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Sommersemester 2021
14. Juni 2021

Gaussian Free Field and Liouville Quantum Gravity

Exercise Sheet 8

Due: Monday, 21.06.2021

Exercise 1 (4 Punkte)

Let D be a bounded domain, $z \in D$ and $0 < \varepsilon < \operatorname{dist}(z, \partial D)$. Denote by $\rho_{\varepsilon,z}$ the uniform distribution on $S_\varepsilon(z) \coloneqq \{w \in \mathbb{C} : |z - w| = \varepsilon\}$. Show that $\rho_{\varepsilon,z} \in \mathcal{M}_0$.

Exercise 2 (6 Punkte)

Let D be a bounded domain and h a Gaussian Free Field on D. Fix $z \in D$ and $0 < \varepsilon_0 < \operatorname{dist}(z, \partial D)$. Now define for $t \geq t_0 \coloneqq -\log(\varepsilon_0)$ the process $B_t \coloneqq h_{e^{-t}}(z)$. Show that this process has stationary increments.

Exercise 3 (5 Punkte)

Let $r < 1$ and ρ be the uniform distribution on $\partial D_r(0)$. Show that

$$\int_{\mathbb{D}} \int_{\mathbb{D}} G_0^{\mathbb{D}}(x, y) \rho(\mathrm{d}x) \rho(\mathrm{d}y) = \int_{\mathbb{D}} G_0^{\mathbb{D}}(x, 0) \rho(\mathrm{d}x).$$