



# Conservation laws and non-reciprocity

May 5 – 8, 2025

Münster

## Organisers

Markus Bär

André Schlichting

Angela Stevens

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**05 - 08 May 2025, in Münster, Germany**

	<b>Monday 5 May</b>	<b>Tuesday 6 May</b>	<b>Wednesday 7 May</b>	<b>Thursday 8 May</b>
9:00-9:30		Erwin Frey	Markus Schmidtchen	Klemens Fellner
9:30-10:00		Sebastian Hensel	Michael Wilczek	Alberto Dinelli
10:00-10:30			coffee	
10:30-11:00		Suropriya Saha	Alessia Nota	Charlotte Dietze
11:00-11:30		Rishabh Gvalani	Massimiliano Esposito	Raphael Wittkowski
11:30-12:00		Fridtjof Brauns	Raphael Winter	Johannes Zimmer
12:00-14:00			lunch	
14:00-14:30	Robert Jack	Martin Burger	Aljaz Godec	14:15-15:15 <i>Kolloquium Talk</i> (lecture hall M4)
14:30-15:00	Artur Stephan	Daniel Greve	Maximilian Engel	Govind Menon
15:00-15:30	Klaus Kroy	Philippe Laurençot	Ramin Golestanian	
15:30-16:15			coffee	
16:15-16:45	Poster Pitch Talks and Posters	Fernando Peruani	Benjamin Gess	
16:45-17:15		Franco-Velazquez	Sarah A. M. Loos	
18:30	Reception: SRZ Foyer 2nd floor	Conference Dinner: Caputo's		



# General information

**Venue.** The main workshop venue is the MM-conference center located on the second floor of the Seminarraumzentrum (SRZ) at Orléans-Ring 12, 48149 Münster (see map on p. 4). You will find the registration there. Moreover, the coffee breaks and poster sessions take place in the lounge of the seminar building SRZ (second floor) right in front of the seminar room.

You can find the latest information on the webpage:

[www.uni-muenster.de/MathematicsMuenster/go/conservation-laws\\_2025](http://www.uni-muenster.de/MathematicsMuenster/go/conservation-laws_2025)

**Wi-Fi access.** If you are part of the eduroam community, you may connect to the network “eduroam” as usual. Otherwise you can connect to the SSID “GuestOnCampus” and start any web browser. You will automatically be redirected to the login page. Confirm the terms of use and click on “log in for free”. 1 GB data volume is available per device and day. Please note that the connection is not encrypted.

**Coffee break/Lunch.** We provide coffee and snacks during the coffee breaks.

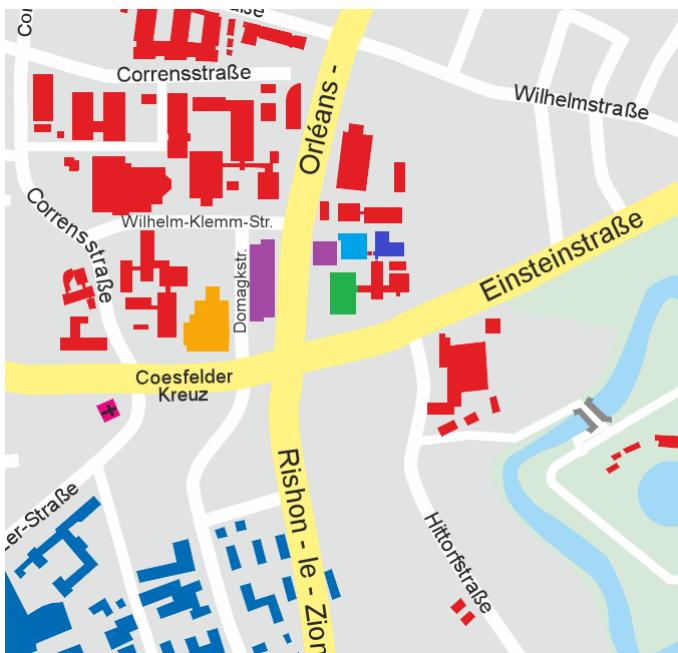
There are a couple of restaurants for lunch in the vicinity:

- Canteen – Mensa am Ring, Domagkstraße 61 (most convenient option, even if not the most idyllic place)
- Ristorante Milano (Italian), Wilhelmstraße 26 (closed Mondays)
- Il Gondoliere (Italian), Von-Esmarch-Straße 28 (closed Mondays)

- Buddha Palace (Indian), Von-Esmarch-Straße 18 (closed Tuesdays)
- Gustav Grün (Green Fast Food), Wilhelmstraße 1
- Áro (Green Fast Food), Neutor 3

**Public transportation.** You can check the bus schedule on the website of [Stadtwerke-Münster](#) (in German and English), or use Google maps.

**Free time activities.** Here are some suggestions for the little free time around the busy schedule of the workshop: Go see the castle (University main building), the park behind it and the embedded botanic garden. Visit a museum, e.g. the LWL Museum of Art and Culture (special exhibition on “The Fascination of Lacquer”) or the Picasso-Museum (right now there is a Chagall exhibition). Have a walk around the lake ‘Aasee’ or make yourself familiar with European history at the Historical City Hall which is one of the two places where 1648 the Peace of Westphalia was signed.



SRZ (workshop venue, 2nd floor)

Lecture building

Math Department

canteen

multi-storey car park.

# **Book of abstracts**

# Multiscale (in-)stability of traveling waves in the non-reciprocal Cahn–Hilliard model

Fridtjof Brauns      Tue 11:30

Non-reciprocal versions of the Cahn–Hilliard model for phase separation (short NRCH models) have gained great interest to understand dynamic patterns in mass-conserving systems. In contrast to classical phase separation which exhibits uninterrupted coarsening, the traveling patterns in NRCH exhibit remarkable multistability of wavelengths spanning orders of magnitude. Here, we will address some of the dynamic processes selecting a particular wavelength out of the many stable ones as well as the nature of the instabilities by which stability is eventually lost at very large wavelengths. Remarkably, these instabilities are spatially localized and may be understood in terms of the local dispersion relation.

# Cross-diffusion and -interaction: from variational to non-variational problems

Martin Burger      Tue 14:00

We discuss some multi-species models from biology, social and data sciences with reciprocal or non-reciprocal interactions. We discuss possibly symmetrization and gradient flow structures as well as generalizations thereof such as GENERIC or Port-Hamiltonian. In a special class of non-reciprocal interactions frequently used for consensus or clustering processes, we highlight the limitations of variational approaches and discuss some open questions.

# Focusing dynamics of 2D Bose gases in the instability regime

Charlotte Dietze      Thu 10:30

We consider the dynamics of a 2D Bose gas with an interaction potential of the form  $N^{2\beta-1}w(N^\beta \cdot)$  for  $\beta \in (0, 3/2)$ . The interaction may be chosen to be negative and large, leading to the instability regime where the corresponding focusing cubic nonlinear Schrödinger equation (NLS) may blow up in finite time. We show that to leading order, the  $N$ -body quantum dynamics can be effectively described by the NLS prior to the blow-up time. Moreover, we prove the validity of the Bogoliubov approximation, where the excitations from the condensate are captured in a norm approximation of the many-body dynamics.

This is joint work with Lea Boßmann and Phan Thành Nam.

# Motility-induced Self-organization of Microbial Ecosystems: From Non-reciprocity to Weak Random Interactions

Alberto Dinelli      Thu 09:30

The self-organization of microbial ecosystems involves a wide variety of mechanisms, ranging from biochemical signaling to population dynamics. Among these, the role of motility regulation has been little studied, despite the importance of active migration processes. To bridge this gap, I will first discuss how non-reciprocal motility regulation can drive the spatial organization of a 2-strain bacterial system. While interactions at the particle scale are inherently non-reciprocal, I will

show that non-reciprocity has a subtle fate across scales, as it may fade upon coarse-graining. By studying the conditions for its survival at the macroscopic scale, I will discuss the dramatic impact of non-reciprocity on the emergent phase behavior of the system.

In the second part of my talk, I will then consider microbial ecosystems comprising a large number of strains of motile bacteria. I will show how weak, random motility regulation suffices to induce a rich spatial organization in the ecosystem. Using a combination of analytical theory and numerical simulations, I will show how increasing the heterogeneity of the interactions between strains generically induces a fragmentation transition, leading the ecosystem to self-organize into distinct communities. All in all, our work highlights the important role of motility regulation in the emergent organization of bacterial ecosystems.

## Separation of time scales in weakly interacting diffusions

Maximilian Engel      Wed 14:30

We study metastable behaviour in systems of weakly interacting Brownian particles with localised, attractive potentials which are smooth and globally bounded. In this particular setting, numerical evidence suggests that the particles converge on a short time scale to a “droplet state” which is metastable, i.e. persists on a much longer time scale than the time scale of convergence, before eventually diffusing to 0. In this talk, we provide rigorous evidence and a quantitative characterisation of this separation of time scales. Working at the level of the empirical measure, we show that (after quotienting out the motion of the centre of mass) the rate of convergence to the quasi-stationary

distribution, which corresponds with the droplet state, is of order 1 as the inverse temperature goes to infinity. Meanwhile, the rate of leakage away from its centre of mass is of exponentially small order as the inverse temperature goes to infinity. Furthermore, we can determine the length scale of the quasi-stationary distribution. We thus provide a partial answer to a question posed by Carrillo, Craig, and Yao (2019) in the microscopic setting.

## Conservation Laws and Nonreciprocity in Nonequilibrium Thermodynamics

Massimiliano Esposito      Wed 11:00

I will discuss the crucial role of broken conservation laws in determining the nonconservative forces that prevent a system from relaxing to equilibrium in nonequilibrium thermodynamics. I will then show how nonreciprocity emerges as a generic feature in the presence of such nonconservative forces. These ideas will be formulated and illustrated within the frameworks of stochastic thermodynamics [1-3], and if time permits, also in hydrodynamics [4] and reaction-diffusion systems [5].

- [1] R. Rao and M. Esposito, "Conservation Laws shape Dissipation", *New J. Phys.* 20, 023007 (2018)
- [2] T. Aslyamov and M. Esposito, "General Theory of Static Response for Markov Jump Processes", *Phys. Rev. Lett.* 133, 107103 (2024)
- [3] G. Falasco and M. Esposito, "Macroscopic Stochastic Thermodynamics", *Rev. Mod. Phys.* 97, 015002 (2025)
- [4] D. Forastiere, F. Avanzini, and M. Esposito, "Dissipation in Hydrodynamics from Micro- to Macroscale: Wisdom from Boltzmann and Stochastic Thermodynamics", *New J. Phys.* 26, 063022 (2024)

[5] F. Avanzini, T. Aslyamov, E. Fodor and M. Esposito, "Nonequilibrium Thermodynamics of Non-Ideal Reaction-Diffusion Systems: Implications for Active Self-Organization", *J. Chem. Phys.* 161, 174108 (2024)

## Oscillation in a nonlinear Becker-Döring model for prion dynamics

Klemens Fellner      Thu 09:00

Prions are able to self-propagate biological information through the transfer of structural information from a misfolded/infectious protein in a prion state to a protein in a non-prion state. Prions cause diseases like Creuzfeldt-Jakob. Prion-like mechanisms are associated to Alzheimer, Parkinson and Huntington diseases. We present a fundamental bi-monomeric, nonlinear Becker-Döring type model, which aims to explain experiments in the lab of Human Rezaei showing sustained oscillatory behaviour over multiple hours, [1]. Besides two types of monomers, our model suggests a nonlinear depolymerisation process as crucial for the oscillatory behaviour. Since then, experimental evidence seems to confirm this process. We provide details on the mechanism of oscillatory behaviour and show numerical simulations [2].

- [1] M. Doumic, K. Fellner, M. Mezache, H. Rezaei, A bi-monomeric nonlinear Becker-Döring type system to capture oscillatory aggregation kinetics in prion dynamics, *Journal of Theoretical Biology*, 480 (2019) 241–261.
- [2] M. Doumic, K. Fellner, M. Mezache, J.J.L. Velazquez, Asymptotic Analysis of a bi-monomeric nonlinear Becker-Döring system, to appear in *Nonlinearity*

# Biochemical systems in which detailed balance fails in some reactions

Eugenio Franco & Juan J. L. Velazquez

Tue 16:45

At the fundamental level, we can expect the chemical reactions (in particular the ones in biochemistry) to be given by chemical rates satisfying the so-called detailed balance condition, which is a general property of macroscopic systems which are obtained from an underlying microscopic model which is invariant under time reversal.

However, chemical systems are often modeled by systems of equations for which detailed balance fails. This can be justified if the system is in contact with "reservoirs" which are out of equilibrium, as it is usually the case in biochemical systems. From the mathematical point of view, systems with detailed balance can be formulated as "gradient flows". In several biochemical situations, the failure of detailed balance happens in a "localized way", in some specific reactions.

In this talk we will discuss several problems in which his localized failure of detailed balance plays an important role. Specifically we will comment about the following problems:

- Role of the lack of equilibrium in the classical Hopfield-Ninio kinetic proof-reading model or How the T-cells manage to discriminate with great accuracy between different ligands?
- How to obtain in a rigorous mathematical way models without detailed balance, starting from models in which detailed balance holds?.
- Can you obtain the adaptation property (i.e. response to gradients instead of absolute values of signals) in equilibrium systems or you need to be out of equilibrium?.
- How to determine if detailed balance fails or holds from measurements (if we do not know the reaction rates)?

# Turing Foams and Active Foams

Erwin Frey      Tue 09:00

Non-equilibrium protein pattern formation and the self-organization of motor-filament mixtures, driven by NTPase cycles, are crucial mechanisms for cellular processes like division and polarization. Despite their distinct physical origins, both systems can form remarkably similar structures. Protein diffusion-reaction dynamics lead to foam-like patterns, which we termed "Turing foams," that follow non-equilibrium interface laws similar to equilibrium foams [1]. Motor-filament mixtures similarly self-organize into supramolecular structures, including micelles, bilayers, and foams, driven by instabilities [2]. This talk will discuss the shared non-equilibrium principles governing these systems. It will focus on interface laws linked to thermodynamic-like relations and their applications in designing specific pattern morphologies for synthetic life-like systems.

- [1] Deciphering the Interface Laws of Turing Mixtures and Foams, Henrik Weyer, Tobias A. Roth, and Erwin Frey, [arXiv:2409.20070].
- [2] Supramolecular assemblies in active motor-filament systems: micelles, bilayers, and foams, Filippo De Luca, Ivan Maryshev, and Erwin Frey, Physical Review X 14, 031031 (2024).

# Gradient flow structures and large deviations for porous media equations

Benjamin Gess      Wed 16:15

While the derivation of nonlinear but uniformly parabolic equations from microscopic dynamics, fluctuations around these limits, and the corresponding canonical choice of a gradient flow structure are now well-understood, less is known for equations with either degenerate,

or unbounded diffusivity. Specifically, for the model case of the porous medium equation (PME), multiple gradient flow structures have been identified since the works of Brézis and Otto; however, it remains unclear which, if any, are thermodynamic in nature, meaning that they arise through the large deviations of a microscopic model. In this talk, to demonstrate that the (formal) geometric picture we obtain is thermodynamic, we examine a rescaling of the zero-range process (ZRP) that converges to the PME and prove a full large deviations principle. The proof of this result is complicated by the degeneracy and unboundedness of the diffusivity. We then discuss how the large deviations rigorously identify a gradient flow structure for the PME.

## Thermal relaxation asymmetry: from single molecules to hydrodynamic fluctuations

Aljaz Godec      Wed 14:00

According to conventional wisdom, a macroscopic system placed in an environment with a different temperature relaxes (i.e., warms up or cools down) to the temperature of the surroundings. This relaxation is mediated by the flow of heat that is set only by the temperature difference. However, when rapid changes in temperature push a system far from equilibrium, thermal relaxation becomes asymmetric, which was predicted theoretically and recently confirmed experimentally. That is, under quite general conditions, heating is in fact faster than cooling.

I will first introduce the relaxation asymmetry in reversible and detailed-balance violating finite-dimensional systems. Next I will present a rigorous formulation of thermal relaxation in the framework of hydrodynamic fluctuation theory for short- and long-range interacting systems, respectively. I will show that the asymmetry persists in

the hydrodynamic limit and that in contrast to finite-dimensions a "true" close-to-equilibrium thermal quench with symmetric heating and cooling at all times does not exist.

## Non-reciprocal active matter across scales

Ramin Golestanian      Wed 15:00

**Abstract:** Broken action-reaction symmetry has been recently explored in active matter in the context of nonequilibrium phoretic interactions between catalytically active colloids and enzymes [1], and hydrodynamic interactions [2,3], among others. It has been shown to lead formation of self-propelled active molecules that break time-reversal symmetry [4], oscillating active complexes that break time-translation symmetry [5], chiral bound-states [6], and active phase separation with specified stoichiometry [7,8,9]. Non-reciprocal interactions have been found to lead to rich physical phenomena involving various forms of spontaneous symmetry-breaking [10,11,12] as well as the accompanying defect structures that destroy order [13,14]. Recent applications of non-reciprocal active matter have revealed exotic behaviour such as the appearance of effervescent travelling patterns [15] and shape-shifting multifarious self-organization [16], spontaneous escape of kinetic barriers [17], dynamical pattern formation in quorum-sensing active matter [18,19], as well as implications of the physics of non-reciprocal interactions on the origin of life [20,21,22].

- [1] R. Golestanian, Lecture Notes of the 2018 Les Houches Summer School (OUP, 2022); arXiv:1909.03747 (2019)
- [2] N. Uchida and R. Golestanian, PRL 104, 178103 (2010)
- [3] D. J. Hickey, R. Golestanian, and A. Vilfan, PNAS 120, e2307279120 (2023)
- [4] R. Soto and R. Golestanian, PRL 112, 068301 (2014)

- [5] R. Soto and R. Golestanian, PRE 91, 052304 (2015)
- [6] S. Saha, S. Ramaswamy, and R. Golestanian, NJP 21, 063006 (2019)
- [7] J. Agudo-Canalejo and R. Golestanian, PRL 123, 018101 (2019)
- [8] V. Ouazan-Reboul, J. Agudo-Canalejo, and R. Golestanian, EPJE 44, 113 (2021)
- [9] G. Tucci, R. Golestanian, and S. Saha, NJP 26, 073006 (2024)
- [10] S. Saha, J. Agudo-Canalejo, and R. Golestanian, PRX 10, 041009 (2020)
- [11] G. Pisegna, S. Saha, and R. Golestanian, PNAS 121, e2407705121 (2024)
- [12] L. Parkavousi, N. Rana, R. Golestanian, and S. Saha, PRL 134, 148301 (2025)
- [13] N. Rana and R. Golestanian, PRL 133, 078301 (2024)
- [14] N. Rana and R. Golestanian, NJP 26, 123008 (2024)
- [15] S. Saha and R. Golestanian, arXiv:2208.14985 (2022)
- [16] S. Osat and R. Golestanian, Nature Nano 18, 79 (2023)
- [17] S. Osat, J. Metson, M. Kardar, and R. Golestanian, PRL 133, 028301 (2024)
- [18] Y. Duan, J. Agudo-Canalejo, R. Golestanian, and B. Mahault, PRL 131, 148301 (2023)
- [19] Y. Duan, J. Agudo-Canalejo, R. Golestanian, and B. Mahault, PRR 7, 013234 (2025)
- [20] V. Ouazan-Reboul, J. Agudo-Canalejo, and R. Golestanian, Nat. Commun. 14, 4496 (2023)
- [21] V. Ouazan-Reboul, R. Golestanian, and J. Agudo-Canalejo, PRL 131, 128301 (2023)
- [22] V. Ouazan-Reboul, R. Golestanian, and J. Agudo-Canalejo, NJP 25, 103013 (2023)

# Coexistence of stationary & oscillatory phases in nonreciprocal mass-conserving systems

Daniel Greve     Tue 14:30

Nonreciprocal interactions and conservation laws play a crucial role in out-of-equilibrium pattern formation, as observed in diverse phenomena across biological, chemical, and physical systems [1]. Even at the level of minimal field theories like nonreciprocal Cahn-Hilliard models [2], many aspects of emergent complex behaviors and nonequilibrium phase transitions are yet to be explored. First, we consider the regime of strongly nonreciprocal interactions, where a conserved-Hopf instability dominates structure formation, giving rise to emergent traveling waves. We present a universal nonlocal amplitude equation that captures the slow spatiotemporal evolution of fast oscillations in the weakly nonlinear regime. Analytical and numerical results confirm that this description successfully predicts the suppression of coarsening in oscillatory phase separation, showing excellent agreement with the full nonlinear model [3]. Beyond this limit, i.e., if reciprocal and nonreciprocal inter-species couplings are of comparable strength to the phase separation of the individual species, pattern formation becomes more intricate and features a rich diversity of stationary and quasi-stationary persistent states. In particular, we demonstrate that the interplay of nonreciprocity and conservation laws can result in the robust coexistence of uniform stationary phases with oscillatory phases featuring regular or irregular wave patterns. Despite the complexity of this interplay, we show that phase coexistence and transitions can still be captured using a Maxwell doubletangent construction, provided that the underlying nonequilibrium model retains a spurious gradient dynamics structure [4].

- [1] F. Brauns and M. C. Marchetti, Phys. Rev. X 14, 021014 (2024); A. Dinelli, J. O'Byrne, A. Curatolo, Y. Zhao, P. Sollich, and J. Tailleur, Nat. Commun. 14, 7035 (2023); A. K. Omar, H. Row, S. A. Mallory, and J. F. Brady, Proc. Natl. Acad. Sci. U.S.A. 120, e2219900120 (2023); Y. Duan, J. Agudo-Canalejo, R. Golestanian, and B. Mahault, Phys. Rev. Lett. 131, 148301 (2023).

- [2] S. Saha, J. Agudo-Canalejo, and R. Golestanian, Phys. Rev. X 10, 041009 (2020); Z. H. You, A. Baskaran, and M. C. Marchetti, Proc. Natl. Acad. Sci. U.S.A. 117, 19767 (2020); T. Frohoff-Hülsmann, J. Wrembel, and U. Thiele, Phys. Rev. E 103, 042602 (2021).
- [3] D. Greve and U. Thiele, Chaos 34, 123134 (2024).
- [4] D. Greve, G. Lovato, T. Frohoff-Hülsmann, and U. Thiele, Phys. Rev. Lett. 134, 018303 (2025).

## Breakdown of the mean-field description of interacting systems: Phase transitions, metastability and coarsening

Rishabh Gvalani      Tue 11:00

We present results concerning the qualitative and quantitative description of interacting systems, with particular emphasis on those possessing a phase transition under the change of relevant system parameters.

For this, we first discuss and identify continuous and discontinuous phase for mean-field limits of interacting particle systems on the torus and spheres.

Since phase transitions are intimately related to the metastability of the stochastic particle system, we show how a suitable mountain pass theorem in the space of probability measures can describe the metastable behaviour of the underlying finite particle system.

We also show that the mean-field description of the particle system in the regime of strong local interaction breaks down. In this regime, coarsening is observed, where smaller clusters grow through coagulation events. We provide numerical experiments with a positivity preserving

numerical scheme for a SPDE of Dean-Kawasaki type, consisting of the McKean-Vlasov equation and conservative noise.

Joint work with Nicolai Gerber (U Ulm), Greg Pavliotis (Imperial London), André Schlichting (U Ulm)

## A weak-strong uniqueness principle for the Mullins-Sekerka equation

Sebastian Hensel      Tue 09:30

We establish a weak-strong uniqueness principle for the two-phase Mullins-Sekerka equation in ambient dimension  $d = 2$  and  $3$ : As long as a classical solution to the evolution problem exists, any weak De Giorgi type varifold solution (see for this notion the recent work with Stinson, Arch. Ration. Mech. Anal. 248, 8, 2024) must coincide with it. In particular, in the absence of topology changes such weak solutions do not introduce a mechanism for (unphysical) non-uniqueness. We also derive a stability estimate with respect to changes in the data. I will explain our method which is based on the notion of relative entropies for interface evolution problems, a reduction argument to a perturbative setting, and a stability analysis in this perturbative regime relying crucially on the gradient flow structure of the Mullins-Sekerka equation.

This is joint work with Julian Fischer, Tim Laux and Theresa M. Simon.

# Dynamical patterns in a mixture of active and passive particles, via non-reciprocal effective interactions

Robert Jack      Mon 14:00

We introduce a model mixture of active (self-propelled) and passive (diffusive) particles with non-reciprocal effective interactions, which is amenable to exact mathematical analysis [1]. Specifically, we derive partial differential equations that describe the large-scale (hydrodynamic) behaviour of an underlying lattice model. The (deterministic) hydrodynamic limit exhibits behaviour familiar from active particles alone (motility induced phase separation), as well as dynamical patterns associated with non-reciprocal interactions, including travelling and counter-propagating domain walls. The appearance of these dynamical states is signalled by a protrusion of the spinodal curve through the binodal that is associated with phase separation.

[1] J Mason, RL Jack, and M Bruna, arXiv:2408.03932.

# Pattern Formation in Microswimmer Swarms with Time-Delayed AtTRACTIONS

Klaus Kroy      Mon 15:00

Ensembles of motile particles with time-delayed attractions self-organize into various dynamical mesoscale patterns. The interplay between the interaction range and processing and transmission delays gives rise to finite-size phase transitions via spontaneous chiral and translational symmetry breaking. Numerical simulations reveal the emergence of

persistent kinetic compartments and a bustling “stop-go tap phase” due to “aiming errors” and non-reciprocal “light-cones”.

## The discrete collision-induced breakage equation: well-posedness and stationary solutions

Philippe Laurençot     Tue 15:00

The existence and uniqueness of mass-conserving solutions are shown for the discrete collisional breakage equation, which describes the dynamics of cluster growth when clusters undergo binary collisions with possible matter transfer. When mass transfer is turned on, non-trivial stationary solutions are constructed.

Joint work with M. Ali and A.K. Giri (IIT Roorkee).

## Nonreciprocity and Geometric Frustration

Sarah Loos     Wed 16:45

Dynamical steady states with macroscopic currents in nonreciprocal systems I discuss the emergence of dynamical steady states with macroscopic currents due to nonreciprocal interactions. First, I consider fluctuating field theories with conserved dynamics describing travelling waves in nonreciprocal fluid mixtures. We find that close to PT-symmetry breaking phase transitions, fluctuations not only inflate, as is known from equilibrium criticality, but also become increasingly irreversible. Second, I consider single-species disordered spin systems,

where we find that nonreciprocal interactions lead to dynamical states with coherent oscillations but incoherent phases, but can also give rise to chaotic phases and suppress or enhance glassy behaviour.

Suchanek, Kroy, and Loos, PRL 131, 258302 (2023). PRE 131, 258302 (2023), PRE 108, 064123 (2023).

## *Kolloquium Wilhelm Killing* Towards a geometric theory of deep learning

Govind Menon      Thu 14:15

The mathematical core of deep learning is function approximation by neural networks trained on data using stochastic gradient.

I will present a collection of results on training dynamics for the deep linear network (DLN). The DLN is a phenomenological model of deep learning for linear functions that was introduced by computer scientists. It allows a comprehensive analysis that reveals interesting ties with several areas of mathematics and several lessons for ‘true’ deep learning.

This is joint work with several co-authors: Nadav Cohen (Tel Aviv), Kathryn Lindsey (Boston College), Alan Chen, Zsolt Veraszto and Tianmin Yu (Brown).

# Spontaneous localization in multicomponent Smoluchowski's coagulation equations

Alessia Nota      Wed 10:30

The Smoluchowski coagulation equation is an integro-differential equation of kinetic type that provides a mean-field description of mass aggregation phenomena. In this talk, I will examine the existence and non-existence of stationary solutions for both single-component and multi-component coagulation systems under non-equilibrium conditions, which are induced by the addition of a source term for small cluster sizes. A striking feature of the stationary solutions for multi-component systems is the occurrence of an unusual “spontaneous localization” phenomenon. This localization appears to be a universal property of multicomponent systems. I will discuss the emergence of localization in both stationary and time-dependent solutions to mass-conserving coagulation equations, along with conjectures and open problems in this direction.

## The Physics of Single-Species, Non-Reciprocal Active Particles

Fernando Peruani      Tue 16:15

Active particles whose interactions are constrained by a field of view – and therefore do not obey Newton's third law – exhibit remarkably rich collective dynamics in both one and two dimensions. The physics of these systems is fundamentally different from that of active Brownian particles (ABPs) and flocking models with reciprocal interactions, placing them in a distinct universality class.

In two dimensions, particles self-organize into nematic bands, nematic rings, polar filaments, and other complex collective patterns, some of which feature one or two topological singularities. In one dimension, the system undergoes non-trivial coarsening dynamics driven by fluctuation-induced effects that emerge from the non-reciprocity of interactions.

In this talk, I will review the most relevant results related to the physics of single-species, non-reciprocal active particles, highlighting their unique behaviors and open questions.

## Multifaceted dynamics in the Non-reciprocal Cahn-Hilliard Model

Suropriya Saha      Tue 10:30

The dynamics of non-reciprocally coupled conserved densities described minimally by the Non-reciprocal Cahn-Hilliard (NRCH) model is rich and varied. In my talk I will explore the pattern forming abilities inherent to the model beyond that travelling waves, which manifest due to a simultaneous breaking of spatial parity and time reversal symmetry. I will discuss the role played by nonlinearities, multispecies interactions, fluctuations, and the coupling to a fluid, in determining the dynamics. Nonlinear non-reciprocal interactions generically lead to imperfect symmetry breaking where phase separated droplets are nucleated which coexist with oscillating densities producing a new type of spatiotemporal chaos called effervescence. Generalising to many components, I will show that non-reciprocity plays a counter-intuitive role in the multi-species NRCH model enhancing its stability while simultaneously enhancing the diversity in dynamics. Finally, I will explore the role of fluctuations and coupling to the fluid in

determining the stability of the waves and comment on the nature of the spontaneously broken symmetry.

## Excluded Volume Effects and Anisotropic Interactions in a System of Hard Needles

Markus Schmidtchen      Wed 09:00

In this talk, we will present recent work on the macroscopic dynamics of a system of hard Brownian needles in two dimensions, focusing on the role of anisotropic steric interactions. Although the needles themselves have zero volume, their mutual exclusion creates a non-trivial effective volume in configuration space, significantly influencing the system's collective behaviour. Starting from the underlying stochastic hard needle dynamics, we apply the method of matched asymptotic expansions to rigorously derive a nonlinear, non-local partial differential equation governing the evolution of the needle density in position and orientation. In the regime of strong rotational diffusion, we obtain a reduced equation for the spatial density alone and use it to compare the effective excluded volume of needle systems with that of hard spheres. Finally, we investigate spatially homogeneous solutions and demonstrate an isotropic-to-nematic transition at increasing densities, in agreement with Onsager's theory. Our work highlights the interplay between geometry, hard-core interactions, and the emergence of order in anisotropic systems.

# Fast-slow chemical reactions: Convergence of Hamilton-Jacobi equation and variational representation

Artur Stephan      Mon 14:30

Microscopic behaviors of chemical reactions can be described by a random time-changed Poisson process, whose large-volume limit determines the macroscopic behaviors of species concentrations, including both typical and non-typical trajectories. Motivated by real-world applications, where chemical reactions appear naturally on different time-scales, we are interested in a fast-slow limit to reduce the complexity of the system. When the reaction intensities (or fluxes) exhibit a separation of fast-slow scales, the macroscopic typical trajectory is governed by a system of  $\varepsilon$ -dependent nonlinear reaction rate equations (RRE), while the non-typical trajectories deviating from the typical ones are characterized by a  $\varepsilon$ -dependent exponentially nonlinear Hamilton-Jacobi equation (HJE).

In my talk, we study the fast-slow limit as  $\varepsilon \rightarrow 0$  for the viscosity solutions of the associated HJE with a state-constrained boundary condition. We identify the limiting effective HJE on a slow manifold with an effective variational representation for the solution. Through the uniform convergence of the viscosity solutions and the  $\Gamma$ -convergence of the variational solution representations, we rigorously show that all non-typical (and also typical) trajectories concentrate on the slow manifold and the effective macroscopic dynamics are described by coarse-grained RRE and HJE, respectively. This approach for studying the fast-slow limit is applicable to, but not limited to, reversible chemical reactions described by gradient flows.

The talk is based on joint work with Yuan Gao (Purdue U).

# A comparative Study of the Collective States of Active Particles

Michael Wilczek      Wed 09:30

Systems of interacting active particles can show diverse collective states, including motility-induced phase separation, flocking, etc. To what extent these collective states are influenced by the details of the microscopic interaction is not fully understood.

To shed light on this question, we investigate a class of active particle models, in which the particles differ in shape and center of mass. Depending on the particles' properties, even simple steric interactions can then induce torques on the active particles. We find that the emergent collective states can differ significantly between the different types of particles. Our study therefore helps to classify the influence of microscopic interactions on the emergent collective states in active matter and bears implications for the development of continuum theories.

Joint work with Colin-Marius Koch.

## Bifurcations and stability of equilibria for the Vicsek-BGK equation for collective motion

Raphael Winter      Wed 11:30

The Vicsek-BGK equation describes the collective motion of agents with local alignment. We show that the spatially inhomogeneous system undergoes a phase transition from disoriented motion to collective motion. Due to the presence of active particles and lack of conservation laws, the limiting equilibrium cannot be (easily) predicted from the initial data.

Joint work with Sara Merino Aceituno and Christian Schmeiser.

# Efficient computer simulations based on conservation laws

Raphael Wittkowski      Thu 11:00

Conservation laws are widespread in physics and computer simulations are frequently used to study the behavior of systems involving conservation laws. The complexity of the partial differential equations that need to be solved is often high and the simulations can be computationally very expensive. An example is fluid dynamics. Its fundamental equations involve computationally demanding conservation laws and computational fluid dynamics is of enormous relevance for research and development in physics, engineering, and other fields. This talk presents the software AcoDyn, which allows for highly efficient computer simulations based on partial differential equations involving conservation laws. While this talk presents examples mainly from acoustofluidics, AcoDyn is highly flexible and applicable also to other partial differential equations.

# Fluctuating hydrodynamics for Vlasov-Fokker-Planck equations

Johannes Zimmer      Thu 11:30

We consider systems of interacting particles which are described by a second order Langevin equation, i.e., particles experiencing inertia. We introduce an associated equation of fluctuating hydrodynamics ("Dean-Kawasaki-type equation"), which can be interpreted as stochastic version of a Vlasov-Fokker-Planck equation. We show a dichotomy

previously known for purely diffusive systems holds here as well: Solutions exist only for suitable atomic initial data, but not for smooth initial data. The class of systems covered includes several models of active matter. The talk will attempt to sketch the wider picture of mathematical aspects of fluctuating hydrodynamic, including implications for numerical methods for such equations.

Joint work with Fenna Müller and Max von Renesse.

# **Posters**

A simple model for conserved intracellular protein-lipid dynamics exhibits multiscale pattern formation, traveling domains and arrested coarsening

Markus Bär

The Nonreciprocal Ising Model with Zero, One, and Two Conservation Laws

Kristian Blom

Extending Kinetic Theory To Generalized Dynamics:  
non-reciprocal interactions and (hidden) conservation laws

Horst-Holger Boltz

Phase Diagram of the Non-Reciprocal Cahn-Hilliard Model and the Effects of Symmetry

Martin Kjøllesdal Johnsrud

Of Gyrators and Anyons

Ram Mummadavarapu

Non-reciprocal mixtures in suspension: the role of hydrodynamic interactions

Giulia Pisegna

## Defect dynamics in the Nonreciprocal Cahn-Hilliard Model

Navdeep Rana

## Effect of Non-reciprocal couplings on the Potts Universality

Soumya Kanti Saha

## Motility-induced crystallization and rotating crystallites

Alina Barbara Steinberg

## Active pattern formation emergent from single-species nonreciprocity

Michael te Vrugt

## Chemo-mechanical motility modes of partially wetting liquid droplets

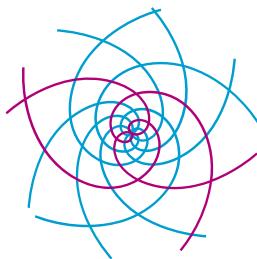
Florian Voss

## Emergence of memory in equilibrium versus nonequilibrium systems

Xizhu Zhao



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