# Workshop

## Interactions between expanders, groups and operator algebras

## Titles and abstracts

- **Goulnara Arzhantseva**

  *Generalised expanders*

  In this expository talk, we discuss the interplay between generalized expander graphs, coarse amenability, and coarse embeddings.

- **Uri Bader**

  *Group algebra criteria for vanishing of cohomology*

  I will discuss discuss cohomological properties of groups that could be considered as higher generalization of property (T). In particular I will discuss a group algebra criterion for vanishing of cohomology groups with unitary coefficients in a certain degree. This talk is based on a joint work with Piotr Nowak.

- **Oren Becker**

  *Stability of approximate group actions*

  An approximate unitary representation of a group $G$ is a function $f$ from $G$ to U(n) such that $f(gh)$ is close to $f(g)f(h)$ for all $g, h$. Is every approximate unitary representation just a slight deformation of a unitary representation? The answer depends on $G$ and on the norm on U(n). If $G$ is amenable, the answer is positive for the operator norm on U(n) (Kazhdan '82). The answer remains positive if we use the normalized Hilbert-Schmidt norm and allow a slight change in the dimension $n$ (Gowers-Hatami '15, De Chiffre-Ozawa-Thom '17). For both norms, the answer is negative if $G$ is a nonabelian free group (or a nonelementary word-hyperbolic group). In this talk we shall discuss a similar notion where U(n) is replaced by Sym(n) with the normalized Hamming metric. We study the cases where $G$ is either free, amenable or equal to $\mathrm{SL}_r(\mathrm{Z})$, $r \geq 3$. When $G$ is finite, a slight variation of our main theorem provides an efficient probabilistic algorithm to determine whether a function $f$ from $G$ to Sym(n) is close to a homomorphism when $|G|$ and $n$ are both large. The talk is based on a joint work with Michael Chapman.

- **Cornelia Druţu**

  *Median geometry for lattices*

  Median geometry is closely connected with versions of property (T) and of a-T-menability. In this talk I shall explain how various types of lattices of isometries of (products of) hyperbolic spaces present various degrees of compatibility with the median geometry and its discrete version, the CAT(0) cubical complex geometry. A byproduct of these results is that they provide evidence that Rips-type theorems cannot hold for actions on median spaces, not even under strong hypotheses. This is joint work with Indira Chatterji.

- **Alexandros Eskenazis**

  *Nonlinear spectral gaps and coarse non-universality*

  I shall discuss two extremal geometric properties of nonlinear spectral gaps pertaining to the bi-Lipschitz and coarse geometry of the underlying expanders. After a brief survey of the expander problem for CAT(0) spaces, I will explain a joint result with M. Mendel and A. Naor establishing the coarse non-universality of this class of spaces. Finally I shall mention a conjectured functional inequality whose validity would resolve the corresponding bi-Lipschitz problem.

- **Maria Paula Gomez Aparicio**

  *Pure braid groups and the Baum-Connes conjecture*

  The Baum-Connes conjecture, which computes in topological terms the K-theory of the reduced $C^*$-algebra of a locally compact group, is still open for some discrete groups, the most famous being $\mathrm{SL}_3(\mathrm{Z})$. Nonetheless, this conjecture has been proven for a large class of discrete groups such as a-T-menable groups as well as discrete subgroups of Lie groups having property RD. The case of pure braid groups Pn was solved by Oyono-Oyono who gave some permanence properties of the conjecture for groups acting on oriented trees. I will show how to give an explicit computation of the conjecture in the case of P4. This is a joint work with Azzali, Browne, Ruth and Wang.

- **Amitay Kamber**

  *Cutoff on* $\mathrm{SL}_2$

  In recent years there is a growing interest in various families of Cayley graphs and Schreier graphs of $\mathrm{SL}_2(F_p)$, as $p$ changes. For example, consider the graphs generated by the projection of fixed elements of $\mathrm{SL}_2(Z)$, or the graphs generated by choosing 2 random elements and their inverses. A big breakthrough of Bourgain and Gamburd implies that those graphs are expanders, and in particular their diameter is logarithmic in their size. Conjecturally, much more should be true - those graphs should satisfy the cutoff phenomena, which implies in particular that the distance between most of the pairs of vertices is optimal up to a $1 + o(1)$ factor. We will present a general approach to the problem, which is based on the work of Sarnak and Xue on automorphic forms. In particular, we are able to prove cutoff for some Schreier graphs coming from the action of $\mathrm{SL}_2(F_p)$ on the projective line over $F_p$. Based on joint work with Konstantin Golubev.

- **Ana Khukhro**

  *A new characterisation of virtually free groups*

  A finite graph that can be obtained from a given graph by contracting edges and removing vertices and edges is called a minor of this graph. Minors have played an important role in graph theory, ever since the well-known result of Kuratowski that characterised planar graphs as those that do not admit the complete graph on 5 vertices nor the complete bipartite graph on (3,3) vertices as minors. In this talk, we will explore how this concept interacts with some notions from geometric group theory, and describe a new characterisation of virtually free groups in terms of minors of their Cayley graphs.

- **Masato Mimura**

  *The Green–Tao theorem for number fields*

  The Szemeredi theorem asserts that an upper dense subset of the set of integers contains arbitrarily long arithmetic progressions. Furstenberg revealed that this theorem can be derived from the multi-recurrence theorem for measure preserving invertible actions on probability standard measure spaces. Furstenberg and Katznelson showed the multi-recurrence theorem for actions of free abelian groups of finite rank; it then implies the multi-dimensional Szemeredi theorem. It is then an interesting question to ask what happens for "sparse" subsets of a free abelian group of finite rank. The celebrated theorem of Green and Tao asserts that a subset relatively upper dense to the set of rational primes contains arbitrarily long arithmetic progressions. In this talk, we generalize this theorem to the setting of an arbitrary number field, which may be seen a multi-dimensional Szemeredi theorem for sparse subsets.

  This is a joint work with colleagues in Tohoku University: Kai Wataru, Akihiro Munemasa, Shin-ichiro Seki and Kiyoto Yoshino. See https://arxiv.org/abs/2012.15669 for the preprint. No serious background of number theory is assumed for this talk.

- **Piotr Nowak**

  *On property (T) for Aut($F_n$)*

  The goal of this talk is to present the recent proof that $\mathrm{Aut}(\mathrm{F}_n)$, the automorphism group of the free group on $n$ generators, has Kazhdan's property (T) for $n \geq 5$. This is joint work with Marek Kaluba and Taka Ozawa ($n = 5$) and with Kaluba and Dawid Kielak ($n \geq 6$). Our proof uses a characterization of property (T) via an algebraic notion of positivity in the group ring, due to Ozawa, and computer assistance in the form of semidefinite programming (i.e. convex optimization over positive definite matrices). As applications we confirm the explanation of the effectiveness of the Product Replacement Algorithm predicted by Lubotzky and Pak, as well as obtain new asymptotically optimal estimates of Kazhdan constant for $\mathrm{Aut}(\mathrm{F}_n)$ and $\mathrm{SL}_n(\mathrm{Z})$.

- **Izhar Oppenheim**

  *The fixed-point spectrum of a random group*

  A discrete group $G$ is said to have property $FL_p$, if every affine isometric action of $G$ on an $L_p$ space admits a fixed point. The fixed-point spectrum of $G$ is the values of $p$ for which $G$ has property $FL_p$. If G is Gromov-hyperbolic and has property (T), it is known that there is a number $p_G$ such that the fixed-point spectrum is of the form $[1, p_G]$ or $[1, p_G]$. For a random group $G$ in the triangular model with density between 1/3 and 1/2, it holds with overwhelming probability that $G$ is Gromov-hyperbolic and has property (T), thus it is natural to study the growth of $p_G$ (as a function of the number of generators of $G$). In my talk, I will discuss a sharp result regarding the growth of $p_G$ that improves on previous results of Drutu and Mackay and de Laat and de la Salle.

- **Mikael de la Salle**

  *$\tilde{A}_2$-geometry, spectral gap at large scales and weak amenability*

  After the celebrated work of Haagerup on free groups and many later developments, it is now well understood that the different approximation properties of group $C^*$ (and von Neumann) algebras are very much connected to the geometry of the spaces on which the group acts. I will illustrate this through a new example : groups acting geometrically on $\tilde{A}_2$ buildings fail to be weakly amenable. In particular, this provides a new geometric proof of the known fact that lattices in $SL_3(\mathbf{Q}_p)$ fail to be weakly amenable. The idea is to exploit spectral gap properties on large scale averaging operators on the building. This has other consequences in terms of approximation properties, strong property (T) and vanishing of cohomology. This is based on a joint work with Jean Lécureux and Stefan Witzel.

- **Ján Špakula**

  *Measured asymptotic expanders and rigidity of Roe algebras*

  Let $X$ be a countable discrete metric space, and think of operators on $\ell^2(X)$ in terms of their $X$-by-$X$ matrix. Band operators are ones whose matrix is supported on a "band" along the main diagonal; the norm-limits of these form a $C^*$-algebra, called the uniform Roe algebra of $X$. This algebra "encodes" the large-scale (a.k.a. coarse) structure of X. "Rigidity of Roe algebras" refers to the question whether isomorphism of Roe algebras implies coarse equivalence of the underlying spaces. I will discuss the notion of measured asymptotic expanders, and how do they provide (yet another) geometric criterion for rigidity of Roe algebras. If time permits, I will expand on some of the ingredients and techniques that we use. (Based on joint work with K. Li and J. Zhang.)

- **Jeroen Winkel**

  *Geometric property (T) for non-discrete spaces*

  Geometric property (T) was defined by Willett and Yu, first for sequences of graphs and later for more general discrete spaces. Increasing sequences of graphs with geometric property (T) are expanders, and they are examples of coarse spaces for which the maximal coarse Baum-Connes assembly map fails to be surjective. Here, we give a broader definition of bounded geometry for coarse spaces, which includes non-discrete spaces. We define a generalisation of geometric property (T) for this class of spaces and show that it is a coarse invariant. Additionally, we characterise it in terms of spectral properties of Laplacians. We investigate geometric property (T) for manifolds and warped systems.