Fixing the gauge fixing procedure: a non-perturbative concern

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From perturbative to non-perturbative QFT - Münster

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Outline

Introduction

- One of the biggest challenges in theoretical physics: comprehension of non-perturbative aspects of Yang-Mills theories.
- A full control of this regime should provide a fundamental understanding of the confinement of quarks and gluons.
- Many approaches to this problem: Dyson-Schwinger equations, functional renormalization group, lattice, effective models, holographic techniques,. . .
- These approaches give complementary information, but no full picture is available.

- Yang-Mills theories are invariant under *gauge* transformations.
- For a given field configuration A^a_μ, there are infinitely many equivalent configurations connected through gauge transformations.
- Possible treatment of such a redundancy in the continuum: gauge-fixing.
- In the path integral formulation we employ the Faddeev-Popov procedure, which seems to work well at the perturbative level.

Faddeev-Popov procedure: toy version

Let us consider a real function $f(x)$ that has n roots, $\{x_1, \ldots, x_n\}$. Formally, we can write

$$\delta(f(x)) = \sum_{i=1}^{n} \frac{\delta(x - x_i)}{|f'(x_i)|}\,, \tag{1}$$

with $f'(x_i) \neq 0 \; \forall\, i$. Integrating over x, we have

$$\int dx\; \delta(f(x)) = \sum_{i=1}^{n} \frac{1}{|f'(x_i)|}\,, \tag{2}$$

which implies

$$\frac{1}{\sum_{i=1}^{n} \frac{1}{|f'(x_i)|}} \int dx\; \delta(f(x)) = 1\,. \tag{3}$$

If $f(x) = 0$ has just one solution $\tilde{x}$ and $f'(\tilde{x}) > 0$, the previous result reduces to

$$f'(\tilde{x}) \int dx\; \delta(f(x)) = 1\,. \tag{4}$$

This is the analogue of the Faddeev-Popov trick if we consider $f(x) = 0$ as the gauge condition and $f'(\tilde{x})$ as the Faddeev-Popov determinant.

The "real" Faddeev-Popov procedure

Choosing the gauge condition $F[A]=0$, we rewrite the path integral in flat Euclidean space (and $SU(N)$ gauge group) as

$$\mathcal{Z}=\int\mathcal{D}g\mathcal{D}A\,\delta\left[F[A]\right]\Delta_{\mathrm{FP}}\;\mathrm{e}^{-S_{\mathrm{YM}}}\,, \tag{5}$$

where Δ_{FP} is the Faddeev-Popov determinant. To do so, we assumed:

- $F[A]=0$ has a unique solution *i.e.*, for each gauge orbit, the gauge condition picks just one representative.
- Δ_{FP} does not develop zero-modes and is positive, such that we can eliminate the absolute value.

To check if the previous assumptions are reasonable for a specific gauge choice, we start with Landau gauge, namely, $\partial_\mu A^a_\mu=0$.

Uniqueness

- Let us consider a gauge field configuration A^a_μ which satisfies the Landau gauge condition $\partial_\mu A^a_\mu = 0$.
- Now, we perform a gauge transformation on $A^a_\mu \to A'^a_\mu$.
- If the gauge condition is ideal then $\partial_\mu A'^a_\mu \neq 0$.
- However, if we restrict ourselves to infinitesimal gauge transformations, $A'^a_\mu = A^a_\mu - D^{ab}_\mu \xi^b$, with $D^{ab}_\mu = \delta^{ab}\partial_\mu - gf^{abc}A^c_\mu$ being the covariant derivative in the adjoint representation of the gauge group and ξ^b, the infinitesimal parameter of the transformation, we see that

$$\partial_\mu A'^a_\mu = 0 \;\Rightarrow\; \underbrace{-\partial_\mu D^{ab}_\mu}_{\textit{FP operator}}\,\xi^b = -(\delta^{ab}\partial^2 - gf^{abc}A^c_\mu\partial_\mu)\xi^b = 0\,. \qquad (6)$$

- *Therefore, at the infinitesimal level, the gauge condition selects one representative per orbit if the FP operator does not develop zero-modes.*

Positivity of the FP operator

- To lift the FP determinant to the action, we assume it is positive.
- In this case, we have to show $-\partial_\mu D^{ab}_\mu \xi^b > 0$.
- *If this is true, then we can localize the FP determinant in the usual way* **and** *we automatically avoid the zero-modes.*

WARNING!

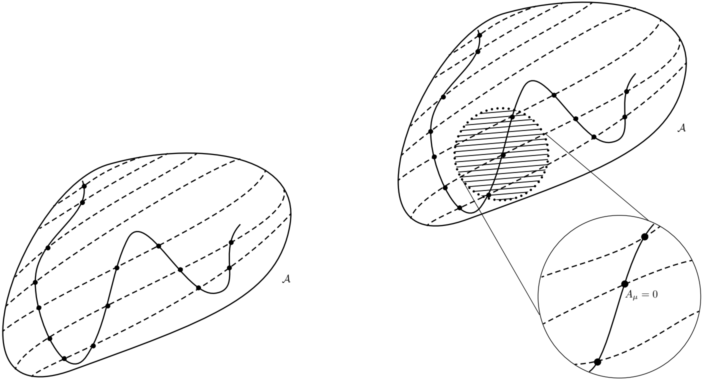
- In his seminal paper, V. Gribov proved $-\partial_\mu D^{ab}_\mu$ has zero-modes. *[Gribov '78] (see also [Sobreiro and Sorella '05])*
- The existence of zero-modes tells us that Landau gauge is not ideal. So, we have a residual gauge freedom.
- Also, the presence of zero-modes makes de FP procedure ill-defined.
- Such configurations, present after the imposition of the gauge condition, are known as Gribov copies and their appearance define the so-called Gribov problem.
- To implement the Faddeev-Popov procedure consistently, we should be able to remove these copies.

Perturbation theory

- Although Gribov copies exist, perturbative computations are performed without taking them into account and give consistent results. So, are these copies harmless?
- Performing perturbative computations around $A^a_\mu = 0$, we can see that the FP operator is positive,

$$-\partial^2\xi^a + \underbrace{gf^{abc}A^c_\mu\partial_\mu\xi^b}_{\textit{small in perturbation theory}} > 0 \tag{7}$$

- Hence, at the perturbative level, our assumptions about gauge-fixing are safe.
- However, as long as we go away from the perturbative regime, it is not possible to ensure those conditions $\Rightarrow$ improve the FP procedure to remove copies?
- It suggests these copies can play a role at the non-perturbative level.


$\mathcal{A}$
$\mathcal{A}$
$A_\mu = 0$

- So far, we just mentioned copies generated by infinitesimal gauge transformations.
- Those are not the full story! We also have copies connected via finite gauge transformations. *[van Baal '92]*
- Up to now, we know how to reasonably control infinitesimal copies only. Although it is not the complete scenario, the elimination of infinitesimal copies is an improvement of the FP procedure.

- In 1978, Singer showed that the Gribov problem is not a peculiarity of Landau gauge, but of all continuous (in field space) gauge choices when suitable regularity conditions are imposed to the gauge fields. *[Singer '78]*
- The origin of the problem is precisely the construction of a global section on a non-trivial fiber bundle.
- Although present in a huge class of gauges, a practical (partial) solution of the Gribov problem depends on the properties of the chosen gauge condition.
- A possible strategy is to try to deal with the problem in one particular gauge and try to extract general features (if any).

The Gribov region

Gribov's proposal

- Gribov pointed out the existence of copies, but *also* introduced a way to (partially) eliminate them!
- The idea is simple: We should define a suitable region in field space where the Faddeev-Popov operator is positive (and, thus, does not develop zero-modes) and contains all physical configurations ($\Rightarrow$ All gauge orbits must cross this region) and restrict the path integral domain to this region.
- A first proposal is known as the Gribov region and is defined by

$$\Omega = \left\{ A^a_\mu,\ \partial_\mu A^a_\mu = 0 \,\middle|\, -\partial_\mu D^{ab}_\mu > 0 \right\} . \tag{8}$$

Properties of Ω

- **Remark 1:** The operator $-\partial_\mu D^{ab}_\mu$, in the Landau gauge, is Hermitian. This makes the definition of a region where it is positive a meaningful task.
- The Gribov region Ω has remarkable geometrical features: *(i) It is bounded in every direction, (ii) It is convex and (iii) All gauge orbits cross it.* *[Dell'Antonio and Zwanziger '91]*
- Therefore, Ω is a suitable candidate to implement Gribov's idea.
- **Remark 2:** This region is NOT free of all Gribov copies, but at least of all infinitesimal ones. *[van Baal '92]*
- **Remark 3:** The boundary of Ω, denoted by $\partial\Omega$, is the so-called *Gribov horizon*. At the horizon, the Faddeev-Popov operator develops zero-modes.

The modified path integral

The path integral restricted to the Gribov region is formally written as

$$\mathcal{Z} = \int \mathcal{D}\Phi \mathcal{V}(\Omega) \mathrm{e}^{-S} \,, \tag{9}$$

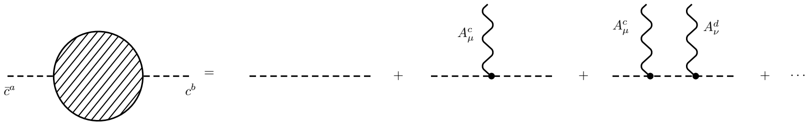
where Φ denotes all fields of the theory, $\mathcal{V}(\Omega)$ is responsible for the restriction of the integration domain and S is defined by

$$S = S_{\mathrm{YM}} + S_{\mathrm{gf}} + S_{\mathrm{ghosts}} \,, \tag{10}$$

with

$$S_{\mathrm{YM}} = \frac{1}{4} \int d^d x \; F^a_{\mu\nu} F^a_{\mu\nu} \,, \;\; S_{\mathrm{gf}} = \int d^d x \; b^a \partial_\mu A^a_\mu \,, \;\; S_{\mathrm{ghosts}} = \int d^d x \; \bar{c}^a \partial_\mu D^{ab}_\mu c^b \,. \tag{11}$$

- A practical way to write an explicit expression for $\mathcal{V}(\Omega)$ is to require that the FP ghosts two-point function, which correspond to the inverse of the FP operator, has no poles. This is the so-called *Gribov's no-pole condition*. *[Gribov '78]*
- Zwanziger was the first one to implement this condition at all orders in perturbation theory, but in a slightly different way. *[Zwanziger '89]*
- Working out the no-pole condition to all orders, it was possible to prove the result is equivalent to Zwanziger's one. *[Capri, Dudal, Guimaraes, Palhares and Sorella '13]*



The action free of (infinitesimal) copies

- The mofidication in the path integral can be effectively absorbed in the action.

$$S_{\mathrm{GZ}} = S + \gamma^4 H(A) - dV\gamma^4(N^2-1)\,, \tag{12}$$

where γ is the so-called Gribov parameter and H, the horizon function,

$$H(A) = g^2 \int d^dx d^dy\; f^{abc} A^b_\mu(x) \left[\mathcal{M}^{-1}(x,y)\right]^{ad} f^{dec} A^e_\mu(y)\,, \tag{13}$$

where $\mathcal{M}^{ab} \equiv -\partial_\mu D^{ab}_\mu$ is the FP operator.

- The Gribov parameter is not free and is fixed by a gap equation,

$$\langle H(A)\rangle = dV(N^2-1)\,. \tag{14}$$

- The horizon function is non-local $\Rightarrow$ the Gribov-Zwanziger action is non-local!

Local Gribov-Zwanziger action

- Remarkably, the Gribov-Zwanziger action can be cast in local form by the introduction of auxiliary fields.
- We introduce a pair of commuting $(\varphi^{ab}_{\mu}, \bar{\varphi}^{ab}_{\mu})$ and anti-commuting fields $(\omega^{ab}_{\mu}, \bar{\omega}^{ab}_{\mu})$.
- The local Gribov-Zwanziger action is

$$S_{\mathrm{GZ}} = S + \int d^d x \, \left(\bar{\varphi}^{ac}_{\mu} \mathcal{M}^{ab} \varphi^{bc}_{\mu} - \bar{\omega}^{ac}_{\mu} \mathcal{M}^{ab} \omega^{bc}_{\mu} + g\gamma^2 f^{abc} A^a_{\mu} (\varphi + \bar{\varphi})^{bc}_{\mu} \right) - dV\gamma^4 (N^2 - 1) \,. \tag{15}$$

- In local form, the gap equation is given by

$$\frac{\partial \mathcal{E}_v}{\partial \gamma^2} = 0 \;\;\Rightarrow\;\; \langle g f^{abc} A^a_{\mu} (\varphi + \bar{\varphi})^{bc}_{\mu} \rangle = 2dV\gamma^2 (N^2 - 1) \,. \tag{16}$$

- This action is local and renormalizable at all orders in perturbation theory. *[Zwanziger '89]*

Comments on γ

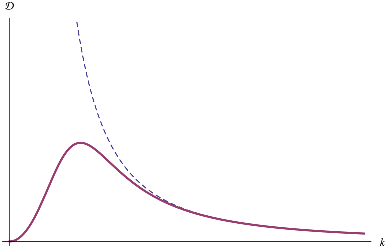
- The gap equation, at one-loop order, leads to $\gamma^2 \propto \mathrm{e}^{-\frac{1}{g^2}}$.
- In the UV, namely, $g \to 0$ implies $\gamma \to 0$.
- In the limit $\gamma \to 0$, we recover the usual (standard) FP action.
- This agrees with the fact that at the perturbative regime, the Faddeev-Popov operator is automatically positive and we don't need to take care of copies.
- The Gribov parameter is associated with the restriction of the path integral, *i.e.*, is related with a boundary introduced in field space.

Gluon propagator

In the Gribov-Zwanziger scenario, the gluon propagator is given by

$$\langle A^a_\mu(k)A^b_\nu(-k)\rangle = \delta^{ab}\mathcal{D}(k)P_{\mu\nu} = \delta^{ab}\frac{k^2}{k^4+2g^2\gamma^4 N}\left(\delta_{\mu\nu}-\frac{k_\mu k_\nu}{k^2}\right) . \quad (17)$$

- The FP action gives a divergent gluon propagator for $k = 0$.
- Taking into account the non-perturbative parameter γ, we obtain a vanishing gluon propagator at zero momentum (in fact, this is true at all orders in perturbation theory).



Due to the presence of the Gribov parameter, the gluon propagator violates reflection positivity $\Rightarrow$ we cannot interpret gluons as excitations in the physical spectrum (*signature of confinement?*).

- It is possible to show that the Gribov-Zwanziger theory has dynamical instabilities, namely, the formation of condensates is energetically favoured.
- Those instabilities are proportional to the Gribov parameter. In the usual FP action we could not see those effects in perturbation theory.
- Taking into account those effects leads to the *Refined Gribov-Zwanziger* action, which reproduces a *decoupling-massive* gluon propagator, and therefore, in agreement with *the most recent lattice data*. *[Dudal, Gracey, Sorella, Vandersickel and Verschelde '08]*
- This action was also derived under general algebraic arguments. *[**ADP** and Sobreiro '13 and **ADP** and Sobreiro '14]*

Refinement of the Gribov-Zwanziger action

For $d = 3, 4$, we take into account the existence of dimension two condensates. This gives rise to the Refined Gribov-Zwanziger action,

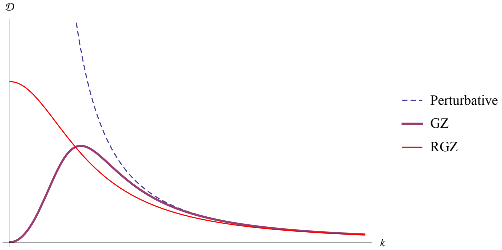
$$
\begin{aligned}
S_{\mathrm{RGZ}} &= S_{\mathrm{YM}} + \int d^dx \left(b^a \partial_\mu A^a_\mu + \bar{c}^a \partial_\mu D^{ab}_\mu c^b \right) \\
&+ \int d^dx \left(\bar{\varphi}^{ac}_\mu \mathcal{M}^{ab} \varphi^{bc}_\mu - \bar{\omega}^{ac}_\mu \mathcal{M}^{ab} \omega^{bc}_\mu + g\gamma^2 f^{abc} A^{h,a}_\mu (\varphi + \bar{\varphi})^{bc}_\mu \right) \\
&+ \frac{m^2}{2} \int d^dx \, A^a_\mu A^a_\mu - M^2 \int d^dx \left(\bar{\varphi}^{ac}_\mu \varphi^{ac}_\mu - \bar{\omega}^{ac}_\mu \omega^{ac}_\mu \right) . \qquad (18)
\end{aligned}
$$

Gluon propagator from the RGZ action

In $d = 3, 4$, the gluon propagator is

$$\langle A^a_\mu(k) A^b_\nu(-k)\rangle_{d=3,4} = \delta^{ab} \frac{k^2 + M^2}{(k^2+m^2)(k^2+M^2) + 2g^2\gamma^4 N} \left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right) , \tag{19}$$

which has a *decoupling-massive* behavior.



BRST symmetry

Faddeev-Popov action

- A very important concept in the FP quantization is the invariance of the gauge fixed Yang-Mills action under BRST transformations.
- This symmetry plays a very important role in the proof of perturbative renormalizability of the FP action and perturbative unitarity and gauge parameter independence of observables.

- The BRST transformations are given by

$$
\begin{aligned}
sA^a_\mu &= -D^{ab}_\mu c^b\,, & sc^a &= \frac{g}{2} f^{abc} c^b c^c\,, \\
s\bar{c}^a &= b^a\,, & sb^a &= 0\,,
\end{aligned} \tag{20}
$$

with $s^2 = 0$ and

$$
sS = s(S_{\rm YM} + S_{\rm gf} + S_{\rm ghosts}) = 0\,. \tag{21}
$$

BRST transformations and the Gribov-Zwanziger action

Considering the local form of the Gribov-Zwanziger action, we have the following transformations,

$$
\begin{aligned}
sA^a_\mu &= -D^{ab}_\mu c^b\,, & sc^a &= \frac{g}{2} f^{abc} c^b c^c\,, \\
s\bar{c}^a &= b^a\,, & sb^a &= 0\,, \\
s\varphi^{ab}_\mu &= \omega^{ab}_\mu\,, & s\omega^{ab}_\mu &= 0\,, \\
s\bar{\omega}^{ab}_\mu &= \bar{\varphi}^{ab}_\mu\,, & s\bar{\varphi}^{ab}_\mu &= 0\,.
\end{aligned}
\tag{22}
$$

and

$$
sS_{\mathrm{GZ}} = \gamma^2 \int d^d x \, \left(g f^{abc} D^{ae}_\mu c^e (\bar{\varphi}^{bc}_\mu + \varphi^{bc}_\mu) + g f^{abc} A^a_\mu \omega^{bc}_\mu \right)\,. \tag{23}
$$

- *The Gribov-Zwanziger action breaks the BRST symmetry explicitly!*
- Although explicit, the breaking is *soft*. When we go to the UV, $\gamma \to 0$ and we recover BRST invariance.
- However, when γ is not negligible, *i.e.*, when we are far from the UV regime, standard BRST symmetry seems to be broken within this framework.

BRST *soft* breaking

- At present, a full understanding of the BRST breaking is still lacking. *[Cucchieri et al., Lavrov et al., Sorella et al., Zwanziger et al.]*
- In 2014, the computation of a BRST exact correlator in Landau gauge was perfomed in large lattice simulations pointing towards a breaking of the BRST symmetry. *[Cucchieri, Dudal, Mendes and Vandersickel '14]*

- Intuitively, the breaking of the BRST symmetry is associated to the introduction of a boundary in field space.
- This framework suggests that the non-perturbative regime of Yang-Mills theories is characterized by a soft breaking of the standard BRST symmetry.

How to reconcile BRST with the Gribov horizon? Is it possible?

Gauge invariant A^h field

- Let us consider the transverse field A^h, $\partial_\mu A^h_\mu = 0$, obtained from the minimization of

$$f[U;A] = \mathrm{Tr} \int d^dx \; A^U_\mu A^U_\mu \,. \tag{24}$$

[Zwanziger '90, M. Lavelle and D. McMullan '97]

- This field is gauge invariant order by order in g and can be formally written as

$$A^h_\mu = \left(\delta_{\mu\nu} - \frac{\partial_\mu \partial_\nu}{\partial^2}\right)\left(A_\nu - ig\left[\frac{1}{\partial^2}\partial A, A_\nu\right] + \frac{ig}{2}\left[\frac{1}{\partial^2}\partial A, \partial_\nu \frac{1}{\partial^2}\partial A\right]\right) + O(A^3)\,. \tag{25}$$

- Its gauge invariance implies $sA^h = 0$.
- The form of the horizon function $H(A)$ and of A^h allow us to write the following expression Phys.Rev. D92 (2015) no.4, 045039

$$H(A) = H(A^h) - R(A)(\partial A)\,. \tag{26}$$

The "new" Gribov-Zwanziger action

The non-local Gribov-Zwanziger action is rewritten as

$$\tilde{S}_{\mathrm{GZ}} = S_{\mathrm{YM}} + \int d^d x \left(b^{h,a} \partial_\mu A^a_\mu + \bar{c}^a \partial_\mu D^{ab}_\mu c^b \right) + \gamma^4 H(A^h) \,, \tag{27}$$

with

$$b^{h,a} = b^a - \gamma^4 R^a(A) \tag{28}$$

and

$$H(A^h) = g^2 \int d^d x d^d y \; f^{abc} A^{h,b}_\mu(x) \left[\mathcal{M}^{-1}(A^h) \right]^{ad} f^{dec} A^{h,e}_\mu(y) \,. \tag{29}$$

As before, we introduce the localizing auxiliary fields. The resulting action is

$$\begin{aligned} S_{\mathrm{GZ}} &= S_{\mathrm{YM}} + \int d^d x \left(b^{h,a} \partial_\mu A^a_\mu + \bar{c}^a \partial_\mu D^{ab}_\mu c^b \right) \\ &+ \int d^d x \left(\bar{\varphi}^{ac}_\mu \left[\mathcal{M}(A^h) \right]^{ab} \varphi^{bc}_\mu - \bar{\omega}^{ac}_\mu \left[\mathcal{M}(A^h) \right]^{ab} \omega^{bc}_\mu + g \gamma^2 f^{abc} A^{h,a}_\mu (\varphi + \bar{\varphi})^{bc}_\mu \right) . \end{aligned} \tag{30}$$

BRST invariance

- We define the BRST transformations, as follows

$$\begin{aligned} sA^a_\mu &= -D^{ab}_\mu c^b\,, & sc^a &= \frac{g}{2} f^{abc} c^b c^c\,, \\ s\bar{c}^a &= b^{h,a}\,, & sb^{h,a} &= 0\,, \\ s\varphi^{ab}_\mu &= 0\,, & s\omega^{ab}_\mu &= 0\,, \\ s\bar{\omega}^{ab}_\mu &= 0\,, & s\bar{\varphi}^{ab}_\mu &= 0\,. \end{aligned} \tag{31}$$

- These transformations define a symmetry of the Gribov-Zwanziger action, namely,

$$sS_{\mathrm{GZ}} = 0\,. \tag{32}$$

- However, due to the presence of A^h, S_{GZ} is still a non-local action.

An important check is that

$$\frac{\partial S_{GZ}}{\partial \gamma^2} \neq s(\text{something})\,, \tag{33}$$

i.e., the Gribov parameter is not akin to a gauge parameter. Therefore, it will enter in correlation functions of physical quantities. Also, written in terms of A^h, the equation that fixes γ, the gap equation, is

$$\langle H(A^h)\rangle = dV(N^2-1)\,, \tag{34}$$

which is *gauge invariant!*

Local action

It is possible to cast the BRST-invariant action in local form through the introduction of a Stueckelberg-like field ξ. PRD 94 (2016) no.2, 025035, PRD 95 (2017) no.4, 045011, PRD 96 (2017) no.5, 054022

$$\begin{aligned} S_{\rm GZ} &= S_{\rm YM} + S_{\rm FP} - \int d^4x \left(\bar{\varphi}^{ac}_{\mu}\mathcal{M}^{ab}(A^h)\varphi^{bc}_{\mu} - \bar{\omega}^{ac}_{\mu}\mathcal{M}^{ab}(A^h)\omega^{bc}_{\mu}\right) \\ &- \gamma^2\int d^4x\, gf^{abc}(A^h)^a_{\mu}(\varphi+\bar{\varphi})^{bc}_{\mu} + \int d^4x\, \tau^a\partial_{\mu}(A^h)^a_{\mu} - \int d^4x\, \bar{\eta}^a\mathcal{M}^{ab}(A^h)\eta^b \end{aligned} \tag{35}$$

The field A^h is now written as

$$A^h_{\mu} = h^{\dagger}A_{\mu}h + \frac{i}{g}h^{\dagger}\partial_{\mu}h\,, \tag{36}$$

with

$$h = \mathrm{e}^{ig\xi^aT^a} \equiv \mathrm{e}^{ig\xi}\,. \tag{37}$$

Note: The GZ action is written in local form, but it is not polynomial on the local fields.

The complete BRST transformations are given by

$$
\begin{aligned}
sA^a_\mu &= -D^{ab}_\mu c^b\,, & sc^a &= \frac{g}{2} f^{abc} c^b c^c\,, \\
s\bar{c}^a &= b^a\,, & sb^a &= 0\,, \\
s\varphi^{ab}_\mu &= 0\,, & s\omega^{ab}_\mu &= 0\,, \\
s\bar{\omega}^{ab}_\mu &= 0\,, & s\bar{\varphi}^{ab}_\mu &= 0\,, \\
sh^{ij} &= -igc^a (T^a)^{ik} h^{kj}\,, & sA^{h,a}_\mu &= 0\,, \\
s\tau^a &= 0\,, & s\bar{\eta}^a &= 0\,, \\
s\eta^a &= 0\,, & s^2 &= 0\,.
\end{aligned}
\tag{38}
$$

The Gribov-Zwanziger expressed in the new variables is local, renormalizable to all orders in perturbation theory and BRST-symmetric. Also, for gauge-invariant operators $\mathcal{O}$ one has

$$
\langle \mathcal{O}(x)\mathcal{O}(y)\rangle\Big|^{\text{Landau}}_{\text{Original GZ}} = \langle \mathcal{O}(x)\mathcal{O}(y)\rangle\Big|^{\text{Landau}}_{\text{“New” GZ}}
\tag{39}
$$

Linear covariant gauges

Linear covariant gauges

- This class of gauges contains a gauge parameter α and is defined by

$$\partial_\mu A^a_\mu = -i\alpha b^a \,. \tag{40}$$

- In this case, the construction of a Gribov region is not as clear as in the Landau gauge.
- For general α (different from zero), the Faddeev-Popov operator is not Hermitian.

Proposal

- We propose to restrict the path integral to Ω^h,

$$\Omega^h = \left\{ A^a_\mu \;\middle|\; \partial_\mu A^a_\mu = -i\alpha b^a \,,\;\; \mathcal{M}^{ab}(A^h) > 0 \right\} \,. \tag{41}$$

- This region is free of “regular” zero-modes and

$$\lim_{\alpha\to 0} \Omega^h = \Omega^{\mathrm{Landau}} \tag{42}$$

- In view of the results discussed before in the Landau gauge, we extend our results to linear covariant gauges. PRD 92 (2015) no.4, 045039, PRD 93 (2016) no.6, 065019,

Gribov-Zwanziger action in linear covariant gauges

The BRST symmetric Gribov-Zwanziger action in linear covariant gauges is constructed as PRD 96 (2017) no.5, 054022

$$\begin{aligned} S_{\mathrm{GZ}}^{\mathrm{LCG}} &= S_{\mathrm{LCG}}^{\mathrm{FP}} - \int d^4x \left(\bar{\varphi}_\mu^{ac} \mathcal{M}^{ab}(A^h) \varphi_\mu^{bc} - \bar{\omega}_\mu^{ac} \mathcal{M}^{ab}(A^h) \omega_\mu^{bc} \right) \\ &- \gamma^2 \int d^4x \, g f^{abc} (A^h)_\mu^a (\varphi + \bar{\varphi})_\mu^{bc} + \int d^4x \, \tau^a \partial_\mu (A^h)_\mu^a - \int d^4x \, \bar{\eta}^a \mathcal{M}^{ab}(A^h) \eta^b \,. \end{aligned} \tag{43}$$

- This action is local, BRST-invariant and renormalizable to all orders in perturbation theory. PRD 96 (2017) no.5, 054022
- It effectively restricts the path integral domain to a region which removes a class of copies in linear covariant gauges.

- As in the Landau gauge, further non-perturbative effects can be taken into account in this case. PRD 93 (2016) no.6, 065019, PRD 96 (2017) no.5, 054022
- In $d = 3, 4$ the Gribov-Zwanziger action in linear covariant gauges can be "refined" by the inclusion of the following operators.

$$\frac{m^2}{2}\int d^d x\, A^{h,a}_{\mu} A^{h,a}_{\mu} \qquad \text{and} \qquad -M^2 \int d^d x \left(\bar{\varphi}^{ab}_{\mu}\varphi^{ab}_{\mu} - \bar{\omega}^{ab}_{\mu}\omega^{ab}_{\mu}\right) . \qquad (44)$$

- Very good agreement with the most recent lattice data.

The BRST invariance allows for the following proofs: PRD 95 (2017) no.4, 045011

- Correlation functions of gauge-invariant operators are independent of α.
- The mass parameters (γ^2, m^2, M^2) are α-independent and can enter physical correlators.
- The longitudinal part of the gluon propagator is tree-level exact.
- The pole mass of the transverse component of the gluon propagator is α-independent.
- The poles of the gauge-invariant correlation function $\langle A^{h,a}_\mu(-p)A^{h,b}_\nu(p)\rangle$ are the same as those of the transverse part of the gluon propagator.
- Also,

$$\langle A^{h,a}_\mu(-p)A^{h,b}_\nu(p)\rangle = \langle A^{h,a}_\mu(-p)A^{h,b}_\nu(p)\rangle_{\alpha=0} = \langle A^{a}_\mu(-p)A^{b}_\nu(p)\rangle_{\rm Landau}\,. \quad (45)$$

Universality

One can consider more general covariant, color invariant and renormalizable gauges as Phys.Lett. B781 (2018) 48-54

$$
\begin{aligned}
S_{gf} &= \int d^4x \, s\left(\bar{c}^a(\partial_\mu A^a_\mu - \mu^2\xi^a + \frac{g}{2}\beta f^{abc}\bar{c}^b c^c) - i\frac{\alpha}{2}\bar{c}^a b^a\right) \\
&= \int d^4x \left(ib^a\partial_\mu A^a_\mu + \frac{\alpha}{2}b^a b^a - i\mu^2 b^a\xi^a + ig\beta f^{abc} b^a\bar{c}^b c^c\right. \\
&+ \left.\frac{g^2}{4}\beta f^{abc} f^{cmn}\bar{c}^a\bar{c}^b c^m c^n\right) + \int d^4x \left(\bar{c}^a\partial_\mu D^{ab}_\mu(A)c^b + \mu^2\bar{c}^a g^{ab}(\xi)c^b\right) \,,
\end{aligned}
$$

and the following action

$$
\begin{aligned}
\tilde{S} &= S_{\rm YM} + S_{gf} - \int d^4x \left(\bar{\varphi}^{ac}_\mu \mathcal{M}^{ab}(A^h)\varphi^{bc}_\mu - \bar{\omega}^{ac}\mathcal{M}^{ab}(A^h)\omega^{bc}_\mu\right) \\
&- \gamma^2 \int d^4x \, gf^{abc}(A^h)^a_\mu(\varphi+\bar{\varphi})^{bc}_\mu + \int d^4x \, \left(\tau^a\partial_\mu(A^h)^a_\mu - \bar{\eta}^a\mathcal{M}^{ab}(A^h)\eta^b\right) .
\end{aligned}
$$

For gauge invariant local operators $\mathcal{O}(x)$, one has

$$\langle\mathcal{O}(x)\mathcal{O}(y)\rangle\Big|_{\tilde{S}} = \left(\langle\mathcal{O}(x)\mathcal{O}(y)\rangle\Big|_{\tilde{S}}\right)_{\sigma_i=0} = \langle\mathcal{O}(x)\mathcal{O}(y)\rangle\Big|^{\text{Landau}}_{\text{“new” GZ}} = \langle\mathcal{O}(x)\mathcal{O}(y)\rangle\Big|^{\text{Landau}}_{\text{GZ}},$$

with $\sigma_i = (\alpha, \beta, \mu^2)$.

In this sense, Zwanziger's horizon function gains universal character.

What about gravity? Brief comments...

- The treatment of quantum gravity as an ordinary quantum field theory also demands a gauge-fixing procedure.
- It is known that in this case, Gribov copies will appear as well.
- Expanding the metric $g_{\mu\nu}$ as

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu} \tag{46}$$

  the copies equation would correspond to

$$\bar{\nabla}^{\mu}\nabla_{\mu}\xi_{\nu} + \bar{\nabla}^{\mu}\nabla_{\nu}\xi_{\mu} = 0\,, \tag{47}$$

  in Feynman-de Donder gauge, where ξ parameterizes the gauge transformation.
- This equation is difficult to solve in general since it involves the background metric as well as the fluctuations $h_{\mu\nu}$.
- For some specific cases, it was shown that this kind of copies equation has solutions! *[Das and Kaku '79, Esposito, Pelliccia and Zaccaria '04, Anabalon, Canfora, Giacomini and Oliva '11, Holdom and Ren '16]*

- Again, the existence of copies is severely related to the chosen background metric $\bar{g}_{\mu\nu}$.
- This naturally raises the following question: Does the account of Gribov copies in gravity spoil the Slavnov-Taylor identities? And what about background independence?
- This suggests that, as in Yang-Mills, a solution to the problem might produce a non-perturbative BRST-invariance which would be able to control background dependence after all.
- Up to now we don't know how to handle Gribov copies in gravity (in fact, even in the background field method in Yang-Mills theories). Phys.Rev.D 102 (2020) 074029, Phys. Rev. D 106 (2022) 2, 025015

Conclusions

- A reformulation of the Gribov-Zwanziger action in Landau gauge with gauge invariant variables was proposed.
- A new set of BRST transformations was constructed. These transformations correspond to a symmetry of the Gribov-Zwanziger action.
- The new BRST symmetry feels the restriction of the path integral to the Gribov region and contains the non-perturbative parameter γ.
- The reformulation puts the Gribov parameter as a manifestly gauge invariant quantity.
- The BRST-invariant formulation can accomodate matter fields - dressed matter fields are also available in this framework. Phys. Rev. D 107 (2023) 11, 114006

Thank You!