

M^2A^2 - **Mathematical Modeling and Analysis**

A Conference in Honor of Angela Stevens

Still $\prod_{k=1}^3 (k+2)$ after all these months

7 – 9 June 2022 | Münster, Germany



Speakers

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| <i>Björn Bringmann (Princeton)</i> | <i>Felix Otto (Leipzig)</i> |
| <i>Martin Burger (Erlangen)</i> | <i>Mariya Ptashnyk (Edinburgh)</i> |
| <i>José Antonio Carrillo (Oxford)</i> | <i>Angkana Rüland (Heidelberg)</i> |
| <i>Bernold Fiedler (Berlin)</i> | <i>Arnd Scheel (Minneapolis)</i> |
| <i>Julian Fischer (Klosterneuburg)</i> | <i>Christian Schmeiser (Wien)</i> |
| <i>Gero Friesecke (TU München)</i> | <i>Hartmut Schwetlick (Bath)</i> |
| <i>Willi Jäger (Heidelberg)</i> | <i>Sebastian Throm (Umeå)</i> |
| <i>Ansgar Jüngel (Wien)</i> | <i>Juan J. L. Velázquez (Bonn)</i> |
| <i>Kyungkeun Kang (Seoul)</i> | <i>Enrique Zuazua (Erlangen)</i> |
| <i>Christof Melcher (Aachen)</i> | |

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Schedule

Tuesday, June 7	Wednesday, June 8	Thursday, June 9
09:00–09:50 welcome & talk: Mariya Ptashnyk	09:00–09:40 José A. Carrillo	09:00–09:40 Kyungkeun Kang
10:00–10:40 Angkana Rüländ	09:50–10:30 Arnd Scheel	09:50–10:30 Gero Friebecke
Coffee Break	Coffee Break	Coffee Break
11:30–12:10 Juan Velázquez	11:10–11:50 Julian Fischer	11:10–11:50 Ansgar Jüngel
Lunch – Reibekuchen	Lunch	Lunch
14:30–15:10 Felix Otto	13:45–14:25 Hartmut Schwetlick	13:00–13:40 Christian Schmeiser
Coffee Break	14:35–15:15 Enrique Zuazua	Coffee Break
16:00–16:40 Martin Burger	Coffee Break	14:20–15:00 Bernold Fiedler
16:50–17:30 Björn Bringmann	16:00–16:40 Sebastian Throm	15:10–16:00 talk & goodbye: Christoph Melcher
18:00 Cheese & Wine Reception	16:50–17:30 Willi Jäger	
	19:30 Conference Dinner	

Tuesday 9:00

Mariya Ptashnyk (Heriot-Watt University, Edinburgh)

Interacting particle systems & Chemotaxis: Angela Revisited

In this talk we present derivation of a fractional cross-diffusion system as the many-particle limit of a multi-species system of moderately interacting particles. Our approach is based on the techniques developed by Oelschläger (1989) and Stevens (2000). A well-posedness result is proved for the regularized macroscopic system, which then yields the same results for the fractional cross-diffusion system in the limit.

This talk is based on joint work with Esther S. Daus and Claudia Raithel.

Daus E., Ptashnyk M., Raithel C., Derivation of a fractional cross-diffusion system as the limit of a stochastic many-particle system driven by Lévy noise, *J Differential Equations*, 309, 386-426, 2022

Tuesday 10:00

Angkana Rüland (Heidelberg University)

Quantitative unique continuation properties of (nonlocal) discrete elliptic equations

While (quantitative) unique continuation properties hold for large classes of local and nonlocal equations and have been intensively studied, for their discrete counterparts only weaker versions of these persist in general. In this talk I discuss quantitative versions of these, both for local and nonlocal equations with optimal correction terms in the discretization error and describe consequences for associated inverse problems.

This is based on joint work with Aingeru Fernández Bertolin, Luz Roncal and Diana Stan.

Tuesday 11:30

Juan Velázquez (University of Bonn)

Oscillatory solutions in coagulation-fragmentation models

In kinetic models satisfying the so-called detailed balance condition, it is possible to construct an entropy functional which can be used to derive convergence to equilibrium results. On the other hand, there are many physical situations (typically open systems) where it is natural to use kinetic equations for which the detailed balance condition fails. In these cases more complicated dynamical behaviours can arise, for instance periodic solutions.

A class of kinetic equations where it is not obvious if temporal oscillations can take place are the coagulation-fragmentation equations. These equations have been extensively used to model chemical systems. In this talk I will describe two examples of coagulation-fragmentation models for which the onset of periodic oscillations can be rigorously proved. One of the models describes the formation of large clusters in a kinetic equation having a source of monomers. The second model is a closed coagulation-fragmentation model without external monomer sources, but in which the detailed balance condition fails at one of the reactions.

Tuesday 14:30

Felix Otto (Max-Planck-Institute for Mathematics in the Sciences, Leipzig)

Regularity structures without Feynman diagrams

Singular stochastic PDE are those stochastic PDE in which the noise is so rough that the nonlinearity requires a renormalization. Hairer's regularity structures provide a framework for the solution theory. His notion of a model can be understood as providing a (formal) parameterization of the entire solution manifold of the renormalized equation. In this talk, I will focus on the stochastic estimates of the model.

I shall present a more analytic than combinatorial approach: Instead of using trees to index the model, we consider all partial derivatives w.r.t. the function defining the nonlinearity (and thus work with multi-indices as index set). Instead of a Gaussian calculus guided by Feynman diagrams arising from the trees, we consider first-order partial derivatives w.r.t. the noise, i.e. Malliavin derivatives.

We employ tools from quantitative stochastic homogenization like spectral gap estimates, which naturally complement the standard choice of renormalization, and annealed estimates, which as opposed to their quenched counterparts preserve scaling.

This is joint work with P. Linares, M. Tempelmayr, and P. Tsatsoulis, based on work with J. Sauer, S. Smith, and Angela's new colleague Hendrik.

Tuesday 16:00

Martin Burger (University of Erlangen–Nuremberg)

Phase Separation in Systems of Active Brownian Particles

Active Brownian Particles have recently emerged as a paradigm for complex system behaviour with minimal interactions. Interestingly, phase separation and cluster formation can be observed in such systems even in the absence of any attractive force. We discuss a mathematical viewpoint on such systems by discussing different microscopic models and their macroscopic counterparts. In particular we are interested in deriving more rigorous results on phase separation phenomena, which turns out to be difficult even to pose as a meaningful mathematical question.

Tuesday 16:50

Björn Bringmann (Institute for Advanced Study, Princeton)

Invariant Gibbs measures for the three-dimensional cubic nonlinear wave equation.

In this talk, we prove the invariance of the Gibbs measure for the three-dimensional cubic nonlinear wave equation, which is also known as the hyperbolic Φ_3^4 -model. This result is the hyperbolic counterpart to seminal works on the parabolic Φ_3^4 -model by Hairer'14 and Hairer-Matetski'18.

In the first half of this talk, we illustrate Gibbs measures in the context of Hamiltonian ODEs, which serve as toy-models. We also connect our theorem with classical and recent developments in constructive QFT, dispersive PDEs, and stochastic PDEs.

In the second half of this talk, we give a non-technical overview of the proof. As part of this overview, we first introduce a caloric representation of the Gibbs measure, which leads to an interplay of both parabolic and hyperbolic theories. Then, we discuss our para-controlled Ansatz and a hidden cancellation between sextic stochastic objects.

This is joint work with Y. Deng, A. Nahmod, and H. Yue.

Wednesday 9:00

José A. Carrillo (University of Oxford)

Nonlocal Aggregation-Diffusion Equations: entropies, gradient flows, phase transitions and applications

This talk will be devoted to an overview of recent results understanding the bifurcation analysis of nonlinear Fokker–Planck equations arising in a myriad of applications such as consensus formation, optimization, granular media, swarming behavior, opinion dynamics and financial mathematics to name a few. We will present several results related to localized Cucker–Smale orientation dynamics, McKean–Vlasov equations, and nonlinear diffusion Keller–Segel type models in several settings. We will show the existence of continuous or discontinuous phase transitions on the torus under suitable assumptions on the Fourier modes of the interaction potential. The analysis is based on linear stability in the right functional space associated to the regularity of the problem at hand. While in the case of linear diffusion, one can work in the L^2 -framework, nonlinear diffusion needs the stronger L^∞ topology to proceed with the analysis based on Crandall–Rabinowitz bifurcation analysis applied to the variation of the entropy functional. Explicit examples show that the global bifurcation branches can be very complicated. Stability of the solutions will be discussed based on numerical simulations with fully explicit energy decaying finite volume schemes specifically tailored to the gradient flow structure of these problems. The theoretical analysis of the asymptotic stability of the different branches of solutions is a challenging open problem.

This overview talk is based on several works in collaboration with R. Bailo, A. Barbaro, J. A. Canizo, X. Chen, P. Degond, R. Gvalani, J. Hu, G. Pavliotis, A. Schlichting, Q. Wang, Z. Wang, and L. Zhang. This research has been funded by EPSRC EP/P031587/1 and ERC Advanced Grant Nonlocal-CPD 883363.

Wednesday 9:50

Arnd Scheel (University of Minnesota)

Selection through growth and the marginal stability conjecture

I'll first survey scenarios where spatial growth acts as a selection mechanism in a complex system and then focus on a particular growth scenario, propagation into unstable state. In this context, selection is phrased in terms of the marginal stability conjecture, which predicts that invasion processes that are marginally stable in the leading edge are dynamically selected. I'll present a recent proof of this conjecture that holds for open classes of systems and initial conditions. Crucially, the proof does not rely on properties of the equation such as comparison principles but on conceptual assumptions such as existence and stability of a critical front. I'll explain how this is a significant step towards establishing and predicting selection of patterns in growth processes.

Wednesday 11:10

Julian Fischer (Institute of Science and Technology Austria)

A rigorous approach to the Dean-Kawasaki equation of fluctuating hydrodynamics

Fluctuating hydrodynamics provides a framework for approximating density fluctuations in interacting particle systems by suitable SPDEs. The Dean-Kawasaki equation - a strongly singular SPDE - is perhaps the most basic equation of fluctuating hydrodynamics; it has been proposed in the physics literature to describe the fluctuations of the density of N diffusing weakly interacting particles in the regime of large particle numbers N . The strongly singular nature of the Dean-Kawasaki equation presents a substantial challenge for both its analysis and its rigorous mathematical justification: Besides being non-renormalizable by approaches like regularity structures, it has recently been shown to not even admit nontrivial martingale solutions.

In this talk, we give an overview of recent quantitative results for the justification of fluctuating hydrodynamics models. In particular, we give an interpretation of the Dean-Kawasaki equation as a "recipe" for accurate and efficient numerical simulations of the density fluctuations for weakly interacting diffusing particles.

Wednesday 13:45

Hartmut Schwetlick (University of Bath)

An elastic flow for spline interpolations

We use a geometric flow on the problem of nonlinear spline interpolations in Euclidean spaces. The spline sought in this problem connects a set number of given knot points so that the elastic energy of the whole curve is controlled. The method applies the theory of fourth-order parabolic PDEs to each piece of the curve between two successive knot points at which certain dynamic boundary conditions are imposed. We show the existence of global solutions of the elastic flow in suitable function spaces. In the asymptotic limit, as time approaches infinity, solutions subconverge to a stationary solution of the problem. The method of geometric flows provides a new approach for the problem of nonlinear spline interpolations.

This project is joint work with Chun-Chi Lin and Dung The Tran, National Taiwan Normal University, Taipei.

Wednesday 14:35

Enrique Zuazua (University of Erlangen–Nuremberg)

Control of reaction-diffusion models

Inspired by problems arising in social and biological sciences, with present recent work of our team on the control scalar reaction-diffusion equations with bistable nonlinearities.

The intrinsic nature of the phenomena under consideration imposes positivity constraints on the controlled trajectories, that are hard to be preserved under the action of controls.

We shall show how the employment of travelling wave solutions and continuous paths of steady states allows, in some cases, to control these models, so that, out of the initial configuration, the final desired one is reached, preserving the constraints. But we shall also exhibit some barrier effects that arise when the diffusivity is too small or the domain in which the diffusion process evolves is too large, making such control properties impossible.

These results are in agreement with numerical simulations and experimental observations.

Wednesday 16:00

Sebastian Throm (Umea University)

Uniqueness of scaling profiles for kinetic equations

We consider the question of uniqueness of self-similar profiles for systems of interacting particles relying on perturbation arguments. We will particularly discuss some of the challenges when applying the method to specific models.

Wednesday 16:50

Willi Jäger (Heidelberg University)

The Role of Hypoxia in Inflammation and Sepsis – Mathematical Modeling, Analysis and Simulations

Inflammation is a reaction of the immune system to disturbances in the body caused for example by bacterial or viral infections. A dysregulated response of the body to an infection can lead to life-threatening organ dysfunction - by sepsis, in the case of SARS CoV 2 a viral sepsis. Even if the severe stages of sepsis can be avoided, those infected may suffer long-lasting side effects. The interruption of the supply of crucial resources such as oxygen, of the energy supply to the cellular systems, that suffer from the stressful situations, and the lateral damage affect the vital processes and functioning of the organism on all levels and at all levels. Mathematical modelling, analysis and numerical simulation of parts of the complex, multi-scale system are required to gain a better understanding and control of the processes. This presentation focuses on the role of hypoxia as a crucial factor in the dynamics of inflammation and sepsis and formulates some of the arising mathematical challenges. The results presented here were achieved in particular in collaboration with

Maria Neuss-Radu, Markus Gahn, Jonas Knoch

in a sub-project of the sepsis network SCIDATOS funded by the Klaus Tschira Foundation.

Thursday 9:00

Kyungkeun Kang (Yonsei University, Seoul)

Analysis of some differential equations related to chemotactic nonlinearity

In this talk, some works of Angela with me and her collaborators are reminded. We firstly discuss kinetic models for chemo-sensitive movement taking into account effects of gradient fields of chemical stimuli, and rigorous derivation of the macroscopic diffusion limit. Secondly, we analyze a one-dimensional Keller-Segel model with a logarithmic chemotactic sensitivity and a non-diffusing chemical is classified with respect to its long time behavior. Qualitative behaviors of the system are shown concerning existence of global solutions or Dirac-mass formation. We thirdly study a Keller-Segel type of system including growth and death of the chemotactic species and an elliptic equation for the chemo-attractant. Existence of blow-up solutions and global existence of regular solutions are established depending on values of parameters for hyperbolic-elliptic problem in dimension two and higher.

Thursday 9:50

Gero Friesecke (Technical University of Munich)

The strong-interaction limit of density functional theory

While not reached in nature, the strong-interaction limit of the exact (Hohenberg–Kohn) density functional points the way towards the real physics in strongly correlated many-electron systems missed by the standard approximations used in physics, chemistry and materials science. For example, the strong-interaction limit - unlike the local density approximation, semilocal, or hybrid functionals - gets the dissociation of the H₂ molecule right.

In my talk I cover

- the derivation of the limiting strictly correlated electrons (SCE) functional from the exact functional via Gamma convergence
- equivalent formulations and the mathematical interpretation as optimal transport with Coulomb cost
- rigorous sparsity theorems in the spirit of Brenier’s theorem in both the continuous and discretized case
- the recently introduced GenCol (Genetic Column Generation) algorithm which exploits this sparsity and appears to overcome the curse of dimension, allowing for the first time the accurate computation of the strong-interaction limit for general densities.

References: GF, Augusto Gerolin and Paola Gori-Giorgi, arXiv:2202.09760, 2022 (recent survey article); GF, Andreas Schulz and Daniela Voegler, arXiv:2103.12624, to appear in SIAM JSC, 2022 (algorithm).

Thursday 11:10

Ansgar Jüngel (TU Wien)

Mean-field limits in stochastic interacting particle systems for multiple species

Nonlinear diffusion equations can be rigorously derived from stochastic interacting many-particle systems in the mean-field limit, as proved by Jourdain/Méléard, Oelschläger, and Stevens. We extend the ansatz of Oelschläger for moderately interacting many-particle systems to the case of multiple species, leading to cross-diffusion systems for the limiting probability density functions. First, if the drift depends on the interactions, cross-diffusion systems with quadratic nonlinearity are rigorously derived, possessing a double entropy structure. Second, if the diffusion coefficients of the particle system depend on the interactions, more general cross-diffusion systems are obtained, including the well-known Shigesada-Kawasaki-Teramoto population system. We prove not only the convergence of the stochastic processes but also provide an estimate for the mean-squared error of the stochastic processes. Finally, we present a random-batch method for the efficient numerical solution of the interacting particle systems and some numerical results.

Thursday 13:00

Christian Schmeiser (University of Vienna)

On two models for non-instantaneous binary alignment collisions

It is a standard assumption in kinetic transport theory that collision processes are instantaneous. We shall report on the initiation of a research program, where this idealization is lifted. The idea for modeling binary, non-instantaneous collisions is to replace instantaneous jumps in the state space of a pair of particles by continuous processes taking finite time. The duration of the collision process can be deterministic or stochastic. This results in a system of equations for the particle distribution between collisions and for the distribution of particle pairs during collision processes. Results will be presented on two spatially homogeneous models for non-instantaneous alignment collisions, where in one case the collision time is governed by a simple Poisson process, whereas in the other case collisions end after reaching complete alignment in finite time. For both models existence, long-time behavior, and the instantaneous limit towards a standard kinetic model will be discussed. (joint work with Laura Kanzler and Veronica Tora)

Thursday 14:20

Bernold Fiedler (Free University of Berlin)

Steady state sensitivity and oscillations in reaction networks

We present some attempts to understand the dynamics of metabolic and gene regulatory networks, based on their graph structure, only. We describe steady state sensitivity to perturbed reaction rates based on joint work with Bernhard Brehm, and by Nicola Vassena. We also explore sufficient conditions for global Hopf bifurcation of time-periodic oscillations. Specific examples include the citric acid cycle, and genetic circadian clocks in mammals.

See also <http://dynamics.mi.fu-berlin.de/>

Thursday 15:10

Christof Melcher (RWTH Aachen University)

Emergent spin-orbit coupling and rotating skyrmions

We examine skyrmionic field configurations on a spherical magnet. Exploiting the Hamiltonian structure and concepts of angular momentum, we present a new family of localized solutions to the Landau-Lifshitz equation that are topologically distinct from the ground state and break rotational symmetry. The approach illustrates emergent spin-orbit coupling arising from the loss of individual rotational invariance in spin and coordinate space – a common feature of condensed matter systems with topological phases.