# Young Set Theory Workshop

Münster, Germany May 29 - June 3, 2023

Stefan Hoffelner Aleksandra Kwiatkowska Sandra Müller Farmer Schlutzenberg

#### Organizers Local Organizers

Alessandro Codenotti Azul Lihuen Fatalini









### **General Information**

**Venue.** The conference will take place at the University of Münster, Germany, at the Department of Mathematics and Computer Science, which is situated at the north-east corner of the intersection of Einsteinstraße and Orleans-Ring. The talks will be held in lecture hall M2, Einsteinstraße 64 (Hörsaal building). This is the building closest to the intersection just mentioned. Coffee breaks will be on the second floor of the adjacent building, Orléans-Ring 12.

**Program and Registration.** The workshop will start the morning of Monday, 29 May, and run until the afternoon of Saturday, 3 June. The schedule can be found on the next page. Registration will be on the ground floor of the building at Orléans-Ring 12 (next to the building where talks are held) between 8:30 and 9:30 on **Tuesday, 30 May**.

**Poster Session and Reception.** The poster session and the reception will take place on Tuesday right after talks in the afternoon in the same place as the coffee breaks, on the second floor of Orléans-Ring 12.

**Wifi.** If your institution is part of the eduroam community you can simply connect to the "eduroam" network. Otherwise you can connect to the "GuestOnCampus" network. Starting any web browser will then redirect you to a login page where you have to accept the terms of use and then click "log in for free". On the latter network there is a 1GB per device per day data limit and the connection is not encrypted.

**Conference Dinner.** The conference dinner will be on Wednesday at 7pm at the Schlossgarten (Schlossgarten 4). All registered participants are welcome at the dinner, the food and water will be covered from the conference budget.

**Conference Photo.** The conference photo will take place on Thursday at the beginning of the lunch break (12.30) in front of the building at Einsteinstraße 64.

#### **Excursions.**

We are organizing bike trips on Wednesday afternoon. You can borrow a bicycle at the main train station (Hauptbahnhof): www.radstation.de/mieten. You will need ID to borrow a bike and you should pre-reserve the bike before picking it up. The website is in German, most likely you want to choose "Hauptbahnhof" as the location and "Fahrrad". You can borrow a bike for a day (return same day), 24h, 3 days, or a week. Please ask any of the local people if you need help with filling out the form.

Those of you who are interested, can walk (35 min) or go by bus with some local people on Tuesday after the reception to the train station to borrow bikes. Radstation is open from 7am until 11pm.

You should have a bike ready before the excursions.

**Excursion 1.** A short trip to Pleister Mühle restaurant. We will leave immediately after talks and have lunch at the restaurant. There is an option of playing a round of mini-golf there. After going back to the city we will have a beer at Aasee lake.

**Excursion 2.** A round trip of 25-30km around Münster. We will stop by at the castle Burg Hülshoff and pass through Münster Old Town. At the end of the trip we plan to get a beer at Aasee lake.

We will leave at 1:30pm from the entrance to the building at Einsteinstraße 64.

**Sightseeing.** Below is a list of further suggestions for Wednesday afternoon (and not only).

- self guided walking tours
  www.gpsmycity.com/gps-tour-guides/munster-2897.html
- Münster hop on/hop off bus. One can wear headphones to hear the guide in English.
   www.muensterbus.ms/de/stadtrundfahrt-hop-on-hop-off/
- art and culture: www.stadt-muenster.de/en/tourismus/art-and-culture
- biking to Vogelschutzgebiet 'Rieselfelder Münster' (bird reserve)
- biking along Werse river

**Grocery on Sunday and Monday.** Note that on Sunday and Monday grocery stores will be closed due to Pentecost holidays. It will be possible to get basic groceries at the "REWE To Go" at the gas station at Steinfurter Str. 1-3, and also at the main train station. Restaurants and some of the bakeries will remain open.

**Restaurants.** Below is a list of restaurants (relatively) close to the math department.

- Mensa am Ring (university canteen), Domagkstraße 61 (closed on Monday and weekends)
- Ristorante Milano (Italian), Wilhelmstraße 26 (closed on Monday)
- Aleppo Grill (Kebab, oriental), Steinfurter Straße 33a (cheap and easy)
- Il Gondoliere (Italian), Von-Esmarch-Straße 28 (closed on Monday)
- Buddha Palace (Indian), Von-Esmarch-Straße 18
- La Gondola D'oro (Italian), Hüfferstraße 34
- Gustav Grün (Green Fast Food, vegan), Wilhelmstraße 1
- Áro (Green Fast Food, with vegan options), Neutor 3
- Nordstern (German, roast chicken, serves food 4pm til late), Hoyastrasse 3
- Krawummel (vegan), Ludgeristraße 62 (further)

#### More fancy options, e.g. for a nice dinner.

- Royals and Rice (Vietnamese Fusion), Frauenstraße 51
- Großer Kiepenkerl (German), Spiekerhof 45
- Pinkus Müller Gasthaus (German), Kreuzstraße 4
- La Taverna (Italian), Hoyastrasse 5

### Schedule

	Monday	Tuesday	Wednesday	Thursday
9:30-10:45	V. Fischer I	Z. Kostana	V. Fischer II	O. Ben-Neria II
10:45-11:15	Coffee Break			
11:15-12:30	O. Ben-Neria I	N. Dobrinen I	T. Tsankov II	T. Gilton
12:30-14:30	Lunch Break			Lunch Break
14:30-15:45	T. Tsankov I	S. Jackson I	Excursion	N. Dobrinen II
15:45-16:00	Coffee Break	EMS Presentation		Coffe Break
16:00-16:15		Poster Session and Reception		
16:15-17:30	W. Chan			T. Tsankov III
17:30-18:00				
18:00-18:30				Outlook: Tsankov

	Friday	Saturday	
9:30-10:45	O. Ben-Neria III	S. Jackson III	
10:45-11:15	Coffe Break		
11:15-12:30	V. Fischer III	N. Dobrinen III	
12:30-14:00	Lunch Break		
14:00-15:15	J. Zomback	M. Eskew	
15:15-15:30	Coffe Break		
15:30-15:45	Cojje Dreak	Outlooks: Jackson, Dobrinen	
15:45-16:30	S. Jackson II		
16:30-17:00	5. Jackson n		
17:00-17:30			
17:30-18:30	Outlooks: Ben-Neria, Fischer		

Book of Abstracts

### Tutorials

### Universality problems in set theory

#### **Omer Ben-Neria**

Hebrew University of Jerusalem

A collection *K* of certain structures (e.g., graphs, groups, topological spaces) has a universal object *M* in *K* if every member of *K* embeds into *M*. By a fundamental result in model theory, if *K* is the collection of all models of a certain complete first order theory *T* and of some regular size  $\kappa$ , then the continuum hypothesis implies the existence of a universal object for *K*.

However, if the continuum hypothesis fails or if the structures in K are not described by first order properties (e.g., Aronszajn trees) then K need not always have a universal object.

The goal of the tutorial is to study several such univerality problems and forcing methods that produce universal objects.

## Ramsey theory of infinite structures

#### Natasha Dobrinen

University of Notre Dame

The simplest form of the infinite Ramsey theorem states that, given any coloring of all pairs of natural numbers into two colors, there is an infinite subset of natural numbers in which all pairs have the same color. When moving from sets to structures, some surprising phenomena occur: For example, there is a coloring of pairs of rational numbers into two colors such that both colors persist in any subset of the rationals forming a dense linear order (Sierpiński 1933). Likewise for colorings of edges in the Rado graph (Erdős- Hajnal-Posa 1975). The study of optimal bounds for colorings of copies (or embeddings) of a finite substructure inside an infinite structure is the subject of big Ramsey degrees. Optimal bounds are connected with structural expansions which produce analogues of the infinite Ramsey theorem.

This tutorial will introduce big Ramsey degrees and discuss currently known results and components intrinsic to characterizations. We will examine proof methods for upper bounds and study Milliken's strong tree theorem, Harrington's forcing proof of the Halpern-Läuchli Theorem, the method of coding trees, and forcing on coding trees. We will also delve into infinite-dimensional Ramsey theory, starting with the Baire space, covering some examples of topological Ramsey spaces, and moving into recent work on Ramsey spaces of Fraïssé structures. We will conclude with open problems.

An expository paper introducing this area is [1], which has many relevant references for this tutorial. References from 2022 onward will be included throughout the tutorial.

#### References

[1] Natasha Dobrinen. "Ramsey theory of homogeneous structures: Current trends and open problems". In: *arXiv preprint arXiv:2110.00655* (2021).

### Combinatorial sets of reals

#### Vera Fischer

University of Vienna

In these tutorial lectures, we will consider recent advances in the study of the so called combinatorial sets of reals, i.e. sets which are usually associated the combinatorial cardinal characteristics of the continuum. Of particular interest for us will be their spectra, i.e. possible cardinalities, their projective complexity, as well as their higher Baire spaces analogues. The study of those sets is closely accompanied by the development of new forcing techniques, as well as significant advancements of already existing techniques, on which we will comment throughout the lectures.

## Determinacy, combinatorics, and partition relations

#### **Stephen Jackson**

University of North Texas

In this tutorial we will present the AD theory of combinatorics and partition relations with applications to structural properties of sets and cardinalities. We will start with the basic properties of sets and measures in the determinacy context, in particular focusing on sets associated with smaller cardinals. We will progress from this to more recent results involving partition properties and applications to general cardinalities. As time permits, other topics such as higher infinity Borel codes, partition relations at larger cardinals, and descriptions will be covered.

### Topological dynamics of Polish groups

#### **Todor Tsankov**

University Claude Bernard - Lyon 1

Topological dynamics is the study of actions of groups (or semigroups) on compact spaces. Traditionally, one is interested in actions of the group of integers but, more recently, a theory has emerged about the new phenomena present for actions of some big, non-discrete Polish groups. For some of them, objects such as the universal minimal flow, that are usually untractable, become easier to describe and understand and carry interesting combinatorial information. In this minicourse, I will cover the basics of abstract topological dynamics, the enveloping semigroup, the topometric structure of the universal minimal flows, and various tractability conditions such as metrizability and the presence of a comeager orbit.

### Invited talks

### The cardinality and combinatorics of infinite sets under determinacy

#### William Chan

University of North Texas

We will discuss the size of familiar infinite sets under the axiom of determinacy. The talk will especially focus on the known structure of the cardinalities under the injection relation below the power set of the first uncountable cardinal under various determinacy assumptions. We will also formulate a notion of regularity and cofinality for cardinalities in this choiceless setting and survey the known results. Partition properties, partition measures, almost everywhere continuity of functions on partition spaces, and other combinatorial properties will be relevant in studying the structure of cardinalities. This talk includes joint work with Stephen Jackson and Nam Trang.

## Chang's Conjecture for triples revisited

#### **Monroe Eskew**

University of Vienna

A prominent open question is whether  $\aleph_{\omega}$  can be Jonsson. By an argument due to Silver, if the continuum is not too large, then  $\aleph_{\omega}$  being Jonsson is equivalent to some infinitary Chang principle involving the  $\aleph_n$ 's. As a step towards getting this property, Foreman showed in his thesis that one can force from a 2-huge cardinal the principle

 $(\aleph_3,\aleph_2,\aleph_1)\twoheadrightarrow(\aleph_2,\aleph_1,\aleph_0),$ 

which we will abbreviate by  $(3,2,1) \rightarrow (2,1,0)$ . He asked whether we can get closer to Jonssonness by having  $(n+2, n+1, n) \rightarrow (m+2, m+1, m)$  simultaneously for all finite n > m. We partially answer his question by

producing a model in which this holds for all n > m such that m is even. This is joint work with Masahiro Shioya.

# Club Stationary Reflection and other combinatorial principles

#### **Thomas Gilton**

University of Pittsburgh

Club Stationary Reflection (CSR) is a substantial strengthening of stationary reflection, and it asserts roughly that every appropriate stationary set reflects "almost everywhere." Magidor first showed that this principle is consistent on  $\omega_2$ .

In this talk we will introduce this reflection principle after a brief, general discussion of stationary reflection. We will discuss limitations on when CSR can hold. Then we will discuss how to obtain models in which CSR holds, focusing especially on models in which additional combinatorial principles of interest hold simultaneously with CSR.

## The existence and properties of rectangular structures

#### Ziemowit Kostana

Bar-Ilan University

In the 1981 Avraham, Rubin, and Shelah proved that consistently there exists an  $\omega_1$ -dense set of reals A such that every function  $F : A \longrightarrow A$  is non-decreasing on some uncountable set. The key notion in their proof is that of an *increasing linear order*. During the talk we will try to generalize this notion to other structures, mainly graphs. The applications are twofold:

- First, we can study some uncountable counterparts of the classical (countable) Fraïssé limits. For example, what homogeneity properties are true/consistent?
- Second, we can prove more results in the spirit of the Avraham-Shelah Theorem. For example, we prove that it is consistent with

*ZFC* (and even *ZFC*+ Martin's Axiom) that there exists a separable, uncountable, rational metric space *X* such that every injective  $f: X \longrightarrow X$  is an isometry on an uncountable set.

#### Ergodic theorems along trees

#### Jenna Zomback

Williams College

In the classical pointwise ergodic theorem for a probability measure preserving (pmp) transformation T, one takes averages of a given integrable function over the intervals  $(x, Tx, T^2x, ..., T^nx)$  in front of the point x. We prove a "backward" ergodic theorem for a countable-to-one pmp T, where the averages are taken over subtrees of the graph of T that are rooted at x and lie behind x (in the direction of  $T^{-1}$ ). Surprisingly, this theorem yields forward ergodic theorems for countable groups, in particular, one for pmp actions of free groups of finite rank, and can be extended to yield ergodic theorems for pmp actions of free semigroups as well. In each case, the averages are taken along subtrees of the standard Cayley graph rooted at the identity. This is joint work with Anush Tserunyan.