

## Stochastic Analysis meets QFT - critical theory

June 12 – 14, 2023 Münster

**Book of Abstracts** 

Organisers

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Stochastic Analysis meets QFT - critical theory

Reception: Wine + Cheese SRZ Foyer 2nd floor Wednesday June 14 Léonard Ferdinand Nguyen Viet Dang Vincent Rivasseau Thomas Krajewski Alexander Hock Sabine Harribey Zhituo Wang Annette-Allee 3, 48149 Münster **Margherita Disertori** Conference Dinner Tuesday June 13 Nikos Zygouras Yvain Bruned Ilya Chevyrev Coffee break Martin Hairer Coffee break A2 Restaurant, Hao Shen Break Lunch Break Break Reception: Wine + Cheese SRZ Foyer 2nd floor Francesco De Vecchi Nikolay Barashkov Nicolas Perkowski Monday June 12 Rongchan Zhu Xiangchan Zhu Razvan Gurau Luca Fresta Registration 10:20-11:10 14:00-14:50 15:20-16:10 11:10-11:30 11:30-12:20 12:20-14:00 14:50-15:20 16:10-16:30 16:30-17:20 17:20-17:30 17:30-18:20 18:30-21:00 19:00-21:00 9:50-10:20 8:30-9:00 9:00-9:50

# 12-14 June 2023, in Münster, Germany

## **General information**

**Venue.** The main workshop venue is the MM-conference center located on the second floor of the Seminarraumzentrum (SRZ) at Orléans-Ring 12, 48149 Münster (see map on p. 3). You will find the registration there. Moreover, the coffee breaks and the reception take place in the lounge of the seminar building SRZ (second floor) right in front of the seminar room.

You can find the latest information on the webpage: www.uni-muenster.de/MathematicsMuenster/events/2023/stochastic-analysismeets-QFT.shtml

**Wi-Fi access.** If you are part of the eduroam community, you may connect to the network "eduroam" as usual. Otherwise you can connect to the SSID "GuestOnCampus" and start any web browser. You will automatically be redirected to the login page. Confirm the terms of use and click on "log in for free". 1 GB data volume is available per device and day. Please note that the connection is not encrypted.

**Reception.** There is a Wine + Cheese reception for all participants of the workshop on Monday and Wednesday evening starting at 18:30 in the lounge of the MM-conference center (Seminarraumzentrum (SRZ), second floor, Orléans-Ring 12).

**Conference dinner.** The conference dinner takes place on Tuesday at 19:00 at the <u>Restaurant A2 am Aasee</u> (Annette-Allee 3, 48149 Münster).

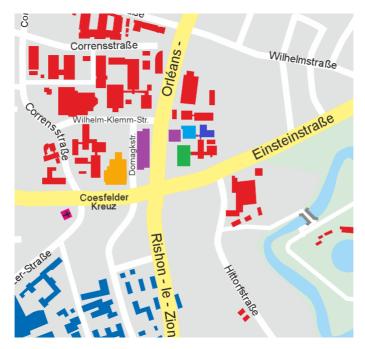
**Coffee break/Lunch.** We provide coffee and snacks during the coffee breaks.

There are a couple of restaurants for lunch in the vicinity:

- Canteen Mensa am Ring, Domagkstraße 61 (all non-local participants receive vouchers (1 meal + 1 drink per day) for the canteen during registration)
- Ristorante Milano (Italian), Wilhelmstraße 26 (closed Mondays)
- King Kebab (Fast Food), Corrensstraße 80
- Il Gondoliere (Italian), Von-Esmarch-Straße 28 (closed Mondays)
- Buddha Palace (Indian), Von-Esmarch-Straße 18 (closed Tuesdays)
- La Gondola D'oro (Italian), Hüfferstraße 34
- Gustav Grün (Green Fast Food), Wilhelmstraße 1
- Áro (Green Fast Food), Neutor 3

**Public transportation.** You can check the bus schedule on the website of <u>Stadtwerke-Münster</u> (in German and English), or use Google maps.

**Questions.** In case of further questions, please use: Email: sek.ag.weber@uni-muenster.de



SRZ (workshop venue, 2nd floor) Math Department canteen multi-storey car park.

## **Book of abstracts**

# Gluing and Markov property for $Phi_3^4$ on Cylinders

Nikolay Barashkov Mon 9:00

The Markov property is an important property of Random Fields that allows to use them to construct a Quantum Field Theory. It is closely connected to Segal's Axioms, which describe how to assemble Random Fields on a bigger manifold from Fields on smaller pieces. In this talk I will describe how to establish these properties for the  $\Phi_3^4$  model on cylinders. This is joint work with T. Gunaratnam.

#### Convergence of the renormalised model for the generalised KPZ equation via preparation maps

Yvain Bruned Tue 10:20

In this talk, we will present the convergence of the renormalised model of the generalised KPZ equation via local transformations that are governed by preparation maps. The main idea is an extension of a result on the convergence of a class Feynman diagrams given by Hairer and Quastel in [3]. With this extension, one is able to perform local transformations that make appear the renormalisation given for a model defined recursively via preparation maps in the context of Regularity Structures. This approach works both in the discrete [2] and continuous [1] settings and could lead to a general convergence theorem.

#### References

- 1 I. Bailleul, Y. Bruned. Random models for singular SPDEs. arXiv:2301.09596, (2023).
- 2 Y. Bruned and U. Nadeem, Convergence of space-discretised gKPZ via Regularity Structures. arXiv:2207.09946, (2022).
- 3 M. Hairer and J. Quastel. A class of growth models rescaling to KPZ. Forum Math. Pi, 6(e3):1–112, (2018).

#### Invariant measure and universality of the 2D Yang-Mills Langevin dynamic (II)

Ilya Chevyrev Tue 16:30

In this talk, I will present a recent work on the invariance of the 2D Yang-Mills measure for its Langevin dynamic. The Langevin dynamic both in 2D and 3D had previously been constructed in joint work with Chandra-Hairer-Shen, but it was an open problem to show the existence of an invariant measure even in 2D. In establishing this invariance, we follow Bourgain's invariant measure argument by taking lattice approximations, but with several twists. An important one, which I will focus on, is that the approximating invariant measures require gauge-fixing, which we achieve by developing a rough version of Uhlenbeck compactness combined with rough path estimates of random walks. I will also present several corollaries of our main result, including a representation of the YM measure as a perturbation of the Gaussian free field, and a new universality result for its discrete approximations. Based on joint work with Hao Shen.

## The $\phi_3^4$ measure on compact Riemannian 3-manifolds

Dang Nguyen Viet Wed 9:00

I will describe a joint work with Bailleul, Ferdinand and To where we construct the  $\phi_3^4$  quantum field theory measure on compact Riemannian 3-manifolds, as invariant Gibbs measure of some stochastic partial differential equation.

## Non-commutative *L<sup>p</sup>* spaces and Grassmann stochastic analysis

Francesco De Vecchi Mon 14:00

We introduce a theory of non-commutative  $L^p$  spaces suitable for non-commutative probability in a non-tracial setting and use it to develop stochastic analysis of Grassmann-valued processes, including martingale inequalities, stochastic integrals with respect to Grassmann Itô processes, Girsanov's formula and a weak formulation of Grassmann SDEs. We apply this new setting to the construction of several unbounded random variables, including a Grassmann analog of the  $\Phi_2^4$  Euclidean QFT in a bounded region. The talk is based on a joint work with Luca Fresta, Maria Gordina and Massimiliano Gubinelli.

#### The non-linear supersymmetric hyperbolic sigma model on a complete graph with hierarchical interactions

Margherita Disertori Tue 9:00

Initially introduced as toy model for quantum diffusion, the non-linear supersymmetric hyperbolic sigma model has been attracting much attention in recent years due to its connection to history dependent stochastic processes. In this talk I will present a version of the model with hierarchical interactions. The internal symmetries of the model allow to perform some block-spin renormalization steps exactly. The resulting effective action has renormalized coefficients but no additional interaction terms. I will show the corresponding derivation and some applications.

This is joint work with S. Rolles and F. Merkl.

## Stochastic quantization for a tensorial non-local field theory

Léonard Ferdinand Wed 15:20

I will introduce quartic melonic tensor field theories, a class of field theories built using a non-local quartic interaction term. These resemble the more well-known  $\Phi_d^4$  models but behave differently with regards to power-counting and the structure of their divergences. In particular, these models are conjectured to be non-trivial in their critical dimension, in contrast with  $\Phi_4^4$ . I will then report on recent joint work with Ajay Chandra where we use stochastic analysis methods to construct the  $\Phi_2^4$  and  $\Phi_3^4$  analogs of these models.

#### The forward-backward SDE for subcritical Euclidean fermionic field theories

#### Luca Fresta Mon 17:30

In this talk, I will describe a synergy between the renormalization group (RG) in the form of Polchinski's equation and the stochastic quantisation in the form of a forward-backward stochastic differential equation (FBSDE). This approach can be used for constructing subcritical Grassmann Gibbsian measures and is based on controlling the solution of the FBSDE by means of a flow equation with respect to a scale parameter. However, unlike the standard RG approach, we only need to solve Polchinski's equation in an approximate way, resulting in a great simplification of the analysis. Based on joint work with F. De Vecchi and M. Gubinelli.

## The small-N series in the zero-dimensional O(N) model

#### Razvan Gurau Mon 11:30

I will discuss the application of some constructive field theory inspired techniques to the study of resurgence in the 0 dimensional O(N) model and its small N limit. This is the first step in the program of applying

such techniques to fully fledged higher dimensional quantum field theory.

## The role of symmetry in renormalisation

#### Martin Hairer Tue 14:00

There are several interesting situations where the solutions to singular SPDEs exhibit a symmetry at a formal level that could in principle be broken by the renormalisation procedure required to define them. We'll discuss a relatively simple argument showing that, in many cases, the renormalisation can be chosen in such a way that the symmetry does indeed hold and we'll apply it to the stochastic quantisation of the 3D Yang-Mills theory.

# Extraordinary Interfaces and Boundaries in 4-epsilon dimensional O(N) models

Sabine Harribey Wed 11:30

The critical O(N) models are one of the most thoroughly studied classes of conformal field theories (CFTs) in three dimensions. Indeed, there exists a variety of approximation methods applicable to them, such as epsilon expansion, large N expansion or conformal bootstrap. It is then of interest to study the three dimensional O(N) models with interfaces (co-dimension one defects), as well as on spaces with boundaries. In particular, we are interested in "extraordinary" critical interfaces and boundaries which break the O(N) symmetry to O(N-1). Such models were studied recently with a D dimensional bulk and surface defects quadratic in the fields. In this talk, we will adopt a different approach and study defects that are always of co-dimension one. More precisely, I will present the renormalisation group analysis for a quartic O(N)model in 4-epsilon dimensions with cubic interactions on an interface. For sufficiently large N, we find stable IR fixed points with purely imaginary cubic couplings. I will also review the special case N=1 corresponding to a boundary Yang-Lee model.

#### Non-perturbative results of a just-renormalisable model

#### Alexander Hock Wed 10:20

We consider the scalar  $\phi^4$  model on the 4-dimensional noncommutative Moyal space. This is the critical dimension where the model becomes just-renormalisable. At the self-dual point, this model breaks down to a matrix model, where the noncommutativity of the underlying space is related to the size N of the matrix. Assuming a formal expansion in 1/N, the Dyson-Schwinger equations (after applying Ward identities) decouple which leads to (non-)linear integral equations at each order in 1/N. We will present and discuss from different perspectives the leading order (genus g = 0) result of the 2-point function, which is a resummation of infinitely many Feynman diagrams. We will also discussion the Hopf-algebraic renormalision of this model in the sense of Connes-Kreimer, which has the same complexity as an ordinary just-renormalisable QFT.

#### Loop vertex expansion for random matrices with higher order interactions

Thomas Krajewski 🛛 Wed 16:30

The loop vertex expansion is an alternative to the standard Feynman graph expansion which trades the latter for a convergent expansion over trees. In this talk, we present the general framework and apply it to some random matrix models. As a byproduct, we establish analyticity in the coupling in a domain independent of the size of the matrix, as well as Borel summability. This is based on work in collaboration with V. Rivasseau et V. Sazonov, see https://arxiv.org/abs/1910.13261.

# Energy solutions and generators for singular SPDEs

Nicolas Perkowski Mon 15:20

I will discuss how to use tools from Gaussian analysis and operator semigroups together with some commutator estimates to construct Markov semigroups for some singular SPDEs. This yields in particular uniqueness for Goncalves-Jara-Gubinelli type energy solutions. The method applies to some critical equations and, in finite dimensions, even for some supercritical equations. In infinite dimensions we get Markov semigroups for supercritical equations but we lack a uniqueness result for supercritical energy solutions in infinite dimensions. The main SPDE examples where this works are of Burgers type: quadratic, divergence-free nonlinearity and Gaussian quasi-invariant measure. This is joint work with Lukas Gräfner.

#### Random Tensors, Loop Vertex Representation and Cumulants

Vincent Rivasseau Wed 14:00

First of all, I briefly discuss the tensor track, a particular approach to quantum gravity based on random tensors. Then I review the Loop Vertex Representation (LVR), which combines combinatorial tools such as the BKAR formula, selective Gaussian integration, and the Fuss-Catalan generating function. In the last part I apply this LVR to obtain convergent expansions for the cumulants of matrix models.

## Invariant measure and universality of the 2D Yang-Mills Langevin dynamic (I)

Hao Shen Tue 15:20

In an earlier work with Chandra, Chevyrev and Hairer [CCHS'20], we constructed the local solution to the stochastic Yang-Mills equation on 2D torus, which was shown to have gauge covariance property and thus induces a Markov process on a singular space of gauge equivalent classes. In this talk, we discuss a more recent work with Chevyrev [CS'23], where we consider the Langevin dynamics of a large class of lattice gauge theories on 2D torus, and prove that these discrete dynamics all converge to the same limiting dynamic constructed in [CCHS'20]. A novel step in the argument is a geometric way to identify the limit using Wilson loops. This universality of the dynamics is crucial for obtaining a sequence of important results for 2D Yang-Mills,

including for instance the invariance of the 2D Yang-Mills measure for its Langevin dynamic, which will be discussed by Ilya Chevyrev.

#### Constructive renormalizations in Quantum Many-body system

Zhituo Wang Wed 17:30

In this talk I will present some recent progress on the construction of ground state of the 2-dimensional Hubbard model, which is a prototypical model for studying phase transitions in quantum many-body system. Using fermionic cluster expansions and constructive renormalization theory, we proved that the ground state of the 2-d Hubbard model on the honeycomb lattice with triangular Fermi surfaces is not a Fermi liquid in the mathematical precise sense of Salmhofer. I will also discuss the crossover phenomenon in the 2-d square Hubbard model and universalities. This presentation is based on the work arXiv:2108.10852 and arXiv:2303.13628.

# Large N limit and 1/N expansion of the observables for O(N) linear sigma model

Rongchan Zhu Mon 10:20

In this talk, we talk about the large N problems for the Wick renormalized linear sigma model, i.e. N-component  $\Phi^4$  model, in two spatial dimensions, using stochastic quantization methods and Dyson–Schwinger equations. We identify the large N limiting law of a collection of Wick renormalized O(N) invariant observables. In particular, under a suitable scaling, the quadratic observables converge in the large N limit to a mean-zero (singular) Gaussian field Q with an explicit covariance; and the observables which are renormalized powers of order 2n converge in the large N limit to suitably renormalized n-th powers of Q.

Furthermore, we derive the 1/N expansion for the k-point functions of the quadratic observables by employing a graph representation and carefully analyzing the order of each graph from Dyson-Schwinger equations. Finally, we obtain the next order stationary dynamics.

## A class of singular SPDEs via convex integration

Xiangchan Zhu Mon 16:30

In this talk I will talk about our recent work on a class of singular SPDEs via convex integration method. In particular, we establish global-in-time existence and non-uniqueness of probabilistically strong solutions to the three dimensional Navier–Stokes system driven by space-time white noise. In this setting, solutions are expected to have space regularity at most  $-1/2 - \kappa$  for any  $\kappa > 0$ . Consequently, the convective term is ill-defined analytically and probabilistic renormalization is required. Up to now, only local well-posedness has been known. With the help of paracontrolled calculus we decompose the system in a way which makes it amenable to convex integration. By a careful analysis of the regularity of each term, we develop an iterative procedure which yields global non-unique probabilistically strong paracontrolled solutions. Our result applies to any divergence free initial

condition in  $L^2 \cup B_{\infty,\infty}^{-1+\kappa}$ ,  $\kappa > 0$ , and implies also non-uniqueness in law. Finally I will show the existence, non-uniqueness, non-Gaussianity and non-unique ergodicity for singular quasi geostrophic equation in the critical and supercritical regime.

## SPDEs at the critical dimension

Nikos Zygouras Tue 11:30

I will make an overview of the progress on treating SPDEs at the critical dimension, the current status and further challenges. Examples will include stochastic heat equations and a more recent Allen-Cahn example.

#### Acknowledgements

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 $\begin{array}{c} {\rm ERC\ Grant\ "Global\ Estimates\ for\ Singular\ Stochastic\ PDEs"} \\ {\rm https://erc.europa.eu/homepage} \end{array}$ 

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