Workshop “Representation theory’s hidden motives” Münster–Sydney 2021

Vigleik Angeltveit: **The Picard group of Equivariant Stable Homotopy Theory and the Slice Spectral Sequence**

Equivariant stable homotopy groups are usually graded on the real representation ring. But it is possible to grade them on the Picard group instead. I will present some sample Picard group calculations, and explain how it makes certain calculations more tractable.

Robert Cass: **Perverse mod p sheaves on the affine Grassmannian**

The geometric Satake equivalence relates representations of a reductive group to perverse sheaves on an affine Grassmannian. Depending on the intended application, there are several versions of this equivalence for different sheaf theories and versions of the affine Grassmannian. In this talk we will discuss a version of the geometric Satake equivalence for mod p sheaves in characteristic p. In this case an abstract Tannakian monoid arises rather than a reductive group. We will discuss what is known about this monoid, including a compatibility with Levi restriction via a form of hyperbolic localization. We will also discuss potential relations with other versions of the geometric Satake equivalence. Part of this is joint work with Cédric Pépin.

Kevin Coulembier: **Frobenius exact tensor categories**

Partly motivated by Grothendieck’s original vision for motives, the question arises of when a tensor category (k-linear symmetric monoidal rigid abelian category) is tannakian, i.e. is the representation category of an affine group scheme, or more generally of a groupoid in schemes. Just as interesting is the question what one can say about tensor categories which are not tannakian. For fields k of characteristic 0, these questions were answered very elegantly and conclusively by Deligne. I will report on recent progress on these questions in prime characteristic, based on joint work with Pavel Etingof and Victor Ostrik.

Jens Eberhardt: **Motivic Springer Theory**

Algebras and their representations can often be constructed geometrically in terms of convolution of cycles. For example, the Springer correspondence describes how irreducible representations of a Weyl group can be realised in terms of a convolution action on the vector spaces of irreducible components of Springer fibers. Similar situations yield the affine Hecke algebra, quiver Hecke algebra (KLR algebra) or quiver Schur algebra. In this spirit, we show that these algebras and their representations can be realised in terms of certain equivariant motivic sheaves called Springer motives. On our way, we will discuss weight structures and their applications to motives as well as Koszul and Ringel duality. This is joint work with Catharina Stroppel.

Lie Fu: **Multiplicative McKay correspondence for surfaces**

Given a finite subgroup of $SL(2, \mathbf{C})$, the classical 2-dimensional McKay correspondence provides a connection between the representation theory of the group and the exceptional divisor of the minimal resolution of the quotient singularity that the group naturally gives. I report a joint work with Zhiyu Tian in 2019, where we gave a multiplicative version of this correspondence in terms of the orbifold cohomology of the quotient stack, which fits into the context of crepant resolution conjectures. Our isomorphism is motivic and global (for any surface orbifold).

Victoria Hoskins: **Motives of stacks of sheaves and bundles on curves**

I will present formulae for motives (with rational coefficients) of moduli stacks of coherent sheaves and vector bundles on a smooth projective curve C of fixed rank and degree. The proof involves a rigidification using Flag-Quot schemes parametrising Hecke modifications. This is joint work in progress with Simon Pepin Lehalleur.

Marc Hoyois: **Hilbert schemes in motivic homotopy theory**

Hilbert schemes of affine spaces are highly singular schemes with a complicated geometry, but they exhibit some interesting stability phenomena as the dimension of the affine space goes to infinity. I will explain a computation of the motives of these Hilbert schemes, and the relevance of their Gorenstein and local complete intersection loci in motivic homotopy theory. This is based on joint work with Joachim Jelisiejew, Denis Nardin, Burt Totaro, and Maria Yakerson.

Masoud Kamgarpour: **Langlands correspondence for hypergeometric motives**

Hypergeometric sheaves are rigid local systems on the punctured projective line. Their study originated in the seminal work of Riemann on the Euler–Gauss hypergeometric function and has blossomed into an active field with connections to many areas of mathematics. In the modern era, the subject of hypergeometric (and more generally rigid) local systems was rejuvenated in the works of Katz, who elucidated their motivic nature. The core conjecture of the geometric Langlands program predicts that one can associate to every local system on a curve X, a Hecke eigensheaf on the moduli of bundles (with appropriate level structures) on X. In this talk, I will explain how to construct the Hecke eigensheaves associated to hypergeometric local systems. This is based on joint work with Lingfei Yi (`arxiv.org/abs/2006.10870`)

Shane Kelly: **Motives with modulus over a general base**

This is joint work with Hiroyasu Miyazaki. Motives with modulus, as developed by Kahn, Miyazaki, Saito, Yamazaki is an extension of Voevodsky's theory of motives with the aim of capturing non-$\mathbf{A}^1$-invariant phenomena that is inaccessible to Voevodsky's theory but still "motivic" in nature, such as wild ramification or the unipotent parts of generalised Jacobians. I will give an overview of the theory, and explain how it can be generalised to an arbitrary qcqs base. Both equal and mixed characteristic bases are allowed.

Martina Lanini: **Totally nonnegative Grassmannians, Grassmann necklaces and quiver Grassmannians**

Totally nonnegative (tnn) Grassmannians are subvarieties of (real) Grassmannians which have been widely investigated thanks to the several applications in mathematics and physics. In a seminal paper on the subject, Postnikov constructed a cellularisation of the tnn Grassmannians whose cell closure relation is encoded in the purely combinatorial defined poset of Grassmann necklaces. In this talk I will report on joint work with Evgeny Feigin and Alexander Puetz in which we consider a linear algebraic analogue of Grassmann necklaces that gives rise to a quiver Grassmannian X. Grassmann necklaces encode also in this case some geometric information: they parametrise the fixed point set of an algebraic torus acting on our quiver Grassmannian, and as a poset they coincide with the poset coming from a cellular decomposition of X.

Marc Levine: **Atiyah-Bott localization for Witt sheaf cohomology, with applications**

Atiyah-Bott localization for singular cohomology of a space with a torus action has proven to be an effective tool in many areas, including enumerative geometry. We give here a parallel for cohomology with Witt-sheaf coefficients, which is useful for computing quadratic refinements of classical numerical invariants. Since the torus-equivariant Witt-sheaf cohomology is the same as the non-equivariant cohomology, there is no parallel theory for torus actions. We describe a version for schemes with an SL_2^n or N^n-action, where N is the normalizer of the torus in SL_2. As an application, we compute (joint with Sabrina Pauli) a quadratic count of cubic rational curves on certain smooth hypersurfaces or complete intersections in a projective space; the first example being the case of a quintic hypersurface.

Timo Richarz: **Motivic Satake equivalence**

The geometric Satake equivalence due to Lusztig, Drinfeld, Ginzburg, Mirković and Vilonen is an indispensable tool in the Langlands program. It relates the category of sheaves on an infinite dimensional space called the affine Grassmannian to representations of the Langlands dual group. Versions of this equivalence are known for different cohomology theories such as Betti cohomology, algebraic or arithmetic D-modules and ℓ-adic cohomology. In this talk, I explain how to apply the theory of motivic complexes as developed by Voevodsky, Ayoub, Cisinski-Déglise and many others to the construction of a motivic Satake equivalence with rational coefficients. Under suitable cycle class maps, one recovers the more classical equivalences. This is joint work with Jakob Scholbach.

Nikita Semenov: **Hopf-theoretic approach to motives of twisted flag varieties**

Chow motives were introduced by Alexander Grothendieck in the 1960s, and they have since become a fundamental tool for investigating the structure of algebraic varieties. More recently, Levine and Morel defined a universal oriented cohomology theory, called the algebraic cobordism. It allows one to consider algebraic analogues of well studied topological oriented cohomology theories such as Morava K-theories.

Let G be a split semisimple algebraic group over a field and let A be an oriented cohomology theory in the sense of Levine-Morel. In the talk I will explain a relation between the A-motives of geometrically cellular smooth projective G-varieties and the Hopf algebra structure of $A(G)$. Using this I will provide various applications to the structure of motives of twisted flag varieties. This is a joint work with Victor Petrov.

Wolfgang Soergel: **Geometric interpretation of the homotopy category of special bimodules through mixed Tate motives**

(with M. Wendt and R. Virk) Six-functor-formalisms without extensions between Tate motives give particularily nice tilting equivalences between equivariant derived categories and algebraic structures. This leads to a natural geometric interpetation of Khovanov Kohomology.

Markus Spitzweck: **A representation theorem for integral étale abelian motives**

We show that a full symmetric monoidal stable subcategory of the category of integral étale motives over a base scheme essentially generated by homotopy colimits by tensor powers of an abelian motive (of a certain type) and its dual can be modelled by modules over a commutative algebra in a large category of derived representations of a general linear group over the integers. This generalizes rational results by Iwanari.

David Treumann: **G-spectra of cyclic defect**

Broue's conjecture predicts that a block of $\mathbf{Z}_p[G]$ and of $\mathbf{Z}_p[N]$ have equivalent derived categories, when N is the normalizer of the defect group of the block and this defect group is commutative. In case the defect group is cyclic, there are two old proofs of it, one by Rickard and a while later one by Rouquier. Tony Feng, Allen Yuan and I can construct a similar equivalence between blocks of p-complete G-spectra and p-complete N-spectra, that induces Rouquier's equivalence after taking homology.

Kari Vilonen: **Mixed Hodge modules and representation theory**

I will explain how mixed Hodge modules can be utilized to understand representation theory of real groups. I will discuss a set of conjectures which postulate the existence of a mixed Hodge structure on certain representations of real groups. This is joint work with Wilfried Schmid. I will also explain joint work with Dougal Davis which constitutes some progress towards a proof of these conjectures.

Ting Xue: **Character sheaves, Hecke algebras and Hessenberg varieties**

We discuss character sheaves in the setting of graded Lie algebras. Via a nearby cycle construction irreducible representations of Hecke algebras of complex reflection groups at roots of unity enter the description of character sheaves. Recent work of Lusztig and Yun relates character sheaves to irreducible representations of trigonometric double affine Hecke algebras. We will explain the connection between the work of Lusztig-Yun and our work, and discuss some conjectures arising from this connection, in particular, applications to cohomology of Hessenberg varieties and affine Springer fibres. This is based on joint work with Kari Vilonen and partly with Tsao-Hsien Chen and Misha Grinberg.

Yaping Yang: **The perverse coherent sheaves on toric Calabi-Yau 3-folds and Cohomological Hall algebras**

Let X be a smooth local toric Calabi-Yau 3-fold. On the cohomology of the moduli spaces of certain sheaves on X, there is an action of the cohomological Hall algebra (COHA) of Kontsevich and Soibelman via "raising operators". I will discuss the "double" of the COHA that acts on the cohomology of the moduli space by adding the "lowering operators". We associate a root system to X. The double COHA is expected to be the shifted Yangian of this root system. We also give a prediction for the shift in terms of an intersection pairing. We provide evidence of the aforementioned expectation in various examples.

This is based on my joint work with M. Rapcak, Y. Soibelman, and G. Zhao.

Gufang Zhao: **Frobenii on Morava E-theoretical quantum groups**

This talk is based on joint work with Yaping Yang. We study a family of quantum groups constructed using Morava E-theory of Nakajima quiver varieties. We define the quantum Frobenius homomorphisms among these quantum groups. This is a geometric generalization of Lusztig's quantum Frobenius from the quantum groups at a root of unity to the enveloping algebras. The main ingredient in constructing these Frobenii is the transchromatic character map of Hopkins, Kuhn, Ravenal, and Stapleton. In the talk we explain the construction of the Frobenius homomorphism, as well as an application - a Steinberg type tensor product formula for representations of the quantum groups.

Changlong Zhong: **K-theory stable bases of Springer resolutions**

In enumerative geometry, K-theory stable bases are defined by Maulik and Okounkov and are used to construct a quantum group action on K-theory of quiver varieties. In this talk I will focus on such bases for Springer resolutions. I will talk about the connection between K-theory stable bases, the Hecke algebra, and the wall crossing functors.