror with quicksilver on its convex reverse side. A piece of white cardboard with a small hole in it was fixed to the shutter. By aligning the mirror so that its centre of curvature coincided with the hole in the cardboard it could be expected that all light passing through the hole would be reflected back to it by the mirror. In contrast, he observed 'upon the chart four or five concentric irises or rings of colours, like rainbows, encompassing the hole . . .' ([1], p 290).

This experiment can be reproduced using a commercial cosmetic mirror. When a modern shutter blind is nearly closed so that the gaps between the slats remain open and a white card provided with a circular hole of about 5 mm is fixed in front of it, we have the same setup as Newton. However, after adjusting the mirror so that the light is focused back to the hole, no special effect can be observed. Obviously Newton did not mention one necessary condition for the appearance of the rings: his mirror must have been covered with impurities, causing part of the light to be scattered. To achieve this scattering effect in our experiment, the front of the mirror must

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be covered by a thin layer of scattering particles. Domestic dust does very well, but you have to wait two or three weeks for this! Instead, covering the mirror with a thin film of salad oil instantly gives very good results.

This idea of using a greasy film goes back to the beginning of the 19th century, when Adolphe Quételet and William Whewell were investigating and writing about the phenomenon of coloured rings, and hence the coloured rings are also known as Quételet's rings (see [3], p 158).

## Coloured rings caused by a magic breath

To demonstrate the experiment in a classroom we used a bright electric light source instead of sunlight. Between the light and the mirror we placed a 30 cm × 30 cm screen with a 5 mm circular hole in its middle. The distance between the screen and the mirror has to correspond to the radius of curvature of the concave mirror. This is the case when the light spot reflected to the screen has the same size as the circular hole. (If the spot spreads out due to the manufacturing tolerances of the mirror, a circular aperture covering the rim of the mirror will correct this.)

At the start of the demonstration the students can only see a very small band around the hole, receiving light from the mirror. This is just what one would expect when the light is focused on the hole, but by simply breathing against the mirror colourful rings like rainbows are magically created on the screen (see figure 2). However, the magic quickly fades away as the condensation evaporates, showing directly that this was the reason for the phenomenon.

To show experimentally which element of the mirror contributes to which of the observed rings, the illumination can be restricted to a small section of the mirror surface. This is best done by directing the narrow beam of a laser pointer through the hole of the screen onto different sections of the mirror. Regardless of which element of the mirror is hit by the laser beam, the rings appear in the same form and size as if the whole screen was illuminated. (Of course, when using monochromatic light the coloration due to the spectral distribution is absent.) From this observation it can be concluded that each element of the mirror produces a similar pattern of rings (see figure 4). The bright ring system produced by the completely illuminated mirror can be thought of as a superposition of the elementary rings originating from each element of the mirror surface.

## Virtual coloured rings 'painted' in oil

The projection of the coloured rings on a screen requires a very bright light source. This has the advantage that many students can see the phenomenon at the same time. If a single observer looks directly into the reflected light, only a relatively faint illumination is needed to make the coloured rings visible. This alternative view had already been adopted by Newton when he glanced along the optical axis towards the surface of the illuminated mirror. He recognized that the position and the curvature of the visible sections of the ring systems changed according to his visual angle.

In order to obtain sufficient luminosity for a real projection of the coloured rings, it is necessary that all mirror elements contribute to the superposed image. Obviously, our eyes are sensitive enough to manage with the relatively low light intensity scattered by particles on the small surface element of the mirror. Therefore, this type of observation does not require the concentration of the reflected light by means of a concave mirror, and a plane mirror will be adequate.

A simple hands-on experiment can help to illustrate this by using an ordinary mirror in a darkened room.

The mirror must be misted up (preferably with small droplets of water) and the observer should be looking at it from a distance of about 2.5 m. On holding a flashlight (whose reflector has been removed) in front of the observer's forehead, coloured stripes can be seen around the direct reflection in the mirror. These stripes are sections of rings whose centre is displaced according to the distance between the eye and the flashlight.

Even the flame of a candle is bright enough to produce Quételet's rings. Quételet used candle flames as a light source, and the small size of the candle flame meant that it had the advantage that it could be put between the observer's eye and the mirror without obscuring too much of the visual field. A similar phenomenon based on a different physical principle was recently described by James Bridge in this journal [4].

Ordinary glass panes can also be used for producing interference rings. Quételet's rings can be seen on windows which have not been cleaned for some time (see figure 1). Sometimes

**Figure 1.** Coloured rings can be seen on a dirty window pane around the reflection of the setting sun.

you can find by chance windows that look dirty during daylight but which display colourful stripes at night when illuminated by the headlights of a car (figure 5). This effect can be reproduced by applying a thin coating of salad oil to a pane of glass. This results in an impressive display of coloured rings around the reflection of the sun. They can be up 50 cm in diameter (figure 6).

## A simple model explaining the coloured rings

The origin of the coloured rings will be explained by a simple model. According to the setup of the experiment shown in figure 7, L denotes a point light source and O the observer's eye. Their positions are described within a rectangular coordinate system. The surface of the mirror is lying in the $xy$-plane. In order to simplify the geometry we presuppose that the thickness $t$ of the mirror glass is small compared to the distances $l_z$ and $o_z$. We consider a small element of the surface where P denotes the position of a small particle. Figure 8 shows that the (almost parallel) rays $l_1$ and $l_2$ emitted by L impinge the surface: at point A the ray $l_1$ is refracted into the glass, reflected at the silvered layer $\overline{\mathrm{MM'}}$ and afterwards hits the particle P, at which point it is scattered. As the size of the particles is assumed to be of the magnitude of some micrometres, the forwardly directed Mie scattering prevails. In contrast, the ray $l_2$ is at first scattered at P, and the scattered light is then refracted into the glass.

Figure 9 shows the geometric relationship needed to calculate the phase difference. Starting from the light source L the rays $l_1$ and $l_2$ arrive at the points A and B with the same phase. Among the light rays scattered at the mirror element are the rays $o_1$ and $o_2$, which travel to the eye O of the observer. They have a constant phase shift when leaving the points F and E. The optical retardation

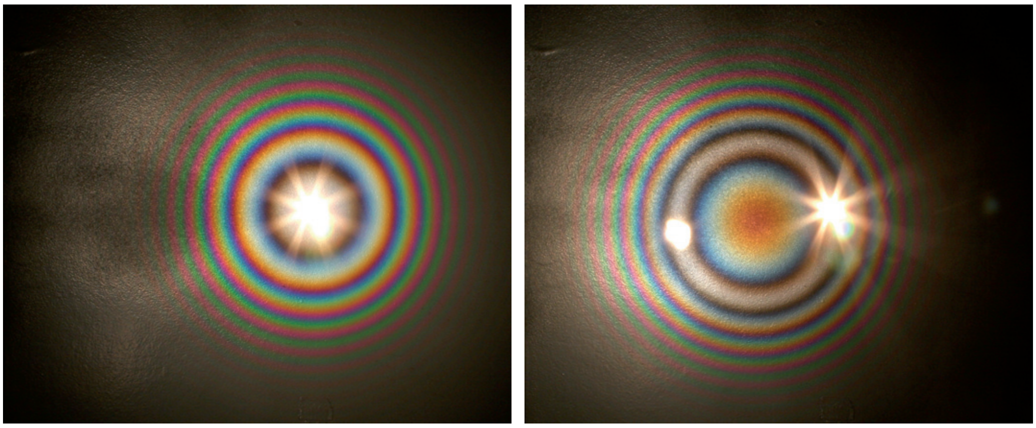

**Figure 2.** Coloured rings produced with the setup shown in figure 3. On the left: concentric rings. On the right: at a slight distortion of the mirror, eccentric rings appear.

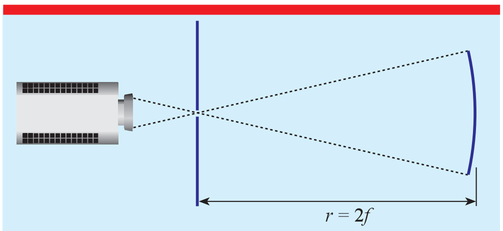


**Figure 3.** Schematic of the setup of the demonstration experiment.

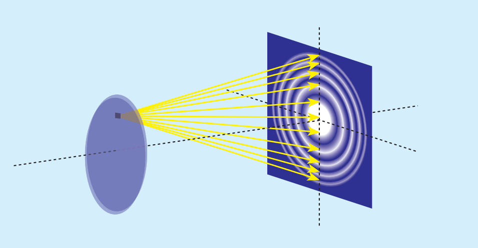

**Figure 4.** Each element of the mirror produces a similar elementary ring system.

$\Delta s$ may then be calculated to give

$$\Delta s = \left(\overline{AP'} + \overline{PF}\right)_{opt} - \left(\overline{BP} + \overline{P'E}\right)_{opt}. \quad (1)$$

In this calculation of the optical path length[1] we make use of the fact that the distance covered by the light travelling from A via the mirror to the (real) particle P is the same as the distance to its virtual image point P′. According to this the light path between $\overline{PE}$ and $\overline{P'E}$ is the same, too.

The geometrical relations show that $\overline{BP} = \overline{AP} \cdot \sin\alpha$ and $\overline{AC} = \overline{AP} \cdot \sin\alpha'$. Making use of the law of refraction and substituting $\sin\alpha = n \cdot \sin\alpha'$ we get: $\overline{BP} = n \cdot \overline{AC}$ and correspondingly $\overline{FP} = n \cdot \overline{DE}$. Due to the identity of these optical path lengths, the calculation of the optical retardation is reduced to

$$\Delta s = \left(\overline{CP'} - \overline{P'D}\right)_{opt}. \quad (2)$$

Now we determine this retardation as a function of the thickness $t$ of the mirror glass and its index of refraction $n$. Crucial for the length scale is that $(\overline{PP'})_{opt} = 2nt$. Regarding as well that $\angle CP'P = \alpha'$ and $\angle PP'D = \beta'$, the lengths in equation (2) can be replaced by equivalent trigonometric expressions to obtain

$$\Delta s = 2nt(\cos\alpha' - \cos\beta'). \quad (3)$$

[1] The optical path length in a medium is the distance which the light would travel in the same time in the vacuum.

**Figure 5.** Coloured rings on a window pane produced by the front light of a car (photo: Eva Seidenfaden).

**Figure 6.** Virtual interference rings on a glass pane illuminated by the sun.

The relations between the trigonometric functions, the law of refraction and approximating for small angles permit us to transform in the following way:

$$\cos\alpha' = \sqrt{1 - \sin^2\alpha'} = \sqrt{1 - (\sin^2\alpha)/n^2} \approx 1 - \frac{\sin^2\alpha}{2n^2}.$$

Applying such transformations to equation (3), we get

$$\Delta s = 2nt\left[\left(1 - \frac{\sin^2\alpha}{2n^2}\right) - \left(1 - \frac{\sin^2\beta}{2n^2}\right)\right]. \quad (4)$$

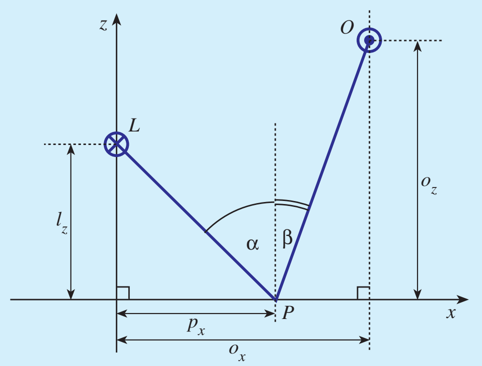


**Figure 7.** Schematic representation of the experimental setup.

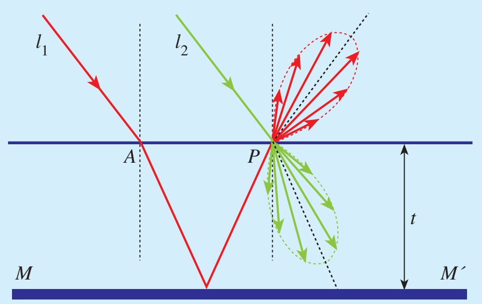


**Figure 8.** Schematic representation of a surface element of a mirror. The light hitting the particle is scattered.

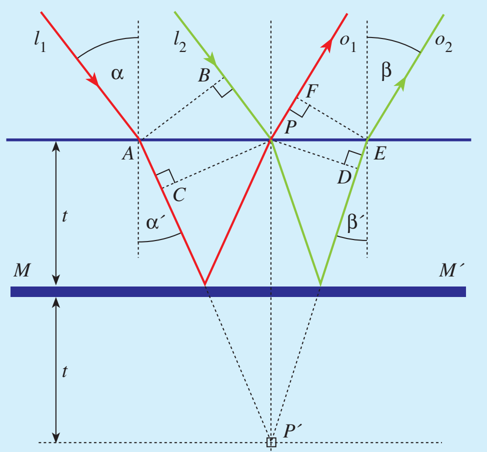


**Figure 9.** Selected path of the rays at the mirror element.

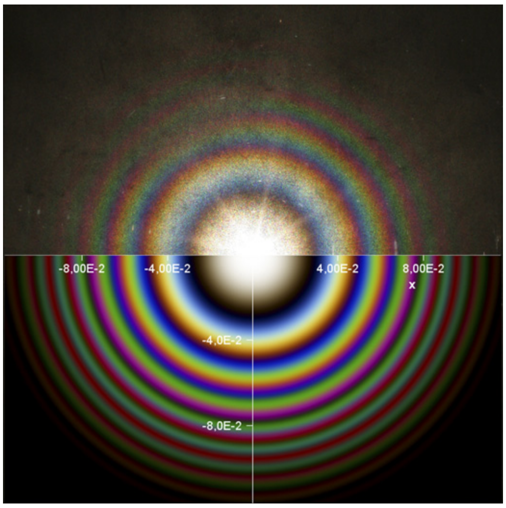


**Figure 10.** Concentric interference rings. Upper part: photograph of an impure mirror. Lower part: representation of the corresponding model calculation.

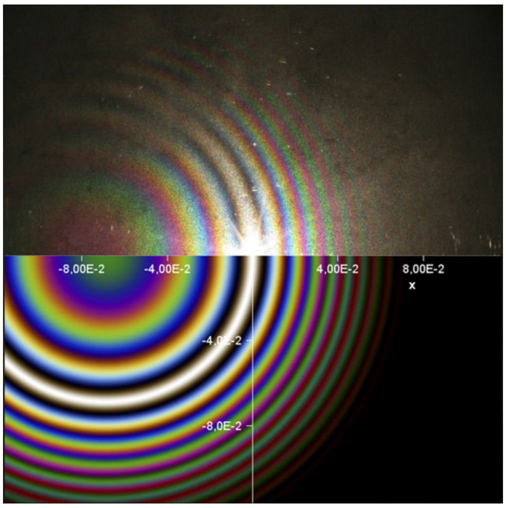


**Figure 11.** Eccentric interference rings. Upper part: photograph of an impure mirror. Lower part: representation of the corresponding model calculation.

Summarizing and cancelling in equation (4) yields

$$\Delta s = \frac{t}{n}(\sin^2 \beta - \sin^2 \alpha). \qquad (5)$$

If instead of the angles only the distances depicted in figure 7 are known, one may benefit from the approximation $\sin^2 \alpha \approx \tan^2 \alpha$, which is valid only for small angles. Because $p_x/l_z = \tan\alpha$ and $(o_x - p_x)/o_z = -\tan\beta$, equation (5) can therefore be expressed as

$$\Delta s = \frac{t}{n}\left[\left(\frac{o_x - p_x}{o_z}\right)^2 - \left(\frac{p_x}{l_z}\right)^2\right]. \qquad (6)$$

## Comparison between experimental results and a model calculation

The model described above has been implemented using a computer program calculating the optical retardation for all points of the mirror surface. According to the RGB colours of the monitor, this calculation was performed for the wavelengths of 605, 545 and 460 nm. The strength of each corresponding R, G, and B component was determined by values proportional to the intensity of the *superposed* light. The respective mirror element was then dyed by the combination colour made up of these components.

The graphic results of these model calculations were compared with photographs taken during experiments which were performed with the same parameters (see figures 10 and 11). In both cases, for the development of concentric and eccentric Quételet rings, we obtained good agreement for the diffraction orders next to the white zeroth order. For higher diffraction orders the sequence of calculated and photographed colour rings corresponds, while their location diverges. This deviation is presumably due to our very simple colour model. This argument is supported by the fact that the photographs of the interference rings obtained with monochromatic light are a surprisingly good match to the calculated rings.

The photographs show that the light intensity of the coloured rings decreases with the distance from the zeroth order of diffraction. This decrease has been modelled by taking into account the restriction of the coherence length of only a few micrometres.

*Received 13 March 2007, in final form 6 April 2007*
*doi:10.1088/0031-9120/42/6/001*

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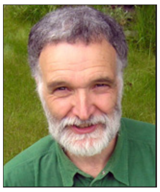

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