Young Geometric Group Theory XI
Research Statements

University of Münster

13–17 February 2023

Contents

Mariam Alhawaj

University of Toronto, Canada

Generalized pseudo-Anosov Maps and Hubbard Trees

The Nielsen-Thurston classification of the mapping classes proved that every orientation preserving homeomorphism of a closed surface, up to isotopy is either periodic, reducible, or pseudo-Anosov [2]. Pseudo-Anosov maps have particularly nice structure because they expand along one foliation by a factor of $\lambda > 1$ and contract along a transversal foliation by a factor of $\frac{1}{\lambda}$ [3]. The number λ is called the dilatation of the pseudo-Anosov. Thurston showed that every dilatation λ of a pseudo-Anosov map is an algebraic unit, and conjectured that every algebraic unit λ whose Galois conjugates lie in the annulus $A_\lambda = \{z : \frac{1}{\lambda} < |z| < \lambda\}$ is a dilatation of some pseudo-Anosov on some surface S.

Pseudo-Anosovs have a huge role in Teichmuller theory and geometric topology. The relation between these and complex dynamics has been well studied inspired by Thurston.

In my research, I develop a new connection between the dynamics of quadratic polynomials on the complex plane and the dynamics of homeomorphisms of surfaces. In particular, given a quadratic polynomial, we show that one can construct an extension of it which is generalized pseudo-Anosov homeomorphism. Generalized pseudo-Anosov means the foliations have infinite singularities that accumulate on finitely many points [3]. We determine for which quadratic polynomials such an extension exists. My construction is related to the dynamics on the Hubbard tree which is a forward invariant subset of the filled Julia set that contains the critical orbit.

[1] Mariam Alhawaj. Generalized pseudo-Anosov Maps and Hubbard Trees. In preparation.

[2] Benson Farb, Dan Margalit. A primer on mapping class groups. 2012 by Princeton University Press, ISBN 978-0-691-14794-9

[3] Andre de Carvalho and Toby Hall. Unimodal generalized pseudo-Anosov maps. Geom. Topol., 8:11271188, 2004.

[4] David Fried. Growth rate of surface homeomorphisms and ow equivalence. Ergodic Theory Dynam. Systems, 5(4):539563, 1985.

[5] William P. Thurston. Entropy in dimension one. In Frontiers in complex dynamics, volume 51 of Princeton Math. Ser., pages 339384. Princeton Univ. Press, Princeton, NJ, 2014.

Marco Amelio

WWU Münster, Germany

Non-split sharply 2-transitive groups of odd characteristic

A group G is said to be sharply n-transitive if there exists a set X with at least n elements on which the group acts in such a way that, for any two n-tuples $(x_1, ..., x_n)$ and $(y_1, ..., y_n)$ of pairwise different elements there is exactly one $g \in G$ with $g.x_i = y_i$, $i = 1, ..., n$.

A sharply 2-transitive group is said to split if it has a non-trivial abelian normal subgroup. Moreover, if involutions (that is, elements of order 2) have fix-points, then there is a G-equivariant bijection between X and the set of involutions of G (considering the action of G on its set of involutions by conjugation). In this case, the characteristic of G is defined as the order of the product of two involutions. If involutions do not have fix-points, the group is said to have characteristic 2.

A typical example of a sharply 2-transitive group is the affine group of a field $AGL(1, K) \cong K_+ \rtimes K^*$ (for its natural action on K). If a sharply 2-transitive group splits it can always be seen as the affine group of a near-field (that is, a field but with only one distributive law).

All finite sharply 2-transitive groups split, as shown in [1] and [2]. In the infinite case, the first constructions of sharply 2-transitive groups that do not split (or alternatively, that do not appear as the affine group of a near-field) were shown in [3] (for characteristic 2) and in [4] (for characteristic 0).

The aim of our work is to construct non-split sharply 2-transitive groups of characteristic other than 0 or 2, using small cancellation techniques introduced in [5].

[1] Hans Zassenhaus. Kennzeichnung endlicher linearer Gruppen als Permutationsgruppen. In *Abhandlungen aus dem Mathematischen Seminar der Universität Hamburg*, volume 11, pages 17-40. Springer, 1935.

[2] Hans Zassenhaus. Über endliche fastkörper. In *Abhandlungen aus dem Mathematischen Seminar der Universität Hamburg*, volume 11, pages 187-220. Springer, 1935.

[3] Eliyahu Rips, Yoav Segev and Katrin Tent. A sharply 2-transitive group without a non-trivial abelian normal subgroup. *J. Eur. Math. Soc. (JEMS)*, 19(10):2895-2910, 2017.

[4] Eliyahu Rips and Katrin Tent. Sharply 2-transitive groups of characteristic 0. *J. Reine Angew. Math.*, 2019(750):227-238, 2019.

[5] Rémi Coulon. Partial periodic quotients of groups acting on a hyperbolic space. *Annales de l'Institut Fourier*, 66(5) 1773-1857, 2016.

Simon André

University of Münster, Germany

Geometric group theory, model theory and multiply transitive group actions

---

Part of my research lies at the interface of geometric group theory and model theory (the study of mathematical structures from the point of view of first-order logic). Deep connections between these two areas have emerged from the resolution by Sela of a famous question asked by Tarski around 1945, establishing that non-abelian free groups cannot be distinguished from each other by means of a first-order sentence. One of the results I proved in this direction can be roughly stated as follows (see [1]).

Theorem. *Let G be a finitely generated group and let G' be a hyperbolic group. If G is indistinguishable from G' by means of a first-order sentence, then G is a hyperbolic group.*

This result shows that hyperbolicity is highly intrinsic (at least among finitely generated groups).

More recently, I have started working on multiply transitive group actions, and more specifically on sharply 2-transitive and sharply 3-transitive groups. Let $n \geq 1$ be an integer; an action of a group G on a set X of cardinality at least n is said to be sharply n-transitive if for any n-tuples $(x_1, \ldots, x_n), (y_1, \ldots, y_n)$ of pairwise distinct elements of X, there exists a unique element of G mapping $(x_1, \ldots, x_n)$ to $(y_1, \ldots, y_n)$. A group G is said to be sharply n-transitive if it acts sharply n-transitively on some set X (with $|X| \geq n$). For instance, for any field K, the natural action of the affine group $K \rtimes K^*$ on K is sharply 2-transitive, and the natural action of $\mathrm{PGL}_2(K)$ on $K \cup \{\infty\}$ is sharply 3-transitive. Until recently, it was an open problem whether all sharply 2 and 3-transitive groups are of this form. The first counterexamples were constructed by Rips, Segev and Tent a few years ago. I proved the following result in collaboration with Tent (see [2]).

Theorem. *There exist infinite simple sharply 2-transitive groups. In particular, they are not of the form $K \rtimes K^*$.*

In collaboration with Guirardel, I proved that these groups can even be finitely generated. Our proof relies on small cancellation theory over relatively hyperbolic groups.

[1] Simon André, Hyperbolicity and cubulability are preserved under elementary equivalence, *Geom. Topol.*, 2020.

[2] Simon André and Katrin Tent. Simple sharply 2-transitive groups. *Trans. Amer. Math. Soc. (to appear)*, 2023.

Naomi Andrew

University of Oxford, UK

Automorphisms, actions on trees, and free-by-cyclic groups

I'm interested in actions on trees, automorphisms, and how these interact: so far this has involved thinking about free groups, free products, free-by-cyclic groups, one ended hyperbolic or relatively hyperbolic groups, amongst others. Another thread (not unique to me) is the analogy between mapping class groups and $\mathrm{Out}(F_n)$, which extends to an analogy between free-by-cyclic groups and fibred three manifolds.

One technique involves using actions on trees that, while they aren't canonical, are "invariant enough" – they are preserved by a finite index subgroup of automorphisms. In this case that subgroup itself admits an action on the same tree, and it can be analysed using this action. We used this to show that certain free-by-cyclic groups have finitely generated outer automorphism group [2], and used similar techniques (and, somewhat surprisingly, some free-by-cyclic groups!) to think about centralisers of certain elements of $\mathrm{Out}(F_n)$ [3].

On the other hand, some groups admit no actions on trees (except, of course, trivial actions with a global fixed point) – this is known as Serre's Property FA. Groups with FA include the automorphism groups of some free products, for instance of four or more copies of the same finite group [1].

I'm currently thinking about automorphisms of Leary–Minasyan groups (HNN extensions of $\mathbb{Z}^n$, generalising Baumslag–Solitar groups), centralisers of outer automorphisms of [various adjectives might go here] hyperbolic groups, homological and cohomological properties of free-by-cyclic groups and their analogies with fibred three manifolds, as well as removing the word "certain" from the results about $\mathrm{Out}(F_n \rtimes \mathbb{Z})$ and centralisers in $\mathrm{Out}(F_n)$.

[1] Naomi Andrew. Serre's property (FA) for automorphism groups of free products. *J. Group Theory*, 24(2):385–414, 2021.

[2] Naomi Andrew and Armando Martino. Free-by-cyclic groups, automorphisms and actions on nearly canonical trees. *J. Algebra*, 604:451–495, 2022.

[3] Naomi Andrew and Armando Martino. Centralisers of linear growth automorphisms of free groups `arXiv:2205.12865` [math.GR]

Raphael Appenzeller

ETH Zürich, Switzerland

Generalized trees and buildings

Trees (from graph theory) and 0-hyperbolic metric spaces can be given a common framework as Λ-trees [1], where $\Lambda = \mathbb{Z}$ for trees and $\Lambda = \mathbb{R}$ for 0-hyperbolic metric spaces. In the general setup, Λ is an ordered abelian group, examples include also $\mathbb{Q}$ and $\mathbb{Z} \times \mathbb{Z}$ with a lexicographical ordering. Affine Λ-buildings are a generalization of both simplicial affine buildings ($\Lambda = \mathbb{Z}$) and Euclidean buildings ($\Lambda = \mathbb{R}$). Affine buildigs are a higher-dimensional version of trees, in fact the one-dimensional affine Λ-buildings are exactly the Λ-trees without leaves.

I am working on the axiomatics of Λ-trees and Λ-buildings [2,3], as well as on a construction of an affine Λ-building that uses algebraic groups and real closed fields [4]. In analogy to Teichmüller theory, where a surface group acts on a symmetric space, one can define actions of surface groups on these affine Λ-buildings and possibly learn something about them this way [5].

Recently I have also been interested in Lean [6], a computer-program that allows to formalize and prove mathematical statements. I have used Lean to formalize the notion of Λ-trees and to prove some statements about them.

[1] I. M. Chiswell, *Introduction to Λ-trees*, River Edge, NJ: World Scientific Publishing Co. Inc., ISBN 981-02-4386-3, 2001.

[2] C. Bennett, P. Schwer, K. Struyve, *On axiomatic definitions of non-discrete affine buildings*, Adv. Geom., **14**(3):381-412, 2014.

[3] R. Appenzeller, *(In)dependence of axioms of Λ-trees*, arXiv:2112.02704, 2021.

[4] L. Kramer, K. Tent, *Affine Λ-buildings, ultrapowers of Lie groups and Riemannian symmetric spaces: an algebraic proof of the Margulis conjecture*, arXiv:math/0209122, 2002.

[5] M. Burger, A. Iozzi, A. Parreau, M. B. Pozzetti, *The real spectrum compactification of character varieties: characterizations and applications*, Comptes Rendus. Mathématique, **359**(4):439-463, doi : 10.5802/crmath.123, 2021.

[6] Lean theorem prover, `https://leanprover.github.io/`.

Macarena Arenas

University of Cambridge, United Kingdom

Non-positively curved cube complexes and large-dimensional hyperbolic groups

Currently my main focus is the study of cubulated, large-dimensional hyperbolic groups. I have a method for producing a wide variety of examples of such groups, and I am currently working on exploiting this method to try to construct families of large-dimensional hyperbolic groups with exotic properties. I am interested in questions regarding different notions of dimension for cubulated hyperbolic groups, and in the finiteness properties of their subgroups.

One of the main tools I utilise is *cubical small-cancellation theory*, which is a generalisation of 'classical' small-cancellation theory. This is a very powerful tool for proving results about quotients of (often hyperbolic) cubulated groups. Some of my work focuses on proving structural results for groups that arise from 'cubical presentations' satisfying good cubical small-cancellation conditions. For instance, understanding when this cubical presentations are aspherical spaces, and when they give rise to hyperbolic quotients.

Lucía Asencio Martín

Newcastle University, UK

Folding groups, monoids and step traces

Hi! :)

For the last few years I have been interested in free groups and in different ways that they can be studied. During my master's thesis I learned about free groups from a geometric and algorithmical point of view, using Stallings automata and Stallings foldings.

I recently moved to the UK to start my PhD in Newcastle University, where I am interested in understanding more about these foldings not only for free groups but also for RAAGs.

The aim is to understand how Stallings ideas can be extended to work with trace monoids, which are the equivalent of RAAGs in the category of monoids, and with step traces, which are a generalisation of trace monoids that is used to model parallel computing problems.

Pranav Asrani
University of Münster, Germany

I am a Masters student and I have studied Geometric Group Theory in courses at Münster during the past year. I have been interested in the interaction of Differential Geometry on manifolds and the algebraic tools used to study them partly motivated by my interest in Gromov's work.

My interest in Geometric Group theory was also motivated by the paradox of Banach and Tarski wherein the proof uses tools from amenability which naturally arises in the context of Geometric Theory.

Pénélope Azuelos

University of Bristol, UK

Actions on (generalisations of) trees

Since starting my PhD in September, I've mostly been thinking about groups which act on trees, CAT(0) cube complexes and median spaces more generally. I'm also interested in learning more about the asymptotic cones of these spaces.

Over the past few months I have also been thinking about subgroup spaces of countable groups. Together with Damien Gaboriau, we are working on finding the perfect kernels of these spaces for groups that admit certain actions on trees, including for example groups with infinitely many ends.

Shaked Bader

Oxford University, United Kingdom

Higher rank hyperbolicity and homological isoperimetric inequalities

---

The Poincaré duality theorem, a classical theorem by Poincaré, states that for a closed, orientable, n-manifold M there are isomorphisms $H^k(M) \cong H_{n-k}(M)$ given by cap product with the fundamental class. By definition of group cohomology, the same holds for the fundamental group of an aspherical, closed, oriented n-manifold. A natural class of groups that arise from this result is the class of *Poincaré duality groups*.

Definition. *A group G is an n-*Poincaré duality group *if $H^n(G, \mathbb{Z}G) \cong \mathbb{Z}$, where $\mathbb{Z}$ is given with the trivial G-module structure, and for all G-modules N, there are isomorphisms $H^k(G, N) \cong H_{n-k}(G, N)$.*

Whether finitely presented Poincaré duality groups are exactly the fundamental gruops of aspherical, closed, oriented n-manifolds is an open question for $n \geq 3$. We will study finitely presented Poincaré duality groups.

In a recent work [1], Kleiner and Lang introduced a notion of higher rank hyperbolicity defined in terms of homological isoperimetric inequalities. A slightly different version of the inequality appears naturally in the study of Poincaré duality groups by Kielak and Kropholler [2]. We aim to show that the Kleiner–Lang definition can be used in place of Kielak–Kropholler's version. Once this is done, the next step would be to investigate the action of a Poincaré duality group on the boundary that Kleiner–Lang defined using their notion of higher rank hyperbolicity.

[1] Kleiner, B., Lang, U. Higher rank hyperbolicity. Invent. math. 221, 597–664 (2020). https://doi.org/10.1007/s00222-020-00955-w

[2] Kielak, Dawid and Kropholler, Peter (2021) Isoperimetric inequalities for Poincaré duality groups. Proceedings of the American Mathematical Society, 149 (11), 4685-4698. (doi:10.1090/proc/15596).

Giuseppe Bargagnati

Università di Pisa, Italy

Simplicial volume of open manifolds

I am a PhD student in topology at Pisa University. I am working in the area of simplicial volume and bounded cohomology, which lies between geometric topology and algebraic topology, and has deep connections with differential geometry and geometric group theory.

The simplicial volume was defined by Gromov in his pioneering 1982 article [Gro82]. Despite being purely homotopic in nature, the simplicial volume is deeply related to the geometric structures that a manifold can carry. It is known that, if a closed manifold admits a hyperbolic metric, then its simplicial volume is strictly positive (and in fact it is proportional to the Riemannian volume, and the proportionality constant depends only on the dimension). On the other hand, the simplicial volume of a closed manifold vanishes if the fundamental group of the manifold is amenable. In the context of non-compact oriented manifolds without boundary, the situation changes; in fact, it turns out that the simplicial volume of the hyperbolic space $\mathbb{H}^n$ vanishes for $n \geq 2$ and that an open manifold with amenable fundamental group may have either vanishing or infinite simplicial volume.

During my PhD I have been studing the simplicial volume of open manifolds; in particular, in a joint work with my advisor Prof. Roberto Frigerio [BF22], we proved that the simplicial volume of an open contractible 3-manifold M vanishes if and only if M is homeomorphic to $\mathbb{R}^3$, and is infinite otherwise. With the same techniques, we also proved that the spectrum of simplicial volume (i.e., the possible values that the simplicial volume can assume) of irreducible 3-manifolds is contained in the one of closed 3-manifolds.

Moreover, I proved that for certain classes of manifolds (namely, inward tame manifolds and simply connected at infinity manifolds) the finiteness of the simplicial volume is guaranteed by the amenability of the fundamental group at infinity [Bar22].

In my future research I would like to make steps toward the computation of the spectrum of simplicial volume of open 3-manifolds, and to explore more connections of simplicial volume and bounded cohomology with geometric group theory.

[Gro82] M. Gromov, *Volume and bounded cohomology* Inst. Hautes Études Sci. Publ. Math., 1982.

[BF22] G. Bargagnati and R. Frigerio, *Simplicial volume of contractible 3-manifolds* Trans. Am. Math. Soc., 2022.

[Bar22] G. Bargagnati, *Simplicial volume of manifolds with amenable fundamental group at infinity* arXiv, https://arxiv.org/abs/2207.10525, 2022.

Oussama Bensaid
Max Planck Institute for Mathematics, Bonn, Germany

Embeddings between groups and metric spaces

I am interested in the coarse geometry of spaces of non-positive curvature, such as symmetric spaces and Euclidean buildings. During my Ph.D. thesis, I have been interested in coarse embeddings between such spaces. Coarse embeddings have been introduced by Gromov [1] in the 80's, and they generalize quasi-isometric embeddings when the control functions are not necessarily affine. A map $f : (X, d_X) \to (Y, d_Y)$ is a *coarse embedding* if there exist functions $\rho_\pm : [0, \infty) \to [0, \infty)$ such that $\rho_-(r) \to \infty$ as $r \to \infty$ and for all $x, y \in X$

$$\rho_-(d_X(x, y)) \leq d_Y(f(x), f(y)) \leq \rho_+(d_X(x, y)).$$

For example, the embedding of $\mathbb{R}^n$ into $\mathbb{H}^{n+1}$ as a horosphere is a coarse embedding that is exponentially distorted. Note also that a subgroup inclusion between finitely generated groups is always a coarse embedding, while it is a quasi-isometric embedding only when the subgroup in question is undistorted. An important feature of CAT(0) spaces, and in particular symmetric spaces of noncompact type and Euclidean buildings, is their *rank*, which is the maximal dimension of an isometrically embedded copy of a Euclidean space. We can show that for these spaces, the rank is monotonous under coarse embeddings:

Theorem. [2] *Let $X = S \times B$ be of rank k, where S is a product of symmetric spaces of noncompact type and B is a product of Euclidean buildings.*
If Y is a proper cocompact CAT(0) space of rank $< k$, then there is no coarse embedding from X to Y.

We can show a similar result for mapping class groups in the target space. When we restrict the target to symmetric spaces and Euclidean buildings, we can allow the domain to have a Euclidean factor of dimension 1. A Euclidean factor of dimension $\geq$ 2 in the domain is not allowed because of the horospherical embeddings $\mathbb{R}^2 \to \mathbb{H}^3$.

[1] M. Gromov, Asymptotic invariants of infinite groups. Geometric group theory, 2:1–295, 1993.

[2] O. Bensaid, (2022). Coarse embeddings of symmetric spaces and Euclidean buildings. arXiv preprint:2201.06442.

Federica Bertolotti

Scuola Normale Superiore di Pisa, Italy

Simplicial volume of mapping tori

I am doing my Ph.D. in Pisa with Roberto Frigerio and I am interested in the simplicial volume of mapping tori. Before saying anything more about my research, let me spend a few words on these two objects.

Let M be an oriented closed manifold and $f : M \to M$ be an orientation preserving homeomorphism. The *mapping torus* with monodromy f and fiber M is given by

$$E_f = M \times [0,1]/\sim,$$

where $(x,1) \sim (f(x),0)$ for every $x \in M$. In particular, the mapping tori are exactly the manifolds fibering over S^1.

The *simplicial volume* is a homotopy invariant that measures the complexity of closed oriented manifolds in terms of singular simplices (to be more accurate, in terms of singular chains). Despite the topological definition, the simplicial volume has several implications on the geometry that a manifold can carry: just to mention an example, if a manifold M admits a negatively curved Riemannian metric, then it has positive simplicial volume and, on the other hand, if the metric has non-negative Ricci tensor, then the simplicial volume vanishes.

Given M an oriented closed connected manifold, I denote by $\mathrm{MCG}(M)$ the mapping class group of M, i.e. the set of homotopy classes of orientation preserving self-homotopy equivalences of M. In order to study the simplicial volume of mapping tori, Roberto Frigerio and I defined in [1] a length function on the mapping class group of M, and we called it *filling volume*:

$$\mathrm{FV}\colon \mathrm{MCG}(M) \to \mathbb{R}_{\geq 0}.$$

This map can be seen as a kind of dynamical invariant that weighs the complexity of the action of a mapping class on the set of singular simplices of M. In the same paper we proved that for every orientation preserving self-homeomorphism $f\colon M \to M$, the filling volume $\mathrm{FV}([f])$ of the mapping class $[f] \in \mathrm{MCG}(M)$ represented by f is given by the simplicial volume of the mapping torus with monodromy f and fiber M. Moreover, thanks to this invariant, we proved some vanishing results on the simplicial volume of mapping tori.

Now I am trying to understand better this map. The hope is to find even more applications and, maybe, to find some conditions on the monodromy guaranteeing the positivity of the simplicial volume of the associated mapping torus.

[1] Federica Bertolotti, Roberto Frigerio (2022) *Length functions on mapping class groups and simplicial volumes of mapping tori*, arXiv:2205.10846.

Lara Beßmann
University of Münster, Germany

Universal groups for right-angled buildings and automatic continuity

Universal groups have been introduced by Burger and Mozes in [1] to study lattices in products of trees. They are subgroups of the automorphism group of a locally finite regular tree such that the action around a vertex is contained in a prescribed permutation group. Varying the permutation group leads to different universal groups that are all acting on the same tree. The automorphism group of the tree can be endowed with the topology of pointwise convergence. A neighbourhood basis of the identity is then given by pointwise stabilisers of finite sets of vertices. Hence, the universal groups are topological groups with respect to the subspace topology. These groups are not only interesting to study lattices but also provide nice examples of locally compact totally disconnected topological groups and moreover also of infinite simple groups.

More recently, they have been generalised to right-angled buildings by De Medts, Silva, and Struyve in [2]. For the definition of a universal group over a right-angled building of type (W, I) are $|I|$ many local permutation groups necessary and they prescribe the action on the panels. The properties of the universal group depend highly on the properties of the local groups. For example, a universal group is discrete if and only if all local groups act freely. Hence, these groups provide even more examples of topological groups with different properties and we can control which properties a universal group has by choosing the local permutation groups in a suitable way.

I study automatic continuity for universal groups defined over right-angled buildings. The main question I work on is, which conditions on the universal group ensure that any abstract group homomorphism from an arbitrary Polish group into this universal group is continuous. This leads then for example to results on the uniqueness of the Polish group topology of the universal group.

[1] M. Burger and S. Mozes. *Groups acting on trees: From local to global structure*. Publications mathématiques de l'IHÉS 92 (2000), no. 1, pp. 113-150.

[2] T. De Medts, A. C. Silva, and K. Struyve. *Universal groups for right-angled buildings*. Groups Geom. Dyn. 12 (2018), no. 1, pp. 231-287.

Nicolás Bitar

Université Paris-Saclay, France

Symbolic dynamics on finitely generated groups

My research takes place in the area of symbolic dynamics over finitely generated groups. The main objects of study are subshifts, that is, sets of maps $x : G \to A$ from a finitely generated group G to a finite alphabet A subject to constraints specified as a collection of forbidden patterns. These spaces have a natural left G-action $gx(h) = x(g^{-1}h)$, called the shift. In fact, this action characterizes subshifts as the closed G-invariant subsets of A^G for the product topology. These spaces were originally conceived as symbolic encodings of group actions, but have become objects of interest in themselves.

In particular, I study how different group properties place structural and algorithmic constraints on the properties of subshifts. Perhaps the quintessential example of this is the Domino Problem. The problem asks if given a finite set of forbidden patterns it is possible to determine if the subshift they define – known as a subshift of finite type – is empty. The decidability of this problem has been shown to be a commensurability invariant for finitely generated groups and a quasi-isometry invariant for finitely presented ones. In fact, the main open problem is the Domino Conjecture: a finitely generated group has decidable Domino Problem if and only if the group is virtually free. It is here that using tools from geometric group theory becomes essential. For instance, as the decidability of the problem is inherited by subgroups, through the Dunwoody-Stallings Theorem, we can restrict ourselves to the study of one-ended groups. In this direction, we proved that, due to their graph of groups decomposition, non-$\mathbb{Z}$ Generalized Baumslag-Solitar groups have undecidable Domino Problem [1].

More generally, I am interested in questions ranging from variants of the Domino Problem to the realizability of subsets of subgroups as stabilizers of shifts of finite type. Most of these problems have been extensively studied in the case of finitely generated free abelian groups. Within this class of groups, there is drastically different behavior between the one dimensional and the higher dimensional cases. What geometrical and combinatorial aspects of $\mathbb{Z}$ and $\mathbb{Z}^d$ account for this difference? I aim to understand this difference and expand these results to finitely generated groups.

The use of tools and ideas borrowed from other domains has already proven to be a successful strategy. As an example, notions from computability theory have proven essential to characterize which entropies are realizable for subshifts of finite type over $\mathbb{Z}^2$. I believe that the interplay between these areas, namely, computability theory, geometric group theory and symbolic dynamics can shed light at the connections between algebraic structures and their capabilities to symbolically encode computation.

[1] Aubrun, N., Bitar, N., & Huriot-Tattegrain, S. (2022). Strongly Aperiodic SFTs on Generalized Baumslag-Solitar groups. arXiv preprint arXiv:2204.11492.

Corentin Bodart

University of Geneva, Switzerland

Regular normal forms for groups, and related properties

I'm a 3rd year PhD student under the supervision of Tatiana Nagnibeda. The central object in my research would be *regular normal forms*. A regular normal form for $G = \langle S \rangle$ is a language $\mathcal{L} \subset S^*$ recognized by a finite state automaton, such that the evaluation map $\bar{\cdot} : \mathcal{L} \to G$ is bijective. I'm usually interested in proving negative results, so a first question was whether we can find groups without any regular normal form. Somehow *regular left-invariant orders* pop up, and things can be done [1]. Even if your favorite group admits regular normal form, just knowing how normal forms look like might be interesting. Indeed, several properties of groups can be restated as the existence of a regular normal form with additional properties. Most notably,

- A group is *automatic* if it admits a regular normal form s.t. any words $v, w \in \mathcal{L}$ ending up at neighboring elements $\bar{v}, \bar{w}$ fellow-travel. Perhaps the most intriguing question regarding automaticy is "Is Thompson's group F (bi)automatic?". More generally, finding new obstructions to (bi)automaticy would be interesting.
- The *complete growth series* of group $G = \langle S \rangle$ is the formal series
$$\Gamma_{(G,S)}(z) = \sum_{g \in G} g \cdot z^{\|g\|_S} \in \mathbb{N}G[[z]].$$
The rationality of $\Gamma_{(G,S)}$ should be thought as an intermediary property between admitting a geodesic regular normal form (i.e., $|w| = \|\bar{w}\|_S$ for all $w \in \mathcal{L}$) and admitting a quasi-geodesic regular normal form (i.e., $|w| \leq C\,\|\bar{w}\|_S$).

  A conjecture is that complete growth series of nilpotent groups should never be rational, unless the group is actually virtually abelian. In joint work with Pierre Bagnoud, we treated the case of Heisenberg groups [2], and more generally of 2-step-nilpotent groups. This work is based on geometric properties of the group and its Malcev completion, and particularly the existence of *dead ends* elements (or related elements). The conjecture as a whole remains to be settled.

Lately, I have also been interested at other types of growths of groups (specifically *cogrowth* and geodesic growth) and their associated languages (the Word Problem and the full language of geodesics). This has renewed my interest into *random walks* on groups, and particularly asymptotics of return probabilities.

[1] Corentin Bodart, *Rational cross-sections, bounded generation and orders on groups*, Preprint arXiv:2210.04219 (2022).

[2] Pierre Alderic Bagnoud and Corentin Bodart, *Dead ends and rationality of complete growth series*, Preprint arXiv:2210.07868 (2022).

Lukas Böke

LMU München, Germany

Quasi-morphisms on groups of diffeomorphisms

We study the group $\mathrm{Diff}_0(M)$ of smooth diffeomorphisms on a compact connected manifold M which are isotopic to the identity. These groups are known to be perfect (this goes back to results of Mather and Thurston in the 1970s). More recently, the question of boundedness of conjugation-invariant norms (and thus uniform perfectness) on these groups was brought up in [2]. For manifolds whose dimension is three or greater than five, by [2] and results due to Tsuboi [3], it is known that their Diff_0 is uniformly perfect.

In two dimensions, the picture is drastically different: The only surface known to allow no unbounded conjugation-invariant norm is the 2-spere. For most surfaces of higher genus, both orientable and not, [1] provides a method to construct non-trivial homogeneous quasi-morphisms on their Diff_0. By the duality theorem of Bavard, this implies that the commutator length is a unbounded conjugation-invariant norm and the group is not uniformly perfect. One of the few remaining surfaces where this question is still open is the Möbius strip. The goal of our research is to construct a homogeneous quasi-morphism on Diff_0 of the Möbius strip by considering the action of these diffeomorphisms on simple closed curves.

[1] Jonathan Bowden, Sebastian Hensel, and Richard Webb. Quasi-morphisms on surface diffeomorphism groups. In *J. Amer. Math. Soc.*, 35:211-231, 2022.

[2] Dmitri Burago, Sergei Ivanov, and Leonid Polterovich. Conjugation-invariant norms on groups of geometric origin. In *Groups of Diffeomorphisms*, volume 52 of *Advanced Studies in Pure Mathematics*, pages 221-250. Math. Soc. Japan, 2008.

[3] Takashi Tsuboi. On the uniform perfectness of diffeomorphism groups. In *Groups of Diffeomorphisms*, volume 52 of *Advanced Studies in Pure Mathematics*, pages 221-250. Math. Soc. Japan, 2008.

Laura Bonn

KIT - Karlsruher Institut für Technologie, Germany

Finiteness properties for non discrete groups

In the field of discrete groups there are some statements about their finiteness properties and there are known examples for groups that are of type F_n but not of type F_{n+1} [1].

In which cases of non discrete groups is it possible to give a good definition of being of type F_n? And how we can find examples of groups which are of type F_∞ or which are of type F_n but not of type F_{n+1}.

The idea is now to find some non discrete groups which are similar to discrete groups for which the finiteness properties are known. One way is to use Thompson like groups, since for the Thompson group F it is known that it is of type F_∞. So we have a prove structure for this group and can try to use this sheme to prove similar statements for Thompson like groups. A construction of Thompson like groups is described in [2].

Another way to construct groups is the construction of Simon Smith [3], the idea here is not to use proofs of the finiteness conditons of similar groups. Here we want to construct groups with specific topological properties. So we can construtct in this way totally disconnected locally compact groups.

A way to define finiteness properties for tdlc groups is given in [4]. Now I have a construction idea for examples and I have a way to define finiteness in the totally disconnected locally compact case. The next step would be to combine both and try to find such examples of groups.

[1] Finitness properties of groups, Kenneth S. Brown, https://core.ac.uk/download/pdf/82012275.pdf

[2] Almos-automorphisms of trees, coloning systems and finitness properties, Skipper and Zaremsky, https://arxiv.org/pdf/1709.06524.pdf

[3] A product for permutation groups and topological groups, Simon M. Smith, https://arxiv.org/pdf/1407.5697.pdf

[4] Finitness properties of totally disconnected locally compact groups, I. Castellano and G. Corob Cook, https://arxiv.org/pdf/1901.08470.pdf

Sira Busch

WWU Münster, Germany

Perspectivities and root groups in buildings

My research focuses on spherical buildings. A building is spherical if its Weyl group is finite. If Δ is a spherical building, the perspectivities in Δ generate a groupoid that encodes the geometry of Δ very well. In my research I strive to provide more insight into this groupoid.

Jacques Tits proved that all higher dimensional spherical buildings have the Moufang property. [2] A spherical building Δ of rank at least two has the Moufang property if for each root α of Δ, the root group U_α acts transitively on the set of all apartments of Δ containing α. [1] A first goal of my research is to try to construct a sequence of perspectivities with which the root automorphisms of subbuildings can be displayed.

[1] Richard M. Weiss, *The Structure of Spherical Buildings*, Princeton University Press, 2003.

[2] Jacques Tits, *Buildings of spherical type and finite BN-pairs*, Lecture Notes in Mathematics **386** (1974).

Rabi Kumar Chakraborty

Westfalische-Wilhelms Universitat Munster, Germany

Group Cohomology

I am currently a 1st year Masters' Student in WWU Münster.

Here, in this sem, I am studying unstable homotopy theory(right now,magic cube is going on),giving a talk on group cohomology(the topological view with applications in groups acting on spheres),Riemannian Geometry I (upto Jacobi fields), and giving a talk in Serre spectral sequence.

I am whole-heartedly interested in Geometry and topology and studying,learning and working out topology from a geometric view-point as far as possible. Previously, I have done an internship in Morse Theory from Milnor's book(upto Reeb's theorem). I am very much eager to learn Geometric topology and classification theory of Riemannian manifolds.

Next semester, I will be taking a course on stable homotopy theory(by the same professor who is taking unstable homotopy theory now).

I already have a Masters' in Mathematics from India, where I read Bredon's, Janich's and Hatcher's book on Algebraic Topology . We learnt about CW and simplicial complexes, fundamental groups,singular homology of spaces ,Kunneth and abelianisation of 1st fundamental group, deck transformations and group actions on spheres; and extensively solved Hatcher's exercise problems apart from the assignments given by our instructor.

Again, in 3rd semester, I read singular cohomology,calculation of cohomology rings of standard spaces,cup and cap products,Poincare Duality; with Higher homotopy theory-cellular approximation,Postnikov towers,Hurewicz(for both single and pair of spaces),fibre bundles,(co)fibrations, fibration sequence and Barrett-Puppe sequence.

In my last MSc from India, I also learnt representation theory of Lie algebras.

Yassin Chandran

The Graduate Center, CUNY, USA

Infinite type surfaces and marked hyperbolic structures

Broadly, I like thinking about:

- surfaces, the hyperbolic/conformal structures they carry, and various related metric/combinatorial spaces (eg. Teichmüller space, curve graph)
- hyperbolic geometry and generalizations of Gromov hyperbolicity (eg. hierarchically hyperbolic spaces, projection complexes)

An *infinite type* surface is a surface whose fundamental group is infinitely generated. In recent years there has been a large surge of energy dedicated to their study and it remains an incredibly fertile and active field. See [3] for a list of open questions and [1] for a survey.

Lately I've been thinking about how (big) mapping classes distort hyperbolic metrics on an infinite type surface, and importing tools from Teichmüller theory of finite type surfaces to the infinite type world.

In ongoing joint work with Ara Basmajian, we investigate the space of marked hyperbolic structures on a surface and show big mapping classes satisfy a trichotomy stated in terms of how they distort hyperbolic metrics. This point of view is very much inspired by Bers' proof of the celebrated Nielsen-Thurston classification for finite type surfaces [2].

[1] Javier Aramayona and Nicholas G. Vlamis. "Big mapping class groups: an overview." In Ken'ichi Ohshika and Athanase Papadopoulos, editors, In the Tradition of Thurston: Geometry and Topology, chapter 12, pages 459–496. Springer, (2020).

[2] Lipman Bers "An extremal problem for quasiconformal mappings and a theorem by Thurston," Acta Mathematica, Acta Math. 141(none), 73-98, (1978)

[3] Yassin Chandran, Priyam Patel, and Nicholas G. Vlamis. "Infinite-type surfaces and mapping class groups: open problems", available at `http://qcpages.qc.cuny.edu/~nvlamis/Papers/InfTypeProblems.pdf`, (2021)

Dorian Chanfi
Justus-Liebig-Universität Gießen, Germany

Finiteness properties of solvable S-arithmetic groups

I am a first-year postdoc in Gießen working in the research group of Stefan Witzel. My research at the moment focuses on the finiteness properties of solvable S-arithmetic subgroups of linear groups. Recall that a group is said to have the finiteness property $\mathcal{F}_n$ if it admits a $K(G,1)$ with finite n-skeleton. More precisely, I am thinking about Σ-invariants of solvable S-arithmetic subgroups of linear groups. The Σ-invariants of a group G, defined in [1], are geometric invariants of G that encode the finiteness properties of all subgroups H of G containing $[G,G]$.

Prior to this, my PhD research centered on constructions of equivariant compactifications of Bruhat-Tits buildings of (quasi-)reductive groups G and the study of the relationships between the geometry of these compactifications and the geometry of various G-spaces [2, 3]. Consequently, I am also broadly interested in linear algebraic groups in general and their associated geometries: flag varieties, wonderful completions, symmetric spaces, buildings etc.

[1] R. Bieri, B. Renz, Valuations on free resolutions and higher geometric invariants of groups, *Commentarii mathematici Helvetici*, 63(3):464–497, 1988.

[2] D. Chanfi, Wonderful compactifications of Bruhat-Tits buildings in the non-split case, arXiv:2011.00349, to appear in *Israel J. Math*.

[3] D. Chanfi, Polyhedral compactifications of Bruhat-Tits buildings of quasi-reductive groups, arXiv:2206.04775.

Hyeran Cho

The Ohio State University, US

Riemannian metric with non-positive curvature on branched coverings

---

My research focuses on 4-manifolds that are modeled on $\mathbb{H}^2 \times \mathbb{H}^2$. They have a form $M = (\mathbb{H}^2 \times \mathbb{H}^2)/\Gamma$ where Γ is a discrete subgroup of $Isom(\mathbb{H}^2 \times \mathbb{H}^2)$ acting geometrically on $\mathbb{H}^2 \times \mathbb{H}^2$. For a codimension 2 totally geodesic submanifold N of M, I consider a branched covering $\overline{M} \to M$ branched over N. I'm interested in non-positively curved Riemannian metrics on the branched covering $\overline{M}$.

In [1], Stadler introduced an interesting example where M is of the form $\Sigma_g \times \Sigma_g$ where Σ_g is a closed surface of genus g and $N = \Delta$ is the diagonal of M. In this case, he proved that there is no Riemannian metric on $\overline{M}$ with non-positive sectional curvature. I'm particularly interested in other cases with different types of submanifold N of $M = \Sigma_g \times \Sigma_g$.

For the case that N is of the diagonal type (i.e., it is locally diagonal. In other words, projections to each factor are surjective), we observed that there is no smooth metric on $\overline{M}$ with non-positive curvature by using Stadler's obstruction in [1]. For the case that N is locally a product $\{point\} \times \mathbb{H}^2$, I constructed a smooth non-positively curved metric on $\overline{M}$ that is equal to the pull-back metric by the branched covering map for sufficiently large radius. The metric construction is motivated by the metric smoothing in [2]. Recently, my advisor Lafont and I work on the remaining case that N is locally a product $\{geodesic\} \times \{geodesic\}$.

Another project that I'm working on with Lafont and Skipper(ENS Paris), is group theoretic version of branched covering spaces.

[1] Stadler, Stephan. "An obstruction to the smoothability of singular nonpositively curved metrics on 4-manifolds by patterns of incompressible tori." Geometric and Functional Analysis 25.5 (2015): 1575-1587.

[2] Constantine, D., et al. "Fat flats in rank one manifolds." Michigan Mathematical Journal 68.2 (2019): 251-275.

Michelle Chu

University of Minnesota–Twin Cities, United States

Hyperbolic manifolds and arithmetic groups

A (finite-volume) hyperbolic manifold M is realized a quotient $\mathbb{H}^n/\Gamma$ where Γ acts on $\mathbb{H}^n$ properly discontinously by isometries. The number theoretical aspects of the elements of Γ as matrices in $\mathrm{Isom}(\mathbb{H}^n) \cong \mathrm{SO}(n, 1; \mathbb{R})$ have many consequences on the geometry of the manifold such as its volume, its symmetries, the lengths of geodesics, or the existence of totally geodesic submanifolds.

There has been a lot of progress in the understanding of hyperbolic 3-manifolds since the first YGGT in 2012. However, very little is known in higher dimensions. A well-known method to construct hyperbolic manifolds in any dimensions is through arithmetic groups which arise from quadratic forms over totally real number fields. Such manifolds are called arithmetic hyperbolic manifolds of simplest type. One may also construct non-arithmetic hyperbolic manifolds by cutting and pasting pieces of arithmetic hyperbolic manifolds. There are many interesting questions one can ask about these manifolds and their fundamental groups. I am particularly interested in understanding the relationship between the geometry of such hyperbolic manifolds and the arithmetic invariants of their fundamental groups.

Gemma Crowe
Heriot-Watt University, Scotland

Decision problems in groups and extensions

I am in my 3rd year of my PhD, and my project focuses on the following question.

Question. *Let G be a virtual right-angled Artin group (RAAG), i.e. there exists a finite index subgroup $H \leq G$ such that H is a RAAG. What can we say about conjugacy in G?*

This is a somewhat vague question, so let me formalise some of the ideas I've been studying so far:

1. Is the conjugacy problem solvable in G?
2. What type of conjugacy growth does G have?
3. What happens when we replace RAAG with graph products?

By the Milnor-Schwarz Lemma, any group G of this form is quasi-isometric to a RAAG. Quasi-isometries are a classic tool in geometric group theory, and many properties, such as hyperbolicity and standard growth, are quasi-isometry invariant. However, the conjugacy problem does not in general pass through a quasi-isometry [1], and it is still open as to whether conjugacy growth is a quasi-isometry invariant.
My work has focused on twisted conjugacy, $\mathrm{CAT}(0)$ groups and formal language theory. Since RAAGs have linear-time conjugacy problem [3], I am also interested in the complexity of the conjugacy problem in G. Other topics of interest include equations in groups and other decision problems.

[1] D. Collins, C. Miller; The conjugacy problem and subgroups of finite index. *Proceedings of the London Mathematical Society*, s3-34(3):535-556, 1977

[2] Crisp, Godelle, Wiest; The conjugacy problem in subgroups of right-angled Artin groups. *Journal of Topology*, 2(3):442-460, 2009

[3] Crowe; Conjugacy languages in virtual graph products (to appear)

Colin Davalo
Universität Heidelberg, Germany

Geometric structures associated to Anosov representations.

I am interested in representations $\rho : \Gamma \to G$ of discrete groups Γ into semi-simple Lie groups G, in particular interested when Γ is a surface group. In particular I like to think about some special unions of connected components of the space of representations of a surface groups that have been shown to only contain discrete and faithful representations: such component are called *Higher Teichmüller components*.

This is the case for fuchsian representations when $G = \text{PSL}(2, \mathbb{R})$, Hitchin representations for $G = \text{PSL}(n, \mathbb{R})$ or any real split simple Lie group, or maximal representations when G is of hermitian type and tube type, for instance $G = \text{Sp}(2n, \mathbb{R})$. These families of examples fit in the more general notion of Θ-positivity .

Representations in all know higher Teichmüller components satisfy some dynamical properties called the *Anosov* properties, which were introduced by Labourie . An interesting question is to determine which Anosov representations a given group can have. I am interested in particular in the case of Borel Anosov representations of surface groups in $\text{Sp}(4, \mathbb{R})$ [1].

I am also interested in geometric structures in the sense of (G, X)-*structures*. A (G, X)-structure on a manifold M induces a holonomy map $\rho : \pi_1(M) \to G$, and in some cases connected components of representations, up to conjugation correspond via this map to spaces of (G, X)-structures on a manifold. For instance fuchsian, i.e discrete and faithful, representations of the fundamental group of a closed orientable surface S_g of genus $g \geq 2$ corresponds to (marked) hyperbolic structures, i.e. $(\text{PSL}(2, \mathbb{R}), \mathbb{H}^2)$-structures on S_g.

Some cocompact domains of discontinuity in flag manifold have been constructed for Anosov representations, therefore such representations are the holonomy of a manifold M that admits Γ as one of it's quotients. I try to show that these domains fiber over the universal cover of the surface, so the manifold M is a fiber bundle over the surface, and I am interested in characterizations of the geometric structures arising through this construction

To work on these questions I like to study symmetric spaces of non-compact type and their compactifications.

[1] Maximal and Borel Anosov representations in $\text{Sp}(4, \mathbb{R})$, `https://arxiv.org/abs/2201.05584`.

Isobel Davies
Otto-von-Guericke-Universität Magdeburg, Germany

A unified approach to Euclidean Buildings and Symmetric Spaces

---

Let X be either a Euclidean building or a symmetric space of non-compact type and consider its ideal boundary ∂X. Currently, I am studying the following problem:

Given ∂X, can one reconstruct the space X?

Given a proper CAT(-1) space X, one can define the cross ratio on four boundary points, using the Gromov product (see [3]). In [1], Bourdon shows how the cross ratio on four boundary points $\omega(\alpha, \beta; \gamma, \delta)$ can be used to find the distance between two points $p(\alpha, \beta; \gamma), p(\alpha, \beta; \delta) \in X$. Using this, I can uniformly reconstruct a rank 1 Euclidean building or rank 1 symmetric space of non-compact type from its ideal boundary.

Looking forward, I wish to uniformly prove that ∂X has the structure of a spherical building and use this structure to find a uniform proof that one can reconstruct X of higher rank.

[1] Marc Bourdon, *Sur le birapport au bord des CAT(-1)-espaces*, Publications mathématiques de l'IHÉS, **83**, 95-104, (1996).

[2] Martin R. Bridson and André Haefliger, *Metric Spaces of Non-Positive Curvature*, Springer, 1999.

[3] Sergei Buyalo and Viktor Schroeder, *Elements of Asymptotic Geometry*. European Mathematical Society, 2007.

[4] Thomas Foertsch and Viktor Schroeder, *Hyperbolicity, CAT(-1)-spaces and the Ptolemy Inequality*, Mathematische Annalen, **350**, 339-356, (2010).

Tim de Laat

University of Münster, Germany

Rigidity properties for group actions

Fixed point properties of affine group actions on Banach spaces and spectral gap of affine group actions on low-dimensional manifolds are fundamental rigidity properties with applications in various areas of mathematics. One such application is the construction of expanders, which are sequences of finite, highly connected, sparse graphs with an increasing number of vertices.

An important source of rigidity properties are actions of groups with Kazhdan's property (T) (for example $\mathrm{SL}(n,\mathbb{R})$ and $\mathrm{SL}(n,\mathbb{Z})$ for $n \geq 3$) or strengthenings of this property, such as Lafforgue's strong property (T). Strong property (T) can be viewed as the trivial representation being isolated among all representations with small exponential growth on Banach spaces with nontrivial type (or on other classes of Banach spaces). In a series of joint works with M. de la Salle, I proved strong property (T) for higher rank Lie groups with respect to various large classes of Banach spaces (see e.g. [1]).

Another source of rigidity properties, in particular of spectral gap, comes from taking into account the geometry of the action. In joint work with G. Arzhantseva, D. Kielak, and D. Sawicki, I constructed the first examples of group actions with spectral gap on surfaces of arbitrary genus > 1. In our actions, the acting group is $\mathbb{F}_2$, which is far from having property (T). A major application of our actions is the construction of a new type of expanders with surprising large-scale geometric features.

[1] T. de Laat and M. de la Salle, *Strong property (T) for higher rank simple Lie groups*, Proc. London Math. Soc. **111** (2015), 936-966.

[2] G. Arzhantseva, D. Kielak, T. de Laat, and D. Sawicki, *Spectral gap on origami surfaces*, preprint (2021), arXiv:2112.11864.

Rodrigo De Pool

Instituto de Ciencias Matemáticas (ICMAT), Madrid

Predoctoral researcher.

In a broad sense, I am interested in interactions between Topology, Geometry and Group Theory (unexpected, eh?). Specifically, I have been studying Mapping Class Groups of finite type surfaces, Curve complexes and other hyperbolic spaces associated to surfaces.

The main problem of my thesis is to extend the classification of homomorphisms between Mapping Class Groups given by Aramayona and Souto in [1]. In this direction, I am currently trying to understand pseudo-Anosov transformations and Teichmüller spaces.

As for my previous work, I have proven the finite rigidity of Non-separating Curve complexes in [3]. Also, jointly with Aramayona and Fernández, we addressed a question of Zaremsky regarding the hyperbolicity of Matching Arc complexes in [2].

[1] Javier Aramayona and Juan Souto. Homomorphisms between mapping class groups. *Geometry & Topology* vol. 16, no. 4 (2013).

[2] Javier Aramayona, Alejandro Fernández and Rodrigo de Pool. *Expositiones Mathematicae* vol. 40, no. 2 (2021).

[3] Rodrigo de Pool. Finite rigid sets of Non-separating Curve complex. *Preprint* (2022). arXiv: 2210.05317 [math]

Jari Desmet
Ghent University, Belgium

Non-associative algebras for exceptional algebraic groups

Linear algebraic groups are matrix groups defined by polynomials. Among the objects of most interest in this theory are the exceptional groups.

Recently, a class of non-associative algebras that have these exceptional groups as symmetries have been discovered [1, 2]. The motivation for these constructions was originally to study the unique equivariant algebra structure on the second smallest irreducible representation of E_8, but it can be used to construct algebras for other types as well.

I have been studying these algebras to find different constructions (such as in [3]) and discern more of their properties. One exciting direction is the field of *axial algebras*, a link that was studied in [1].

[1] Tom De Medts and Michiel Van Couwenberghe. "Non-associative Frobenius algebras for simply laced Chevalley groups." *Transactions of the American Mathematical Society* 374.12 (2021): 8715-8774.

[2] Maurice Chayet and Skip Garibaldi. "A class of continuous non-associative algebras arising from algebraic groups including E_8." *Forum of Mathematics, Sigma*. Vol. 9. Cambridge University Press, 2021.

[3] Jari Desmet. "Non-associative Frobenius algebras for type G_2 and F_4." *arXiv preprint arXiv:2204.05913* (2022).

Ethan Dlugie

University of California, Berkeley, United States

Complex Hyperbolic Geometry and Flat Surfaces

Background: Anyone interested in hyperbolic manifolds is secretly interested in lattices in Lie groups, specifically the Lie group $\mathrm{PO}(1,n) \approx \mathrm{Isom}(\mathbb{H}^n)$. In general, one can find lattices in Lie groups with a certain number theoretic construction. Such lattices are called *arithmetic*, and the celebrated arithmeticity theorem of Margulis says that pretty much any lattice you find your hands on is actually an arithmetic one. The only Lie groups that might contain *nonarithmetic lattices* are $\mathrm{PO}(1,n)$, the isometries of (real) hyperbolic space $\mathbb{H}^n$, and $\mathrm{PU}(1,n)$, the isometries of complex hyperbolic space $\mathbb{CH}^n$.

In this story, Thurston reinterpreted results of Deligne and Mostow by considering the moduli space of *flat cone metrics* on the sphere. This space is topologically isomorphic to the classical moduli space of a punctured sphere, but the data of the flat structure yields a natural complex hyperbolic metric on moduli space. Each choice of flat surface data gives a representation of the mapping class group $\mathrm{MCG}(S_{0,n+3}) \to \mathrm{PU}(1,n)$, and Thurston found that a few special choices of the flat surface data give representations whose images are essentially the only known nonarithmetic lattices here [3].

Current work: Veech showed that Thurston's flat surface story could analogously be told one genus up, by considering flat cone metrics on the torus. The details of the geometry on this moduli space were explored more recently in [2]. At present I am working to flesh out this story, to find which flat surface data on the torus could yield mapping class group representations whose images are lattices in $\mathrm{PU}(1,n)$. Hopefully I will have some answers to this by the time we are meeting in Münster!

Other work: Again working from Thurston's flat cone sphere technology, I explored the connection between Thurston's representations $\mathrm{MCG}(S_{0,m+3}) \to \mathrm{PU}(1,m)$ and specializations of the *Burau representation* of braid groups $B_n \to \mathrm{GL}_{n-1}(\mathbb{Z}[t^{\pm}])$. The complex hyperbolic structures on moduli space occasionally extend to orbifold structures, and I leveraged this to place some restrictions on the kernel of the $n = 4$ Burau representation [1]. This is the last case for which faithfulness of the Burau representation is unknown, and this faithfulness question in particular has strong connections to the question of whether the Jones polynomial detects unknots.

[1] Ethan Dlugie, *The Burau representation and shapes of polyhedra*, arXiv:2210.06561.

[2] Selim Ghazouani, Luc Pirio, *Moduli spaces of flat tori with prescribed holonomy*, Geometric and Functional Analysis **27** (2017), no. 6, 1289-1366.

[3] William P. Thurston, *Shapes of polyhedra and triangulations of the sphere*, The Epstein Birthday Schrift, 1998, 511-549.

Sam Dodds

University of Illinois - Chicago, United States

Rigidity & Flexibility of Random Walks on Hyperbolic Groups

My research is in the intersection of geometric group theory and ergodic theory. Specifically, I am interested in “random” aspects of groups that are “weakly hyperbolic” where both terms are broadly construed.

The central theme of recent projects that I have worked on has been understanding the stationary actions of hyperbolic groups. Given a hyperbolic group Γ and a probability μ on Γ, a space X with a measure ν is a *stationary space* if the measure ν is invariant under the Γ action on average according to μ. Precisely, ν is invariant under convolution: $\mu * \nu = \int_\Gamma g_*\nu \, d\mu(g) = \nu$. If X is compact, then such a μ always exists. When μ is a probability measure, the study of these spaces can almost be divided into two separate problems; understanding measure preserving actions, and understanding *stationary boundaries*. See the article by Furstenberg and Glasner [2] for details. For weak assumptions on Γ and μ, the Gromov boundary of Γ is an example of a stationary boundary, but there are typically many more boundaries. I have studied these spaces primarily through the lens of entropy theory [1].

I am also interested in study random groups: their boundaries; and random walks on random groups. It is well known that for certain parameters in the Gromov model of random groups, the resulting random group almost surely is both hyperbolic, and has property (T). Both of these features have interesting implications for random walks on these groups and the stationary boundaries of these groups.

[1] Samuel Dodds, *On the Lattice of Boundaries and the Entropy Spectrum of Hyperbolic Groups* (2022), available at `arXiv:2211.04554`.

[2] Hillel Furstenberg and Eli Glasner, *Stationary dynamical systems*, Dynamical numbers—interplay between dynamical systems and number theory, Contemp. Math., vol. 532, Amer. Math. Soc., Providence, RI, 2010, pp. 1–28, DOI 10.1090/conm/532/10481. MR2762131

George Domat
Rice University/Fields Institute, USA/Canada

Big mapping class groups in dimensions 1 and 2

I study the groups of topological symmetries of infinite-type surfaces and graphs. An infinite-type surface, S, is a surface that has "infinitely much" topology, i.e. its fundamental group is not finitely generated. For a graph, Γ, infinite-type means locally finite and infinite. The groups I study are $\mathrm{Map}(S)$ and $\mathrm{Map}(\Gamma)$, the **mapping class group** of a surface S and a (locally finite) graph Γ. For surfaces, $\mathrm{Map}(S)$ is the group of orientation-preserving homeomorphisms up to homotopy and for graphs, $\mathrm{Map}(\Gamma)$ is the group of proper homotopy equivalences up to proper homotopy.

These definitions are not dependent on the surfaces or graphs being infinite-type. When S and Γ are of finite-type, $\mathrm{Map}(S)$ and $\mathrm{Map}(\Gamma)$ are exactly the standard mapping class group of a finite-type surface and the outer automorphism group of a free group, $\mathrm{Out}(F_n)$, respectively. These are two groups that have a rich dictionary between them. In the infinite-type setting, $\mathrm{Map}(\Gamma)$, was defined by Algom-Kfir–Bestvina [1] as a "big" analogue of the mapping class group of an infinite-type surface.

I am interested in studying the properties of these groups and how they relate to each other as well as to their finite-type counterparts. In the surface setting, I have studied abelianizations of these big mapping class groups [2]. In the graph setting, together with Hannah Hoganson and Sanghoon Kwak [3], we have studied the coarse geometry of these groups using the framework developed by Rosendal [4].

[1] Yael Algom-Kfir and Mladen Bestvina. Groups of proper homotopy equivalences of graphs and Nielsen realization. *arXiv preprint arXiv:2109.06908*, 2021.

[2] George Domat. Big pure mapping class groups are never perfect. *arXiv preprint arXiv:2007.14929*, 2020. Appendix with Ryan Dickmann, To appear in *Mathematical Research Letters*.

[3] George Domat, Hannah Hoganson, and Sanghoon Kwak. Coarse geometry of pure mapping class groups of infinite graphs. *arXiv preprint arXiv:2201.02559*, 2022. To appear in *Advances in Mathematics*.

[4] Christian Rosendal. *Coarse geometry of topological groups*, volume 223 of *Cambridge Tracts in Mathematics*. Cambridge University Press, Cambridge, 2022.

Sami Douba

Institut des Hautes Études Scientifiques, France

Linear representations of finitely generated infinite groups

I am interested in representations of finitely generated infinite groups and their connections to various fields such as geometry, topology, dynamics, and commutative algebra. Among the types of questions that inspire me are: Given a finitely generated group Γ arising in topology (for instance, as the fundamental group of some compact manifold M), is there an embedding of Γ in a Lie group G that reflects the topological origin of Γ (for instance, an embedding that is the holonomy of some (G, X)-structure on M, where X is a homogeneous space of G)? How much do the linear representations of a given finitely generated group Γ tell us about the group's intrinsic geometry? For example, the group Γ might contain an infinite-order element γ generating an undistorted cyclic subgroup; is this witnessed by some matrix representation ρ of Γ with the property that $\rho(\gamma)$ possesses an eigenvalue of infinite order? Say we are given a finitely generated matrix group Γ; can we elucidate its structure by manipulating Γ, for instance by deforming the generators or Galois-conjugating the entries? If Γ lies in some Lie group G, can we find another embedding $\rho : \Gamma \to G$ that is fundamentally different from the inclusion? For instance, if Γ is discrete, we could ask for ρ to be dense; we could instead stipulate that ρ also be discrete but not continuously deformable to the inclusion; or, if Γ preserves some geometric feature of a homogeneous space of G, we could require that $\rho(\Gamma)$ lack this property. By attaching geometric or dynamical significance to a linear representation ρ of Γ, can we deduce properties of ρ (for instance, that ρ is faithful, or that ρ satisfies the more delicate property that it maps no nontrivial element of Γ to a unipotent, or to a matrix with zero top-right entry) that would be difficult to detect algebraically?

Becky Eastham

University of Wisconsin-Madison, United States

Free and one-relator groups

One of my projects concerns which curves generate the homology of a finite cover of a (possibly punctured) surface. Farb & Hensel asked in [1] if the homology of a finite cover of a punctured surface is always generated by components of lifts of primitive elements. They show that, if the deck group of a regular cover Γ of a punctured surface is abelian or two-step nilpotent, $H_1^{prim}(\Gamma;\mathbb{Q}) = H_1(\Gamma;\mathbb{Q})$. The answer to the general question is no, however: some counterexamples of low index can be found in [3], and in [2] the authors show that, for any union U of finitely many $Aut(F_n)$-orbits, there are covers of punctured surfaces such that the homology generated by lifts of elements in U are not sufficient to span the homology of the cover. In particular, there are covers of punctured surfaces such that the homology generated by lifts of simple closed curves is insufficient to generate the homology of the cover. Autumn Kent has asked whether the homology generated by components of lifts of nonfilling curves is sufficient to span the homology of the cover. I'm trying to answer a weaker question: is the homology of every subgroup of a free group F_n generated by components of lifts of elements in a proper free factor of F_n?

I'm also interested in one-relator groups. A question I've been trying to answer is: is there an infinite chain of proper marked surjections of one-relator groups? A *marked* surjection is one from a group presentation with an ordered basis to another group presentation with an ordered basis such that the map respects the ordering. Baumslag and Solitar showed in [4] that $BS(2,3) \cong \langle a, b \mid ab^2a^{-1}b^{-3} \rangle$ has an infinite chain of marked surjections (which are isomorphisms onto the image) given by $a \mapsto a, b \mapsto b^2$, but the presentations are two-related after the first iteration.

[1] "Finite covers of graphs, their primitive homology, and representation theory," Benson Farb and Sebastian Hensel, arXiv:1610.08819, 2016.

[2] "Simple closed curves, finite covers of surfaces, and power subgroups of Out(Fn)," Justin Malestein and Andrew Putman, arXiv:1708.06486, 2017.

[3] "Graph coverings and (im)primitive homology: some new examples of exceptionally low degree," Destine Lee, Iris Rosenblum-Sellers, Jakwanul Safin, and Anda Tenie, arXiv:2008.13714, 2020.

[4] "Some two-generator one-relator non-Hopfian groups," Gilbert Baumslag and Donald Solitar, Bulletin of the American Mathematical Society, 1962, volume 68, pages 199-201.

Alexandra Edletzberger

University of Vienna, Austria

Quasi-Isometry Problem of Right-Angled Coxeter and Artin groups

---

I am a third year PhD student under the supervision of Dr. Christopher Cashen. I am interested in the Quasi-Isometry (QI) Problem of finitely generated groups admitting a certain splitting as a graph of groups, the so-called JSJ decomposition. This decomposition provides a QI-invariant.

In my first article [3], I give a description of this QI-invariant for a certain class of (in particular non-hyperbolic) Right-Angled Coxeter Groups (RACG) in terms of the defining graph. This generalizes work of Dani-Thomas [2] for the hyperbolic setting. Under some additional assumption, the QI-invariant even gives a complete solution to the QI-problem.

The analogous QI-invariant for Right-Angled Artin groups (RAAGs) was developped by Margolis in [4] and can be used to compare RAAGs with RACGs. I am curious whether this interplay can help to extend the work of Dani-Levcovitz [1] determining when certain RACGs graphs are commensurable and hence QI to a RAAG.

[1] Pallavi Dani and Ivan Levcovitz, *Right-angled Artin subgroups of right-angled Coxeter and Artin groups*, arXiv preprint arXiv:2003.05531 (2020).

[2] Pallavi Dani and Anne Thomas, *Bowditch's JSJ tree and the quasiisometry classification of certain Coxeter groups*, J. Topol. 10 (2017), no. 4, 1066–1106.

[3] Alexandra Edletzberger *Quasi-Isometries for certain Right-Angled Coxeter groups*, arXiv preprint arXiv:2112.10463 (2021).

[4] Alex Margolis, *Quasi-isometry classification of right-angled Artin groups that split over cyclic subgroups*, Groups, Geometry, and Dynamics 14 (2020), no. 4, 1351–1417.

Amandine Escalier

WWU Münster, Germany

Local-to-Global Rigidity and Buildings; Measure and Orbit Equivalence

My research is at the interface of geometric and measured group theories. On the one hand I study groups and graphs at a *local* scale through their microscopic structure, on the other hand I'm interested in their large scale behaviour, that is to say their *asymptotic* geometry.

Concerning the local point of view, the idea is to determine **to what extent the local geometry of a graph determines its global structure.** Formally, a vertex-transitive graph $\mathcal{G}$ is called Local-to-Global rigid if there exists $R > 0$ such that every other graph whose balls of radius R are isometric to the balls of radius R in $\mathcal{G}$ is covered by $\mathcal{G}$. Examples include trees, the usual Cayley graph of $\mathbb{Z}^m$ [1] or the Bruhat-Tits building of $SL_n(\mathbb{Q}_p)$ for $n \neq 3$ [6] . Relying on this last example I showed that torsion-free lattices in $SL_n(\mathbb{Q}_p)$ are LG-rigid [3]. I am now interested in the case of triangular buildings ($n = 3$) and some other graphs such as horospherical products.

From the asymptotic point of view, I study groups through the prism of orbit equivalence. The idea here is to **compare and quantify how close two actions of two groups are.** Formally we say that two groups are orbit equivalent (OE) if there exists a probability space on which they both act freely, measure preservingly and with the same orbits. Ornstein and Weiss showed that all infinite amenable groups are OE to $\mathbb{Z}$. To refine this notion and distinguish amenable groups, Delabie et al. [2] hence introduced a quantified version of OE. I'm interested in building such OE with *prescribed* quantification [4,5] and extend these results into the non-amenable world, for example to right-angled Arting groups.

[1] I. Benjamini and D. Ellis, On the structure of graphs which are locally indistinguishable from a lattice, *Forum of Mathematics*, Sigma 4 (2016).

[2] Delabie et al. *Quantitative measure equivalence between amenable groups*, February 2020, arXiv:2002.00719 [math.GR]

[3] A. Escalier, Local-to-Global-rigidity of lattices in $SL_n(\mathbb{K})$. *Annales de l'Institut Fourier*, Online first, 39 p. arxiv:1512.02775 [math.GR]

[4] A. Escalier. *Building prescribed quantitative orbit equivalence with the group of integers*, February 2022. arXiv:2202.10312 [math.GR].

[5] A. Escalier. *Sofic approximations and optimal quantitative orbit equivalence*, May 2022. arxiv:2205.12137 [math.GR].

[6] M. de la Salle and R. Tessera. Local-to-global rigidity of Bruhat–Tits buildings. *Illinois J. Math.* 60 (2016), no. 3-4, 641–654.

Alex Evetts

University of Manchester, UK

Growth and formal languages

My research interests fall into two broad categories, which intersect non-trivially.

Growth

The *growth function* of a finitely generated group is the function $\gamma(n)$ which counts the number of elements contained in the ball of radius n in the Cayley graph. Up to course bi-lipshitz equivalence, $\gamma(n)$ is a quasi-isometry invariant. Gromov famously characterised the groups for which $\gamma(n)$ is a polynomial as the virtually nilpotent groups. There are many variants of the growth function, including the *conjugacy growth function* $c(n)$, which counts the conjugacy classes intersecting the ball of radius n. In contrast to standard growth, conjugacy growth fails even to be a commensurability invariant, although the known counter-examples are not finitely presented. In recent work [3], I derive asymptotic estimates for the conjugacy growth of certain class 2 nilpotent groups, and conjecture that within the class of virtually nilpotent groups, $c(n)$ is in fact a quasi-isometry invariant. I am also very interested in the formal power series associated to various growth functions, and exploring when these are rational, algebraic, or holonomic.

Formal Languages

Consider fixing a normal form $\eta\colon G \to S^*$ for a group G (generated by S), and study the formal language properties of subsets of $\eta(G)$ or $\eta(G)^k$. For example, we represent the 'multiplication table' of G as the formal language $\{x\#y\#z\colon xy =_G z,\ x, y, z \in \eta(G)\}$. More generally we can study G's *algebraic sets* (solutions sets to equations) or *definable sets* (in the sense of first order logic). There is a growing body of work showing that many of these sets can be described as so-called EDT0L languages (for example [1], [4], [2]). A topic that I am interested in exploring is what algebraic or geometric properties of a group might be related to the formal language properties of the above sets.

[1] L. Ciobanu, V. Diekert and M. Elder, Solution sets for equations over free groups are EDT0L languages, *Internat. J. Algebra Comput.* 26 (2016), no. 5, 843-886.

[2] L. Ciobanu and A. Evetts, Rational sets in virtually abelian groups: languages and growth, arXiv:2205.05621

[3] A. Evetts, Conjugacy growth in the higher Heisenberg groups, *Glasg. Math. J. (to appear)*, arXiv:2111.06443

[4] A.Evetts and A. Levine, Equations in virtually abelian groups: languages and growth, *Internat. J. Algebra Comput.* 32 (2022), no. 3, 411-422.

Tom Ferragut
University of Montpellier, France

Geometry and quasi-isometry classification of horospherical products

---

Horospherical products are a family of metric spaces containing both continuous and discrete examples, from solvable Lie groups $\mathbb{R} \ltimes (N_1 \times N_2)$ to Cayley graphs of lamplighter groups. I first got interested in the coarse geometry of these horospherical products, through a description of their geodesics and visual boundary.
I am currently working on the classification up to quasi-isometry of families of solvable Lie groups, using a geometric rigidity property of their quasi-isometries.

Geometry of horospherical products
Let (X, d_X) and (Y, d_Y) be two Gromov hyperbolic spaces with Busemann functions $h_1 : X \to \mathbb{R}$ and $h_2 : Y \to \mathbb{R}$. The horospherical product $X \bowtie Y$ is defined to be the subset of $X \times Y$ consisting of pairs (x, y) satisfying $h_1(x) + h_2(y) = 0$. Examples are:

- $\mathbb{T}_n \bowtie \mathbb{T}_n = \mathrm{Cay}\,(\mathbb{Z}_{n-1} \wr \mathbb{Z})$
- $\mathbb{T}_n \bowtie \mathbb{H}^2$ is the 2-Cayley complex of $\mathrm{BS}(1, n-1)$.
- $(\mathbb{R} \ltimes_{A_1} N_1) \bowtie (\mathbb{R} \ltimes_{A_2} N_2) = \mathbb{R} \ltimes_{\mathrm{Diag}(A_1, -A_2)} (N_1 \times N_2)$ (hence $\mathbb{H}^2 \bowtie \mathbb{H}^2 = \mathrm{Sol}$)

Where N_1, N_2 are two simply connected nilpotent Lie groups, and where A_1, A_2 are two matrices whose eigenvalues have positive real parts.
We show in [2] that a wide family of distances on $X \bowtie Y$ only differ from a constant. We also give a description of geodesics in $X \bowtie Y$, they are essentially constructed from two consecutive geodesics of X and Y.

Quasi-isometry classification
Eskin, Fisher and Whyte proved in [1] that for $X, Y = \mathbb{T}_n$ or $\mathbb{H}^2$, any quasi-isometry between two horospherical products is close to a map that splits on the X- and Y-coordinates. During my PhD, I generalised this result to a family of horospherical products, including any simply connected, negatively curved Lie group for X and Y.
For example, it implies that if $\mathbb{R} \ltimes_{\mathrm{Diag}(A_1, -A_2)} (N_1 \times N_2)$ and $\mathbb{R} \ltimes_{\mathrm{Diag}(A_1', -A_2')} (N_1' \times N_2')$ are quasi-isometric, then $\frac{tr(A_1)}{tr(A_2)} = \frac{tr(A_1')}{tr(A_2')}$.

[1] A. ESKIN, D. FISHER, K. WHYTE, *Coarse differentiation of quasi-isométries I: Spaces not quasi-isometric to Cayley graphs*. Annals of Mathematics Volume 176 (2012), 221-260.

[2] T. FERRAGUT, *Geodesics and Visual boundary of Horospherical Products*. (arXiv:2009.04698) (2020)

Sam Fisher
University of Oxford, United Kingdom

Algebraic fibring and agrarian invariants

A group G is *algebraically fibred* if there is an epimorphism $G \to \mathbb{Z}$ with finitely generated kernel. The definition is motivated by a theorem of Stallings, which states that if $G = \pi_1(M)$ is the fundamental group of a closed 3-manifold M and there is an algebraic fibration $\varphi\colon G \to \mathbb{Z}$, then φ is induced by a topological fibration $f\colon M \to S^1$ [1]. In other words, M is a surface bundle over the circle and φ is induced by the projection.

In 2020, Kielak showed that algebraic fibring is (virtually) equivalent to the vanishing of the first ℓ^2-Betti number. More precisely, he proved that if G is a finitely generated virtually RFRS group, then G virtually algebraically fibres (i.e. there is a subgroup $H \leqslant G$ of finite index that algebraically fibres) if and only if $\beta_1^{(2)}(G) = 0$ [2]. This generalises and gives an algebraic proof of a celebrated result of Agol [3], which was a key step in proving Thurston's virtually fibred conjecture.

To prove his algebraic fibring theorem, Kielak uses the fact that the pth ℓ^2-Betti number of a RFRS group G (more generally, of a group satisfying the *Atiyah Conjecture*) is the vector space dimension of the group homology $H_p(G; \mathcal{D}(G))$. Here, $\mathcal{D}(G)$ is the *Linnell skew-field* of G, a skew-field with a ring epimorphism $\mathbb{Q}G \hookrightarrow \mathcal{D}(G)$. Thus, ℓ^2-Betti numbers of RFRS groups are examples of *agrarian invariants*.

If R is a ring, G is a group, and $\mathcal{D}$ is a skew-field, then a ring homomorphism $RG \to \mathcal{D}$ is call an *agrarian map*. This turns $\mathcal{D}$ into an RG-module and we can define the homology modules $\operatorname{Tor}_p^{RG}(R, \mathcal{D})$. The *agrarian Betti numbers* or *$\mathcal{D}$-Betti numbers* $\beta_p^{\mathcal{D}}(G)$ are then the $\mathcal{D}$-vector space dimensions of these homology modules. If $\mathbb{F}$ is a skew-field and G is a RFRS group, then Jaikin-Zapirain showed that $\mathbb{F}G$ embeds into a 'nice' skew-field $\mathcal{D}_{\mathbb{F}G}$ [4]. In the case $\mathbb{F} = \mathbb{Q}$, the skew-fields $\mathcal{D}_{\mathbb{F}G}$ and $\mathcal{D}(G)$ coincide. I am interested in these generalisations of ℓ^2-Betti numbers, their connections to algebraic fibring, and in investigating to what extent they can provide natural positive characteristic analogues of usual ℓ^2-invariants.

[1] John Stallings. On fibering certain 3-manifolds. In *Topology of 3-manifolds and related topics*, pages 95-100. Prentice-Hall, Englewood Cliffs, N.J., 1962.

[2] Dawid Kielak. Residually finite rationally solvable groups and virtual fibring. *J. Amer. Math. Soc.*, 33(2):451-486, 2020

[3] Ian Agol. Criteria for virtual fibering. *J. Topol.*, 1(2):269-284, 2008.

[4] Andrei Jaikin-Zapirain. The universality of Hughes-free division rings. *Selecta Math. (N.S.)*, 27(4):Paper No. 74, 33, 2021

Jonas Flechsig

Bielefeld University, Germany

Braid groups and mapping class groups

I am interested in braid groups and mapping class groups. These groups are intimately related by the evaluation map. For Artin's braid groups the evaluation map induces an isomorphism from the mapping class group of a disk with n punctures $\mathrm{Map}(D_n)$ to the n-stranded braid group B_n. This approach to braid groups, yields a lot of tools for the study of braid groups.

In my ongoing PhD project I study braid and mapping class groups over orbifolds Σ_Γ with finitely many orbits of cone points. These orbifold braid groups are of interest since some of them are related to certain Artin groups, see [1]. For the orbifolds Σ_Γ I observed: While the mapping class groups are torsion-free, the braid groups have elements of finite order. In particular, the evaluation map from the mapping class group $\mathrm{Map}(\Sigma_\Gamma^n)$ to the braid group $\mathrm{B}_n(\Sigma_\Gamma)$ is an epi- but not an isomorphism in this case. In my PhD project I study the implications of this difference on the structure of orbifold braid groups.

[1] Daniel Allcock. Braid pictures for Artin groups. Trans. Amer. Math. Soc., 354(9):3455-3474, 2002.

Francesco Fournier-Facio
ETH Zürich, Switzerland

When are wreath products Hopfian?

I am a third-year PhD student at ETH Zürich, in Switzerland. So far my research has focused on bounded cohomology and stability of metric approximations of groups. More recently, I have started getting closer to geometric and combinatorial group theory.

One project along these lines is joint work with Henry Bradford. The (standard, restricted) wreath product is a very natural construction, that given two finitely generated groups Δ, Γ, outputs a third finitely generated group $\Delta \wr \Gamma$. This is a great construction because on the one hand it is very visual and amenable to explicit computations, and on the other hand it produces very interesting examples. So it is a natural question in combinatorial group theory to ask which properties of Δ and Γ pass to $\Delta \wr \Gamma$.

Along these lines, Gruenberg proved in [2] that $\Delta \wr \Gamma$ is residually finite if and only if Δ and Γ are residually finite, and either Γ is finite or Δ is abelian. A related property is the following: we say that a group is *Hopfian* if every self-epimorphism is an isomorphism. The relation comes from the fact, due to Malcev, that finitely generated residually finite groups are Hopfian [3]. Could there be a characterization for Hopficity of wreath products as nice as the one of Gruenberg for residual finiteness? This question was asked in Henry's lightning talk at the last edition of YGGT.

Shortly before the start of this edition, we now know that the problem is infinitely harder. Restricting to the special case in which Δ is finitely generated abelian, we already have an equivalence with *Kaplansky's stable finiteness conjecture*: a longstanding open problem about algebraic properties of group rings. The conjecture is satisfied by huge classes of groups, for instance all sofic groups, all surjunctive groups, and all torsion-free groups that satisfy the Kaplansky's zero divisor or idempotent conjectures.

[1] H. Bradford, F. Fournier-Facio. Hopfian wreath products and the stable finiteness conjecture. *arXiv preprint arXiv:2211.01510*, 2022.

[2] K. W. Gruenberg. Residual properties of infinite soluble groups. *Proceedings of the London Mathematical Society 7:29-62*, 1957.

[3] A. I. Malcev. On isomorphic matrix representations of infinite groups. *Matematicheskii Sbornik 50.3:405-422*, 1940.

Jonathan Fruchter
University of Oxford, United Kingdom

Surface subgroups made to order and profinite rigidity

A famous conjecture of Gromov's, inspired by the Virtual Haken Conjecture, states that every one-ended word-hyperbolic group contains a (quasiconvex) surface subgroup; this is known as the *Surface Subgroup Conjecture*. A prominent class of groups for which the surface subgroup conjecture has been settled by Wilton [2] is the class of hyperbolic fundamental groups of graphs of free groups amalgamated along cyclic subgroups. Central to the resolution of the surface subgroup conjecture in this case is a deep theorem of Calegari [1, Rationality Theorem], stating that the *stable commutator length* function on a free group (which is closely related to maps of surfaces with boundary into graphs) always takes rational values. The proof relies on solving a *linear programming* problem: achieving the best outcome of a carefully-crafted target function under linear constraints. However, only the existence of such a solution (and hence a surface subgroup) is guaranteed, and the surface subgroup it produces remains a mystery.

My research hopes to unravel some of this mystery by introducing improvements on Wilton's techniques, which will produce an array of surface subgroups "made to order". More specifically, if G is a hyperbolic group that splits as a graph of free groups with cyclic edge groups, and g lies in a vertex group of G, then there is a surface subgroup $\pi_1\Sigma \leq G$ containing g^n for some $n \in \mathbb{Z}$. The prime potency of this result lies in its applications to the study of profinite rigidity, specifically within the class of *limit groups*: I aim to use these specialized surface subgroups for detecting splittings of limit groups from their finite quotients. This will also give rise to a rich family of groups that are profinitely rigid within the class of limit groups.

I am also interested in connections between geometric group theory and mathematical logic, and have worked on problems related to the first-order theory of acylindrically hyperbolic groups and right-angled Artin groups.

[1] Danny Calegari, *scl*, MSJ Memoirs, vol. 20, Mathematical Society of Japan, Tokyo, 2009.

[2] Henry Wilton, *Essential surfaces in graph pairs*, Journal of the American Mathematical Society, **31** (2018), no. 4, 893–919.

Jacob Garcia

University of California, Riverside, United States of America

---

Relationships between Conical Limit Points and Stable Subgroups

---

A well known fact about hyperbolic geodesic spaces is the Morse Lemma: given a geodesic $[x, y]$ and a quasi-geodesic ϕ whose endpoints lie on $[x, y]$, ϕ stays within a bounded neighborhood of $[x, y]$ where the bound depends only on the quality of the quasi-geodesic and the hyperbolicity constant. This lemma was then turned into a definition by Matthew Cordes so it can be applied in more settings. Given a proper geodesic metric space X and a geodesic γ, we call γ an N*-Morse geodesic* if, for any (K, C)-quasi-geodesic ϕ whose endpoints lie on γ, we have that ϕ lies in the $N(K, C)$ neighborhood of γ, where $N : \mathbb{R} \times \mathbb{R} \to \mathbb{R}^+$ is a function called the *Morse gauge* of γ. Fixing a base point in X, we can then define the Morse strata $X_e^{(N)}$: the set of all points $x \in X$ so that $[e, x]$ is N-Morse.

The collection of these Morse strata have some remarkable properties, notably, the collection of all Morse strata of a given base point forms a cover of the space X, and each $X_e^{(N)}$ is hyperbolic. Using these ideas, we can construct an analog to the visual boundary of a hyperbolic space for X, called the Morse boundary. The study of Morse geodesic rays and the Morse boundary have been important tools for studying wide classes of spaces, such as mapping class groups and CAT(0) spaces. The Morse strata, in some sense, “sees the hyperbolic directions in the space."

Studying the Morse boundary has been the focus of my research, where I am studying under Matthew Gentry Durham as a fifth year PhD student. I am currently working on generalizing the classifications of quasi-convex isometry groups into the Morse setting. In particular, I am working on the connections between conical limit points in the Morse boundary of a group and its stable subgroups.

Jerónimo García Mejía

Karlsruhe Insitute of Technology (KIT), Germany

Asymptotic geometry of mapping tori of non-positively curved groups

Under the supervision of Claudio Llosa Isenrich I have been working on the asymptotic geometry of *mapping tori of groups* with a particular focus on non-positively curved (NPC) groups. For a finitely presented group $G = \langle S \mid R \rangle$ and an automorphims $\phi\colon G \to G$, the *mapping torus of* ϕ is the group $M_\phi = G \rtimes_\phi \mathbb{Z}$ which admits the following finite presentation

$$M_\phi = \langle S, t \mid R,\ t^{-1}xt = \phi(x),\ \text{for each } x \in S \rangle.$$

An interesting class of NPC groups that has been at the center of my attention is the one of *right-angled Artin groups (RAAGs)*: given a finite simplicial graph Γ one associates to it a group A_Γ with one generator for each vertex $v \in V(\Gamma)$ and a relation $[v, w]$ for each edge $\{v, w\} \in E(\Gamma)$. They naturally interpolate between free and free-abelian groups.

During the first part of my thesis, my focus has been on the complexity of the word problem for the groups $A_\Gamma \rtimes_\phi \mathbb{Z}$ as measured by their *Dehn functions*. Geometrically, the Dehn function of a group $G = \pi_1(M)$ quantifies simple connectivity of the universal cover $\tilde{M}$ of a closed Riemannian manifold: every closed loop in $\tilde{M}$ bounds a disc, the Dehn function measures the area of this filling disc.

Special cases in which the full classification of Dehn functions of $A_\Gamma \rtimes_\phi \mathbb{Z}$ is known are: when A_Γ is a free group (see Bridson and Groves [1]) and when A_Γ is a free-abelian group (see Bridson, Gersten, and Pitett [1,2]). Aside from this very little is known. The most recent progress was achieved by Pueschel and Riley [3], who fully classified the Dehn functions for *(i)* A_Γ a direct products of two free groups and *(ii)* the remaining cases of $|V(\Gamma)| \leq 3$.

As shown by [1–3] there is a close relation between the Dehn function of $A_\Gamma \rtimes_\phi \mathbb{Z}$ and the growth formula of the defining automorphism ϕ: $gr_{\phi,V(\Gamma)}(n) = \max\{|\phi^n(v)| : v \in V(\Gamma)\}$. Motivated by this I have recently become interested in understanding the growth of automorphisms of RAAGs.

[1] The quadratic isoperimetric inequality for mapping tori of free group automorphisms, M. Bridson and D. Groves, American Mathematical Soc., 2010.

[2] Isoperimetric inequalities for the fundamental groups of torus bundles over the circle, M. Bridon and Ch. Pittet, Geometriae Dedicata, 1994.

[3] The optimal isoperimetric inequality for torus bundles over the circle, m. Bridson and S. Gersten, The Quarterly Journal of Mathematics. Oxford. Second Series, 1996.

[4] Dehn functions of mapping tori of right-angled Artin groups, K. Pueschel and T. Riley, T., arXiv:1906.09368, 2019.

Mikel Eguzki Garciarena Perez
University of Salerno, Italy and University of the Basque Country, Spain

Groups of automorphisms of regular rooted trees

Groups of automorphisms of regular rooted trees are a rich source of examples with interesting properties in group theory, and they have been used to solve very important problems. The first Grigorchuk group, defined by Grigorchuk [3] in 1980, is one of the first instances of an infinite finitely generated periodic group, thus providing a negative solution to the General Burnside Problem. It is also the first example of a group with intermediate growth [4], hence solving the Milnor Problem. Many other groups of automorphisms of rooted trees have since been defined and studied. Important examples are the Gupta-Sidki p-groups [5], for p a prime, and the second Grigorchuk group [3]. These are again finitely generated infinite periodic groups and they belong to the large family of the so-called Grigorchuk-Gupta-Sidki groups (GGS-groups, for short).

At the moment I am working on the lower central series of the GGS-groups. In other words trying to understand the terms $\gamma_i(G)$ of the lower central series of the GGS-group G. The terms $\gamma_i(G)$ are examples of verbal subgroups, i.e. subgroups generated by the values of a word in a group. More precisely, $\gamma_i(G)$ is generated by the values in G of the commutator $[x_1, \ldots, x_i]$ of length i. A nice introduction to this topic can be found in [2].

I am also working on some of the properties of another group called Brunner-Sidki-Viera group. The properties I am studying are between others: Hausdorff dimension, p-congruence subgroup property, indices of their maximal subgroups. More abut the topic can be found in [1].

[1] A.M. Brunner, S. Sidki, A.C Vieira, A just-nonsolvable torsion-free group defined on the binary tree.

[2] G. A. Fernández-Alcober and A. Zugadi-Reizabal, GGS-groups: order of congruence quotients and Hausdorff dimension, Trans. Amer. Math. Soc. 366 (2014), no. 4, 1993–2017.

[3] R. I. Grigorchuk, On Burnside's problem on periodic groups, Funktsional. Anal. i Prilozhen. 14 (1980), no. 1, 53–54.

[4] ——, On the Milnor problem of group growth, Dokl. Akad. Nauk SSSR 271 (1983), no. 1, 30–33.

[5] N. Gupta and S. Sidki, On the Burnside problem for periodic groups, Math. Z. 182 (1983), no. 3, 385–388.

Giles Gardam
University of Münster, Germany

GGT, group rings, and computation

The current focus of my research is the series of conjectures about group rings commonly attributed to Kaplansky. For a group G and field K, the group ring $K[G]$ is the K-vector space on basis G with multiplication extending the group product linearly. An illustrative example: if $G = \mathbb{Z} = \langle t \rangle$, we just get the ring of Laurent polynomials in t. An element of the form kg for $k \in K \setminus \{0\}$ and $g \in G$ is called a trivial unit (its multiplicative inverse being $k^{-1}g^{-1}$.)

The Kaplansky conjectures. If G is torsion-free, then the group ring $K[G]$ has

- no non-trivial units
- no non-zero zero divisors
- no idempotents other than 0 and 1.

For any G, possibly with torsion, the group ring $K[G]$ is

- directly finite, i.e. every left-invertible element is right-invertible.

The unit conjecture is known to be false when K has positive characteristic [1] and the direct finiteness conjecture is known to be true when K has characteristic zero by an analytic argument of Kaplansky. Otherwise the conjectures are wide open but some are known to be true when the group has certain properties (sometimes assuming K to have characteristic zero), such as being elementary amenable, virtually special or sofic.

I'm generally interested in approaching questions from a computational perspective. Some other things I like to think about are one-relator groups, free-by-cyclic groups, and profinite groups.

[1] Giles Gardam. A counterexample to the unit conjecture for group rings. *Ann. of Math. (2)*, 194(3):967–979, 2021.

Maria Gerasimova

WWU Münster, Germany

Analysis and probability theory on infinite groups

My research interests lie somewhere between the worlds of functional analysis, geometric group theory and probability theory. In particular, I am interested in probabilistic properties of infinite groups and in harmonic functions with certain properties on infinite groups. I am also interested in stability questions (i.e stability in permutations and Hilbert-Schmidt stability). Another topic of my research are unitary (or uniformly bounded) representations of discrete and locally compact groups and various connections and applications to operator algebras.

[1] Gerasimova M., Gruber D., Monod N., Thom A. (2020). Asymptotics of Cheeger constants and unitarisability of groups. Journal of Functional Analysis, 278(11), 108457.

[2] Gerasimova M., Osin D. (2020). On invertible elements in reduced C^*-algebras of acylindrically hyperbolic groups. Journal of Functional Analysis, 279(7), 108689.

[3] Gerasimova M., Shchepin K. (2021). Virtually free groups are p-Schatten stable. arXiv preprint arXiv:2107.10032.

[4] Amir G., Gerasimova M., Kozma G., Harmonic functions with gradient going to zero (in preparation)

Antoine Goldsborough

Heriot-Watt University, Scotland

Random walks and quasi-isometries

I am in my third year of PhD studies under the supervision of Alessandro Sisto. My research is on random walks on hyperbolic-like groups and how these intereact with quasi-isometries.

The study of random walks on finitely generated groups can be traced back to the seminal work of Kesten in the 1950s [1]. In recent years, there has been a significant amount of work done looking at random walks on hyperbolic groups (see for example [2]) and more generally on acylindrically hyperbolic groups ([3], [4]).

As geometric group theory is devoted to the study of "large-scale" geometry of groups and spaces, a natural question to ask is how do random walks interact with quasi-isometries. Namely, if we have a quasi-isometry $f : G \to H$ where we know the group H, are there any results we can infer about a random walk on G ?

In general, for a random walk on G, there is no corresponding random walk on H that one can study. This issue can be resolved by studying the more general framework of Markov chains as a "quasi-isometry invariant" theory. Looking at these Markov chains allows us to get some interesting results on their behaviour in H and whence in the group G. As an application, we get a Central Limit Theorem of the random walk in G, under some geometric assumptions on the group H.

[1] Harry Kesten, *Symmetric random walks on groups*, Trans. Amer. Math. Soc. **92** (1959), 336–354, DOI 10.2307/1993160. MR109367

[2] Sébastien Gouëzel, *Local limit theorem for symmetric random walks in Gromov-hyperbolic groups*, J. Amer. Math. Soc. **27** (2014), no. 3, 893–928, DOI 10.1090/S0894-0347-2014-00788-8. MR3194496

[3] P. Mathieu and A. Sisto, *Deviation inequalities for random walks*, Duke Math. J. **169** (2020), no. 5, 961–1036, DOI 10.1215/00127094-2019-0067. MR4079419

[4] Joseph Maher and Giulio Tiozzo, *Random walks on weakly hyperbolic groups*, J. Reine Angew. Math. **742** (2018), 187–239, DOI 10.1515/crelle-2015-0076. MR3849626

Laura Grave de Peralta

UC Louvain, Belgium

Combinatorics, groups and more

During my PhD, I studied diverse topics. The general idea was always to study graphs associated to algebraic, geometric or concrete objects. Among the objects I studied :

- combinatorial maps and their generalizations. We proved that such combinatorial objects can have any finite group of automorphisms, and we describe an algorithm to construct such a combinatorial structure with given symmetry group
- combinatorial methods for enumerating subgroups of groups of the form $\Delta_q := \mathbb{Z}_2 * \mathbb{Z}_q$ for $q \geq 3$ prime by studying the Stallings graphs
- objects closely related to Teichmüller space, in particular a combinatorial model that focuses on the so-called *wide part of the moduli space*
- graphs as a model for trophic interactions to try to detect vulnerability in ecological networks modelling microbiomes.

Now for the post-doc, I am working on a project aiming at studying *high-dimensional expanders* and understand their connection with Kac-Moody Steinberg algebras and/or groups.

[1] Bottinelli, R., Grave de Peralta, L., Kolpakov, A.. 2020. Telescopic groups and symmetries of combinatorial maps. Algebraic Combinatorics, Volume 3 (2020) no. 2 p. 483-508.

[2] Grave de Peralta, L.,Mulot, M. Vulnerability detection in trophic networks, a community based approach. (To be submitted, soon!)

[3] Grave de Peralta, L., Kolpakov. A. Expansion properties of pants graphs of surfaces. (In preparation)

[4] Grave de Peralta, L., Subgroup enumeration and statistics of Hecke groups. (In preparation)

Doris Grothusmann

WWU Münster, Germany

The automorphism group of right-angled buildings

Currently, I am working on my master thesis with Prof. Tent. The goal is to give a model theoretic proof, that the automorphism group of right-angled buildings with countable valencies is simple. For finite valencies the result was obtained by Caprace in [1] and De Medts et. al. showed it in [2] for any valencies. However, I will use a different modeltheoretic approach. Tent and Ziegler introduced in [4] stationary independence to prove the simplicity of the automorphism groups of certain countable structures. I am going to apply this to the right-angled buildings as prime models of the free pseudospaces of dimension n as introduced in [3].

After my master thesis, I am considering entering a PhD program in GGT. Therefore, I appreciate connecting with people working in GGT and learning about their research. Especially different approaches to GGT spark my interest, for example from a more topological background.

[1] Caprace, Pierre-Emmanuel. "Automorphism groups of right-angled buildings: simplicity and local splittings." arXiv preprint arXiv:1210.7549 (2012).

[2] De Medts, Tom, Ana C. Silva, and Koen Struyve. "Universal groups for right-angled buildings." Groups, Geometry, and Dynamics 12.1 (2018): 231-287.

[3] Tent, Katrin. "The free pseudospace is n-ample, but not (n+ 1)-ample." The Journal of Symbolic Logic 79.2 (2014): 410-428.

[4] Tent, Katrin, and Martin Ziegler. "On the isometry group of the Urysohn space." Journal of the London Mathematical Society 87.1 (2013): 289-303.

Daniel Groves
University of Illinois at Chicago, United States

(Relatively) hyperbolic groups and CAT(0) cube complexes

I am interested in many things – hyperbolic groups, relatively hyperbolic groups, CAT(0) spaces, mapping class groups, limit groups, 3–manifold groups, etc.

Recently, I have done a lot of working on relatively hyperbolic groups acting on CAT(0) cube complexes, cocompactly but not properly. One result I am proud of is joint work with Jason Manning [3], in which we prove:

Theorem. *Suppose that G is a hyperbolic group acting cocompactly on a CAT(0) cube complex X so that cell stabilizers are quasi-convex and virtually special. Then G is virtually special.*

In papers with Teddy Einstein [1, 2] I have developed the theory of *relatively geometric* actions of relatively hyperbolic groups on CAT(0) cube complexes, and have proved analogues of Agol's Theorem and Haglund–Wise's Canonical Completion and Retraction in this setting. We are working on a relatively geometric version of Wise's Quasi-convex hierarchy theorem.

[1] E. Einstein and D. Groves, Relative cubulations and groups with a 2–sphere boundary, *Compositio Mathematica*, 156 (2020), 862-867.

[2] E. Einstein and D. Groves, Relatively geometric actions on CAT(0) cube complexes, *Journal of the London Mathematical Society*, 105 (2022), 691–708.

[3] D. Groves and J.F. Manning, Hyperbolic groups acting improperly, *Geometry and Topology*, to appear.

Sam Hughes

University of Oxford, UK

Lattices in non-positive curvature

I am interested in lattices (discrete groups with finite covolume) in isometry groups of $\mathrm{CAT}(0)$ spaces. I enjoy discovering exotic lattices with strange properties. For example I constructed the first example of a group acting geometrically on a product of (locally-finite) trees or right-angled buildings which is not virtually torsion-free [1]. I constructed the first examples of irreducible cocompact lattices which fibre over the circle [3]. In joint work with Motiejus Valiunas [4] we constructed a group acting geometrically on the product of a hyperbolic plane and a locally-finite tree which is a hierarchically hyperbolic group but is not biautomatic.

More recently I have become interested in the *Flat Closing Conjecture* and the *Rank Rigidity Conjecture* which predict deep structural information about $\mathrm{CAT}(0)$ groups. The first conjecture states that if a group Γ acts geometrically on $\mathrm{CAT}(0)$ space with an isometrically embedded copy of $\mathbb{E}^n$, then Γ contains a $\mathbb{Z}^n$ subgroup. The second conjecture roughly states that every sufficiently nice $\mathrm{CAT}(0)$ space of higher rank is a symmetric space, a Euclidean building, or splits as a direct product.

I am also interested in group cohomology (e.g. BNSR invariants, coherence, fibring, and ℓ^2-cohomology), hierarchical hyperbolicity (and related notions), K-theory with respect to the isomorphism conjectures, profinite rigidity, and quasi-isometries of pairs.

[1] Sam Hughes. Graphs and complexes of lattices, 2021, submitted. arXiv: 2104.13728 [Math.GR]

[2] Sam Hughes. Irreducible lattices fibring over the circle, 2022, submitted. arXiv: 2201.06525 [Math.GR]

[3] Sam Hughes and Motiejus Valiunas. Commensurating HNN extensions: hierarchical hyperbolicity and biautomaticity, 2022, submitted. arXiv: 2203.11996 [Math.GR]

[4] Sam Hughes. Lattices in a product of trees, hierarchically hyperbolic groups, and virtual torsion-freeness. *Bulletin of the London Mathematical Society* **54**(4), 1413–1419, 2022.

[5] Sam Hughes. A note on the rational homological dimension of lattices in positive characteristic, to appear in *Glasgow Mathematical Journal*.

Giovanni Italiano

Scuola Normale Superiore, Italy

Hyperbolic manifolds that fiber over the circle

I am a fourth year PhD student at Scuola Normale, and my advisor is Bruno Martelli. My main research topic is hyperbolic geometry, particularly constructing manifolds by glueing copies of hyperbolic polytopes.

During my PhD I worked with Bruno Martelli and Matteo Migliorini on the project of investigating the existence of hyperbolic manifolds of dimension $n > 3$ that are also a fibered bundle over S^1. This phenomenon is quite common for hyperbolic 3-manifolds (they all fiber up to a finite cover), but it is also strange, since the fiber is highly not compatible with the hyperbolic structure of the manifold. When the dimension is higher than 3, this condition is even stronger, since Mostow rigidity implies that the fiber cannot admit a hyperbolic metric at all. When the dimension is even, a simple Euler characteristic argument shows that it is impossible for such a fibration to occur, so the first dimension that is worth investigating is $n = 5$.

Adapting a very nice "combinatorial game" from [1] to a 5-dimensional right-angled hyperbolic polytope P_5, we managed to construct a cusped hyperbolic 5-manifold fibering over S^1. The existence of such a manifold answers a long standing open problem in geometric group theory: a finite type group which does not contain any Baumslag–Solitar subgroup $BS(m, n)$ is not necessarily hyperbolic.

[1] K. JANKIEWICZ – S. NORIN – D. T. WISE, *Virtually fibering right-angled Coxeter groups*, Journal of the Institute of Mathematics of Jussieu, 20(3), 957-987.

[2] G. ITALIANO – B. MARTELLI – M. MIGLIORINI, *Hyperbolic manifolds that fiber algebraically up to dimension 8*, accepted for publication in J. Inst. Math. Jussieu

[3] G. ITALIANO – B. MARTELLI – M. MIGLIORINI, *Hyperbolic 5-manifolds that fiber over S^1*, Invent. math. (2022)

Adele Jackson
University of Oxford, United Kingdom

Algorithms and 3-manifolds

A surprising number of algorithmic questions in 3-manifolds have not yet been resolved. For example, it is not known how to determine if a knot is ribbon. While the decidability of the homeomorphism problem for 3-manifolds was resolved by Perelman's proof of Thurston's geometrisation conjecture [1], it is unknown whether the problem is in NP. These sorts of questions have some interesting links here with complexity theory: if determining the genus of a knot is in NP, as a consequence NP and co-NP would be the same complexity class [2], which is widely thought to be false. Similarly, if recognising the unknot were NP-hard, then NP and co-NP would be the same.

As, for 3-manifolds, the piecewise linear category and the topological category are equivalent, the main techniques in this area are to study surfaces in triangulations of 3-manifolds. I am interested in understanding how these triangulations reflect the topology of the 3-manifold, and developing new combinatorial representations of topological data. For example, I worked with others to construct a new type of diagram for links in arbitrary compact orientable 3-manifolds. I have also been working on the size of minimal triangulations of Seifert fibered surfaces. I'm particularly interested in applying geometric group theory techniques to these types of questions, and seeing how 3-manifold techniques can perhaps be used in the group theory setting.

[1] Agol, I., Hass, J. and Thurston, W., 2006. The computational complexity of knot genus and spanning area. *Transactions of the American Mathematical Society*, 358(9), pp.3821–3850.

[2] Brand, J., Burton, B.A., Dancso, Z., He, A., Jackson, A. and Licata, J., 2022. Arc diagrams on 3-manifold spines. *arXiv preprint* arXiv:2202.02007.

[3] Kuperberg, G., 2019. Algorithmic homeomorphism of 3-manifolds as a corollary of geometrization. *Pacific Journal of Mathematics*, 301(1), pp.189–241.

Kasia Jankiewicz

University of California Santa Cruz, United States

Various topics in geometric group theory

Here are the main themes of my research:

- Artin groups. They generalize braid groups, and are closely related to Coxeter groups. I am interested in the existence and non-existence of actions of Artin groups on nonpositively curved spaces, their "NPC-like" features, and their profinite properties.
- Non-positive curvature. I study CAT(0) cube complexes, as well as more general CAT(0) spaces. For example, I am interested in obstructions to actions on such spaces, and rigidty of CAT(0) structures on groups.
- Algebraic fibering and coherence. A group algebraically fibers if it admits and epimorphism to $\mathbb{Z}$ with finitely generated kernel. A group is coherent if every finitely generated subgroup is finitely presented. I study those properties in terms of the geometry of a group.

Héctor Jardón-Sánchez

Universität Leipzig, Germany

Measured-combinatorial generalizations of group properties

My research lies in the broad area of measured group theory. A probability measure preserving (p.m.p.) group action induces an equivalence relation on the underlying space defined by being in the same orbit. I am interested in how group properties translate and generalize to the measured-combinatorial setting of equivalence relations.

In a current project with my PhD advisor Łukasz Grabowski (Leipzig) and Sam Mellick (McGill) we study property (T) of p.m.p. Borel equivalence relations. An example of such object is the equivalence relation induced by a p.m.p. action of a group with property (T). However, we show that examples not coming from group actions can be constructed using Poisson point processes. Once in the measured setting, we show that equivalence relations with property (T) have cost 1, generalizing results from [4].

An equivalence relation may be endowed with further combinatorial information by means of a graphing. A graphing is a graph defined on a standard probability space by measure-preserving Borel automorphisms. For instance, one may obtain such automorphisms from p.m.p. actions of groups. Via graphings, we realize the classes of an equivalence relation as connected components of a graph.

I am interested in those graphings whose connected components embed measurably in a topological manifold. In an on-going project I aim to show that a large class of graphings with planar connected components are treeable, extending results from [1] and [5]. Treeability implies that a graphing is sofic [2], realizes its cost [3], and does not have property (T). In the future, I would like to understand treeability, soficity and property (T) for graphings with connected components embedding measurably in $\mathbb{R}^n$.

Apart from the above, I am also working on other projects related to the Haagerup property, expansion properties of graphings and other equivalence relation invariants such as ℓ_2-Betti numbers.

[1] C. T. Conley, D. Gaboriau, A. S. Marks, and R. D. Tucker-Drob, *One-ended spanning subforests and treeability of groups*, 2021. Available in Arxiv: 2104.07431.

[2] G. Elek and G. Lippner, *Sofic equivalence relations*, Journal Functional Analysis **258** (2010), no. 5, 1692–1708.

[3] D. Gaboriau, *Coût des relations d'équivalence et des groupes*, Inventiones Mathematicae, **139** (2000), 41–98.

[4] T. Hutchcroft and G. Pete, *Kazhdan groups have cost 1*, Inventiones Mathematicae **221**, 873-–891 (2020).

[5] A. Timar, *Unimodular random one-ended planar graphs are sofic*,2022. Available in Arxiv: 1910.01307.

Oli Jones
Heriot-Watt University, United Kingdom

Fixed Point Subgroups in Artin Groups

I am a first year student of Laura Ciobanu.

For any group G and any $\phi \in \mathrm{Aut}(G)$, the set of fixed points of ϕ, $\{g \in G | \phi(g) = g\}$, form a subgroup of G. I am interested in understanding the fixed point subgroups of Artin groups, in particular dihedral Artin groups. This research will use tools from Bass-Serre theory and one-relator group theory.

An Artin group can be defined by a finite graph (V, E) with a labelling function $l : E \to \mathbb{N}$. Given such a graph the associated Artin group has a generator v for each $v \in V$. Each edge $e = \{v, w\}$ is associated to a relation $vwvw... = wvwv...$ where the length of the words on each side of the equality is $l(e)$. A dihedral Artin group is an Artin group defined by a graph with two vertices.

The paper [1] allows us to write explicit actions of the automorphism groups of dihedral Artin groups on trees, so the automorphism groups can be found using Bass-Serre theory. I plan to use this concrete understanding of the automorphisms to start understanding the fixed point subgroups.

I hope to generalise this work with Dihedral Artin groups to a broader class of Artin groups.

[1] Gilbert, Nick & Howie, James & Metaftsis, V. & Raptis, E.. (1999). Tree Actions of Automorphism Groups. Journal of Group Theory. 3. 10.1515/jgth.2000.017.

Martina Jørgensen
ETH Zürich, Switzerland

A combinatorial higher-rank hyperbolicity condition

Generalizations and variations of Gromov hyperbolicity belong to the most present themes in geometric group theory today; among these are *relative hyperbolicity*, *semi-hyperbolicity* and *hierarchical hyperbolicity*. In [1], the authors have built the foundations for a notion of higher rank hyperbolicity, that complements and to some extent encompasses these concepts. Their results hold for higher rank symmetric spaces, and more generally all CAT(0) and Busemann spaces of rank n in an asymptotic sense.

In recent work [2] we continue the investigation of these higher-rank hyperbolicity phenomena, with focus on a more foundational, partly combinatorial aspect. In particular, we explore a coarse $2(n+1)$-point inequality for general metric spaces that reduces to Gromov's quadruple definition of δ-hyperbolicity in the case $n = 1$ and, if $\delta = 0$, to an inequality characterizing metric spaces of combinatorial dimension at most n due to Dress. This unifying condition, referred to as (n, δ)-hyperbolicity, turns out to possess a variety of remarkable properties, tying up higher-rank hyperbolicity with (coarsely) injective metric spaces, injective hulls, and some recent developments in geometric group theory. Some sample results are the following. The ℓ_∞-product of (n_i, δ)- hyperbolic spaces X_i, $i = 1, 2$, is $(n_1 + n_2, \delta)$-hyperbolic. In particular, the ℓ_∞-product of n δ-hyperbolic spaces is (n, δ)-hyperbolic. In general, $(n, *)$-hyperbolicity is preserved under rough isometries, yet quasi-isometry invariance is granted for the class of coarsely injective metric spaces. Further, the asymptotic rank of an (n, δ)-hyperbolic space X is at most n, and the notion also passes directly to the injective hull of X. Every (n, δ)-hyperbolic metric space, without any further assumptions, possesses a slim $(n + 1)$-simplex property analogous to the slimness of quasi-geodesic triangles in Gromov hyperbolic spaces.

Ongoing research focuses on further investigation of the classes of spaces in question. Is there an embedding theorem for $(n, *)$-hyperbolic spaces? Can one say something interesting about boundaries at infinity of $(n, *)$- hyperbolic spaces? Is there a relation between (n, δ)-hyperbolic spaces and coarse median spaces? We aim to contribute in establishing a solid and diverse theory of rank n hyperbolic spaces in the context of generalised nonpositive curvature, with robust characterisations and definitions. [2]

[1] Bruce Kleiner and Urs Lang, Higher rank hyperbolicity, Invent. Math. 221 (2020), no. 2, 597–664.

[2] M. Jørgensen, U. Lang, A combinatorial higher-rank hyperbolicity condition, arXiv:2206.08153v2 [math.MG].

Annette Karrer

McGill University, Canada

Non-positive curved groups and spaces and their boundaries

My work has two main topics: boundaries of finitely generated groups, and non-positively curved complexes such as CAT(0) cube complexes and systolic complexes.

Stalling's theorem, a fundamental theorem in geometric group theory, illustrates that the behavior of a finitely generated group at infinity carries information about the structure of the group. Here, the behavior at infinity is studied by the *space of ends*.

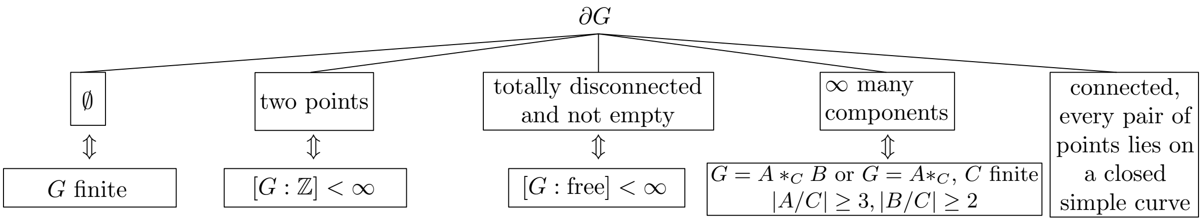

Figure 1: Stalling's theorem about finitely generated groups G.

Boundaries are other topological spaces at infinity that consist of equivalence classes of geodesic rays. An important example is the *Gromov boundary* of *hyperbolic groups*. The research of Gromov, Bowditch, Bestvina-Mess, Whyburn and others result in:

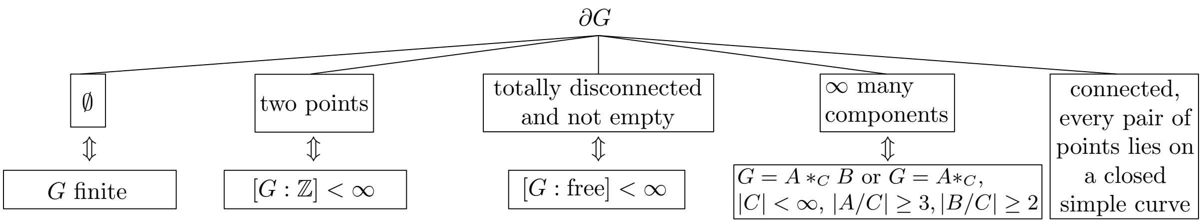

Figure 2: A variant of Stalling's theorem for the Gromov boundary ∂G of a hyperbolic group G.

Recently the Gromov boundary was generalized to the *Morse boundary* of finitely generated groups by Cordes based on the work of Charney–Sultan. It captures the hyperbolic-like behavior at infinity. My research on boundaries is inspired by the problem to generalize Figure 2 to Morse boundaries. In this problem the interplay of Euclidean-like and hyperbolic-like behavior of a group plays an important role. This interplay fascinates me in my research on *visual boundaries* and group actions on non-positively curved complexes such as *CAT(0) and systolic complexes*.

Daniel Keppeler
Westfälische Wilhelms-Universität Münster, Germany

Automatic continuity in a Čech-complete setting

Given two topological groups G and H and a homomorphism $f\colon G \to H$ it is common to ask, if f preserves the topological structure i.e. if f is continuous. While not every homomorphism between arbitrary topological groups is going to be continuous, I am interested in special conditions on G and H under which every abstract homomorphism is automatically continuous. More precisely I study conditions for discrete groups H under which every homomorphism from any Čech-complete (e.g. locally compact or completely metrizable) group to H is continuous. As a starting point for this, I generalized a result of Dudley[1], which states, that any homomorphism from any Čech-complete group to $\mathbb{Z}$ is already continuous and introduced the following definition (based on notations by Connor and Corson):

> A discrete group H is called *Čc-slender* if every Homomorphism from any Čech-complete group to H is continuous.

Čc-slender groups have to be torsion-free (since there are discontinuous homomorphisms from the compact group $\prod_{\mathbb{N}} \mathbb{Z}/p\mathbb{Z} \to \mathbb{Z}/p\mathbb{Z}$), so one might ask, what results we might expect for groups with torsion. More precisely, I am interested in conditions on the torsion subgroups of a discrete group H under which every homomorphism from any Čech-complete group to H is either continuous or has small image. In joined work with Möller and Varghese[3] we showed first results for this in a locally compact setting. In (upcoming) work, I showed first results (in a Čech-complete setting), where H is a Graphproduct and where H is an Automorphismgroup of a right-angled Artingroup.

[1] R. M. Dudley, *Continuity of homomorphisms*. Duke Math. J. 28 ,(1961), 587–594.

[2] G. R. Conner; S. M. Corson, *A note on automatic continuity*. Proc. Amer. Math. Soc. 147,(2019), 1255–1268.

[3] D. Keppeler; P. Möller; O. Varghese, *Automatic continuity for groups whose torsion subgroups are small*. Journal of Group Theory, (2022).

Alice Kerr
University of Bristol, United Kingdom

Coarse geometry of groups and spaces

Definition. *Let (X, d_X) and (Y, d_Y) be metric spaces. A map $f : X \to Y$ is a* coarse embedding *if there exist non-decreasing functions $\rho_-, \rho_+ : [0, \infty) \to [0, \infty)$ such that:*

- *For every $x_1, x_2 \in X$, we have that*

$$\rho_-(d_X(x_1, x_2)) \leqslant d_Y(f(x_1), f(x_2)) \leqslant \rho_+(d_X(x_1, x_2)).$$

- *As $t \to \infty$, we have that $\rho_-(t) \to \infty$.*

Coarse embeddings are a generalisation of quasi-isometric embeddings, where we relax the requirement that the distances in the two spaces have a linear relationship. This allows us to consider natural scenarios that are not covered by quasi-isometric embeddings; for example, the inclusion of a finitely generated subgroup into a finitely generated group may not be a quasi-isometric embedding, but it will always be a coarse embedding.

An obvious question to ask is, given X and Y, does there exist a coarse embedding $f : X \to Y$? Unlike with quasi-isometries, which in many cases have been well studied, there are some seemingly simple examples of spaces for which we do not know the answer to this question. For instance, does there exist a coarse embedding $f : \mathbb{H}^3 \to \mathbb{H}^2 \times T$, where T is a 3-regular tree?

One way of approaching such a question is to look at properties that are invariant under coarse embeddings, such as growth, asymptotic dimension, and separation profiles. Broadly speaking, my current main project is to investigate various coarse invariants, and to find interesting examples where they provide an obstruction to the existence of coarse embeddings.

In previous projects I have studied the geometry of quasi-trees [1], and product set growth in acylindrically hyperbolic groups, especially right-angled Artin groups and mapping class groups [2].

[1] Alice Kerr. Tree approximation in quasi-trees. Preprint, arXiv:2012.10741, 2020.

[2] Alice Kerr. Product set growth in mapping class groups. Preprint, arXiv:2103.12643, 2021.

Lovis Kirschner
Justus-Liebig-Universität Gießen, Germany

Finiteness-Properties of S-Arithmetic Groups

We say a group H is *virtually of type* F, if there are $\Gamma \leq H$ of finite index and a contractible Γ-CW complex X, such that the action is cocompact and free.
Let G be a linear algebraic group over $\mathbb{Q}$ (for example $G = \mathrm{SL}_n$) then a subgroup of $G(\mathbb{Q})$ is called *arithmetic*, if it is commensurable with $G(\mathbb{Q}) \cap \mathrm{GL}_n(\mathbb{Z})$. Considering a ring $\mathbb{Z}[\frac{1}{N}]$ instead of $\mathbb{Z}$ we get the definition of an S*-arithmetic group*. Canonical examples are $\mathrm{SL}_n(\mathbb{Z})$ resp. $\mathrm{SL}_n(\mathbb{Z}[\frac{1}{p}])$.
The natural action of $\mathrm{SL}_n(\mathbb{Z})$ on the symmetric space $X = \mathrm{SL}_n(\mathbb{R})/\mathrm{SO}(n)$ (in the case $n = 2$ for example, we just get the well known action on the hyperbolic plane given by $\left(\begin{smallmatrix} a & b \\ c & d \end{smallmatrix}\right).z = \frac{az+b}{cz+d}$) is not cocompact, but has only finite stabilisers and one can find a torsion free, finite index subgroup Γ. To get a cocompact action, Raghunathan[R68] constructed a smooth, Γ invariant function $X \to [0, \infty[$ with no critical values outside a compact intervall $[0, r]$, and such that the induced funtion $\Gamma \backslash X \to [0, \infty[$ is proper. Now Morse-Theory (see for example [M63]) tells us, that there is a cocompact Γ subspace $Y \subset X$ which is a deformation retract, thus contractible.
The following Theorem is due to Borel and Serre [BS76]:

Theorem. *Any S-arithmetic subgroup of a connected, reductive algebraic group over $\mathbb{Q}$ is virtually of type F.*

Consider the induced action of $H = \mathrm{SL}_n(\mathbb{Z}[\frac{1}{p}])$ on the product of the symmetric space and a so called Bruhat-Tits Building (in case $n = 2$, the latter is just the $p + 1$ regular tree), given by the diagonal embedding $H \to \mathrm{SL}_n(\mathbb{R}) \times \mathrm{SL}_n(\mathbb{Q}_p),\ h \mapsto (h, h)$. Again, one can find a finite index subgroup $\Gamma \leq H$, that acts freely but not cocompactly.
To fix this, Borel and Serre added a boundary to the symmetric space part, in a way that doesn't change the topology, but still is compatible with the action.

My project is to find a new proof of the Borel-Serre Theorem via constructing a Morse function on the product space, in a similar way as Raghunathan did. Later we also want to apply Morse Theory to so called *approximate subgroups*; this is a symmetric subset Λ of a group G that contains the neutral element and fullfills the property $\Lambda\Lambda \subset F\Lambda$ for some finite set $F \subset G$.

[R68] M.S. Raghunathan, A Note on Quotients of Real Algebraic Groups by Arithmetic Subgroups, Invent. Math. (1968), 318-335.

[M63] J. Milnor, Morse theory. Annals of Mathematics Studies, No. 51 Princeton University Press, Princeton, N.J. 1963.

[BS76] A. Borel, J.P. Serre, Cohomologie d'immeubles et de groupes S -aithmetiques, Topology **15** (1976), 211-232.

Dominik Kirstein
Max Planck Institute for Mathematics Bonn, Germany

L^2-invariants in finite characteristic and torsion growth

I am interested in algebraic topology, mostly in the area of L^2-invariants (but I also like to think about algebraic K- and L-theory). At the moment, I am mainly thinking about questions related to the growth of Betti numbers (in finite characteristic) or torsion in integral homology of a space under towers of finite coverings.

L^2-invariants, such as L^2-Betti numbers $b_k^{(2)}$ or L^2-torsion $\rho^{(2)}$, are variants of the classical invariants that are defined for possibly noncompact spaces X taking a cocompact action of a group G into account. They are closely related to questions in geometric group theory and geometry. For example, if M is an odd dimensional closed connected hyperbolic manifold, the L^2-torsion $\rho^{(2)}(\widetilde{M};\pi_1(M))$ is given by its volume (up to a constant depending only on the dimension) so L^2-torsion can be thought of as some volume like invariant.

It is a fundamental question how these L^2-invariants are related to their classical counterparts. The famous approximation theorem by Lück says that the L^2-Betti numbers are given by

$$b_k^{(2)}(X;G) = \lim_{i\to\infty} \frac{b_k(X/G_i;\mathbb{Q})}{[G:G_i]}$$

where the $(G_i)_i$ form a suitable decreasing sequence of finite index normal subgroups of G. At the moment, I am thinking about analogues of this result for Betti numbers with $\mathbb{F}_p$-coefficients or L^2-torsion. One motivation for this is the following problem about profinite rigidity of 3-manifolds:

Suppose that M and N are two closed connected hyperbolic 3-manifolds such that the profinite completions of their fundamental groups are isomorphic. Are M and N then already homeomorphic?

Interestingly, an approximation result for L^2-torsion would at least show that M and N have the same volume. By a result of Thurston, there are only finitely many closed hyperbolic 3-manifolds of the same volume. This would give a partial answer to the question.

Kevin Klinge

Karlsruhe Institute of Technology, Germany

Ore localizations, closures of group rings, and algebraic fibrings

Let me show you some guidelines that I have collected during my three years so far as a PhD student and that have at least some degree of truth to them in the context of my research.

- Groups are torsion free.
- Abelian groups are not interesting.
- The Heisenberg group is cool.
- Everyone has at least one misconception about non-commutative rings.
- While we now distinguish “fields” and “skew fields”, the terms used to be “commutative field” and “field”. The old way was better.
- The best formulation of the Atiyah conjecture does not mention ℓ^2-betti-numbers.

The rest of this text will be an attempt in trying to justify where I came up with these.

For a communitative domain, there is a field of fractions (such as $\mathbb{Z}$ and $\mathbb{Q}$). Generally speaking, this does not work for non-commutative rings. Although there is something called the Ore Condition, which guarantees the existence of a field of fractions.

A good reason to care about non-commutative rings is the study of group homology: The homology is constructed from a sequence of $\mathbb{Z}G$-modules and $\mathbb{Z}G$ is non-commutative if G is non-abelian. We would like to study the dimension of homology. However, it is a priori not even clear, what the dimension of a module should be. On the other hand, we know what the dimension of a vector space is.

A not-so-obscure $\mathbb{Z}G$-module that satisfies the Ore Condition is the Von Neumann Algebra $\mathcal{N}G$. However, $\mathbb{Z}G$ itself only satisfies the Ore Condition if G is amenable. So, in an attempt to scavenge as many of the nice field-like properties of $\mathrm{Ore}(\mathcal{N}G)$, we consider the division closure of $\mathbb{Z}G$ inside $\mathrm{Ore}(\mathcal{N}G)$. This leads to a whole bouquet of interesting questions about division closures. Well known among them is the Atiyah conjecture, which asks if the division closure is again a skew field.

Another nice property of the field of fractions is that it is flat over the base ring. This means that one cannot get “more” homology by changing coefficients from some ring to it’s field of fractions. But since the field of fractions need not exist, the closest we can get is some division closure and we ask again, what properties we can recover. These questions about group homology can be linked to fibrings. That is, is there a map $G \to \mathbb{Z}$ with finitely generated kernel? More recently, I’ve been trying to generalize this notion to maps with other codomains.

Grzegorz Kozera

University of Münster, Germany

I am a student of mathematics at the University of Münster (third semester of my master's degree). My general interests are in algebraic topology and geometric group theory. So far, within these fields, I have taken courses in Münster: Algebraic Topology 1, 2 and Geometric Group Theory (GGT) 1, 2. This semester I am attending the seminar "Topics in Geometric Group Theory" where I will be talking about mapping class groups of surfaces. My first explicit contact with geometric group theory was in the course GGT 1, when we proved that a group G is free if and only if it acts freely and without edge inversion on a tree. At that time I really enjoyed the geometric-topological proof of this theorem. The GGT 2 lecture was entirely devoted to geometric group theory, especially to metric spaces of non-positive curvature. We discussed topics such as: groups as metric spaces, group actions, geodesic spaces, model spaces $\mathbb{E}^n$, $\mathbb{S}^n$, $\mathbb{H}^n$, comparison triangles, CAT(κ) spaces, M_κ^n - polyhedral complexes, Gromov's link condition, Berestovskii's theorem, CAT(0) cube complexes, hyperplanes, halfspace systems. Participation in the conference would be a great opportunity for me to deepen my knowledge in geometric group theory, within which I would like to write my master's thesis.

Julian Kranz

University of Münster, Germany

Classifiability of crossed product C^*-algebras

My research lies at the intersection of C^*-algebras, dynamical systems, and K-theory. A C^*-algebra is a self-adjoint norm-closed algebra of bounded linear operators on a complex Hilbert space. I am particularly interested in determining dynamically when the *crossed product C^*-algebra $C(X) \rtimes G$* associated to an action of a discrete group G on a compact space X is classifiable in the sense of the following theorem:

Theorem (see [2] for a list of references)**.** *Unital, simple, separable, nuclear, $\mathcal{Z}$-stable C^*-algebras satisfying the Universal Coefficient Theorem are classified by K-theory and traces.*

When G is a non-amenable group, there is a set of necessary conditions on the action (amenability, minimality and topological freeness) that become sufficient when combined with a condition called *dynamical comparison*. Interestingly, this condition automatically follows from the necessary ones for many classes of non-amenable groups such as acylindrically hyperbolic groups or Baumslag-Solitar groups [1]. Our proofs use boundary actions associated to geometric actions on various types of non-positively curved spaces.

My goal is to push these results (and similar results for amenable groups) further to a larger class of groups. Since the main input to these types of theorems usually come from geometric group theory, I want to learn more about the subject. This is my motivation to participate in the Young Geometric Group Theory workshop.

[1] Gardella, E., Geffen, S., Kranz, J., and Naryshkin, P. (2022). *Classifiability of crossed products by nonamenable groups.* arXiv:2201.03409.

[2] Winter, W. (2018). *Structure of nuclear C*-algebras: from quasidiagonality to classification and back again.* Proceedings of the International Congress of Mathematicians (ICM 2018) (In 4 Volumes) Proceedings of the International Congress of Mathematicians 2018.

Sanghoon Kwak

University of Utah, Sanghoon Kwak

Mapping Class Groups of Infinite Graphs — "Big Out(F_n)"

Surfaces and graphs are closely related; there are many parallels between the mapping class groups of finite-type surfaces and finite graphs, where the mapping class group of a finite graph is $\mathrm{Out}(F_n)$ — the outer automorphism group of a free group of finite rank. A recent surge of interest in infinite-type surfaces and their mapping class groups begs a natural question: What is the mapping class group of an "infinite" graph? (Hint: It is not $\mathrm{Out}(F_\infty)$; it is quasi-isometric to a point. See [4].)

In [1], Algom-Kfir and Bestvina give an answer by defining the mapping class group of a locally finite, infinite graph Γ as:

$$\mathrm{Map}(\Gamma) = \{\text{Proper homotopy equivalences } \Gamma \to \Gamma\}/\text{proper homotopy}.$$

They showed that $\mathrm{Map}(\Gamma)$ is Polish (separable and completely metrizable), and for infinite-type Γ that $\mathrm{Map}(\Gamma)$ is homeomorphic to $\mathbb{Z}^\infty$, which is not compactly generated.

As geometric group theorists, we are interested in the coarse-geometry of $\mathrm{Map}(\Gamma)$. Here we use Rosendal's framework for coarse geometry of non-locally compact groups. He showed a group that is *generated by a coarsely bounded set* has a well-defined quasi-isometry type equipped with the word metric. Kathryn Mann and Kasra Rafi [2] used this framework to study which mapping class groups of *infinite-type surfaces* have coarsely bounded generating sets.

Motivated by their work, George Domat, Hannah Hoganson, and I [3] took the first step toward the corresponding program for graphs by classifying when the *pure* mapping class group $\mathrm{PMap}(\Gamma)$ is *coarsely bounded*, and when $\mathrm{PMap}(\Gamma)$ is *locally coarsely bounded*(a necessary condition for having coarsely bounded generating set).

Theorem (See [3, Figure 1,2] for the flowcharts). *There are complete classifications of locally finite, infinite graphs Γ that have coarsely bounded $\mathrm{PMap}(\Gamma)$ or that have locally coarsely bounded $\mathrm{PMap}(\Gamma)$.*

As a next step, we are studying possible topological generating sets for $\mathrm{PMap}(\Gamma)$, which would help to classify which Γ has $\mathrm{PMap}(\Gamma)$ with a coarsely bounded generating set.

[1] Y. Algom-Kfir and M. Bestvina. *Groups of proper homotopy equivalences of graphs and Nielsen realization.* arXiv preprint arXiv:2109.06908, 2021.

[2] K. Mann and K. Rafi. *Large scale geometry of big mapping class groups.* preprint arXiv:1912.10914, 2019. To appear in Geom. Topol.

[3] G. Domat, H. Hoganson, and S. Kwak. *Coarse geometry of Pure mapping class groups of Infinite graphs.* preprint arXiv:2201.02559, 2022. To appear in Adv. Math.

[4] G. Domat, H. Hoganson, and S. Kwak. *The automorphism group of the infinite rank free group is coarsely bounded.* New York J. Math. 28 (2022), 1506–1511.

Hermès Lajoinie
Université de Montpellier, France

Rigidity of group actions on spaces with hyperbolic features

I am currently a second year PHD student under the supervision of Thomas Haettel.

I am interested in understanding the rigidity of actions of groups having the strong property (T) .

Vincent Lafforgue defined this strengthening of the Kazdhan property (T) ([1]) in his paper ([2]). Instead of looking at unitary representation of Hilbert space, the strong property (T) implies a fixed point result for representations with a small exponential growth of the norm. De la Salle have proved in ([3]) that higher-rank lattices, for example $SL(n, \mathbb{Z})$ for $n \geq 3$, satisfy this strong property (T).

Lafforgue proved the following result which shows that groups with property strong (T) are more rigid than groups satisfying only Kazdhan property (T) :

Theorem. *Let G be a group with the strong Property (T). Any action of G by isometries on a bounded valency Gromov-hyperbolic graph has a bounded orbit.*

This theorem implies in particular that hyperbolic groups cannot have strong property (T) in contrary with Kazdhan property (T).

So far, I tried to generalized this result for groups that generalized hyperbolicity, like coarse median groups ([4]) or relatively hyperbolic groups ([5]).

[1] B. Bekka, P. de la Harpe & A. Valette – New Mathematical Monographs, Kazhdan's property (T) 11 , Cambridge University Press, Cambridge, 2008.

[2] V. Lafforgue – « Un renforcement de la propriété (T) », Duke Math. J. 143 (2008), no. 3, p. 559–602.

[3] M. de la Salle – « Strong property (T) for higher rank lattices », (2017), arXiv :1711.01900.

[4] B. H. Bowditch – « Coarse median spaces and groups », Pacific J. Math. 261 (2013), no. 1, p. 53–93.

[5] Brian H. Bowditch, Relatively hyperbolic groups, Int. J. Algebra Comput. 22 (2012), no. 3, 1250016 (66 pages).

Corentin Le Bars

Paris-Saclay University, France

Random walks on CAT(0) spaces and boundary theory

I am a PhD student in the Topology and Dynamics team in Laboratoire de Mathématiques d'Orsay, Université Paris-Saclay, and expect to receive my PhD in June of 2023. My field of research belongs to Geometric Group Theory. More specifically, I use techniques from the theory of Dynamical Systems to study rigidity phenomena on groups, boundary theory, and spaces of non-positive curvature.

Let G be a discrete group acting by isometries on a CAT(0) space X, and let μ be a probability measure on G such that its support generates G as a semigroup. Consider the sequence $\omega = (\omega_i)_i$, where the $\omega_i' s$ are chosen independently according to the measure μ. The random walk $(Z_n(\omega))_n$ on G generated by μ is then defined by $Z_n(\omega) = \omega_1 \ldots \omega_n$. Taking $o \in X$, we study the asymptotic behaviour of the random variables $(Z_n(\omega)o)_n$. One of the key ideas in the study of such limit laws is the use of boundary theory. This is also a recurring theme for the study of rigidity phenomena on groups, as for instance, boundary maps and Furstenberg theory are involved in the original proof of Margulis' Super-rigidity Theorem.

The first part of my PhD was devoted to study limit laws of the random walk $(Z_n(\omega)o)_n$ if we assume that G acts with rank one isometries. It is often useful to think of rank one elements as isometries that satisfy hyperbolic-like properties. In this sense, trying to apply methods from the rich theory of hyperbolic spaces (see for example [3]) to the class of CAT(0) spaces who possess rank one geodesics has been a recurring theme in my research. In this setting, I proved that $(Z_n(\omega)o)_n$ converges to a point in the visual boundary $\partial_\infty X$, and that the hitting measure is the unique stationary measure on $\overline{X}$. If we assume further moment conditions on the measure μ, I proved that the drift (the speed at which the random walk goes to infinity) is almost surely positive, and that it satisfies a central limit theorem, see [1] and [2].

Buildings were introduced by Bruhat and Tits in order to give a good framework for the study of semisimple Lie groups over non-Archimedean local fields. Another subject in my thesis was the study random walks on $\tilde{A}_2$-buildings. With the use of recent results in boundary theory for CAT(0) spaces, we can prove that there is a unique stationary measure on the spherical building at infinity and study further limit laws.

[1] Corentin Le Bars, Random walks and rank one isometries on CAT(0) spaces, arXiv:2205.07594 (2022)

[2] Corentin Le Bars, Central limit theorem on CAT(0) spaces with contracting isometries, arXiv:2209.11648 (2022)

[3] Joseph Maher and Giulio Tiozzo, Random walks on weakly hyperbolic groups, J. Reine Angew. Math., vol.742 (2018)

Corentin Le Coz

University of Ghent, Belgium

Coarse geometry and cryptography

My topic of research lies in the intersection of geometric group theory, discrete mathematics, and cryptography.

Separation profile and coarse geometry During my PhD, I've studied separation profile, a coarse geometric invariant defined in term of expansion of subgraphs.

An interesting behaviour of this invariant is the fact that it distinguishes nilpotent and non-nilpotent solvable groups [1]. There is also a relationship with splittings of hyperbolic groups [3]. Using separation profiles, I've been able to prove the following results:

Theorem. [2] *There exist bounded degree graphs of asymptotic dimension one that do not coarsely embed in any finite product of bounded degree trees.*

Theorem. [2] *For any $\epsilon \in (0,1)$, there exists a hyperfinite sequence of bounded degree graphs $(\Gamma_n)_{n\geq 0}$, such that $c_p(\Gamma_n) \succeq_p (\log|\Gamma_n|)^{1-\epsilon}$, where c_p is the L^p compression exponent.*

Post-Quantum Hash functions My group theory experience led me to study hash functions, fundamental tools in cryptography. In [4], we apply Tillich-Zémor scheme to Cayley graphs of $\mathrm{SL}_n(\mathbb{F}_p)$ constructed by Arzhantseva and Biswas, having the properties that they are expander graphs with large girth, two desirables properties for hash functions, that we claim to be resistant to quantum computers.

Future I'm interested in a better understanding of coarse geometric invariants (asymptotic dimension, Poincaré profiles, etc.). A line of research I am currently working on is the relationship between coarse geometry and relative hyperbolicity. I would also like to work more on group-based cryptography. Higher dimensional expanders and expanding linear spaces are two possible direction of development in this domain.

[1] C. Le Coz and A. Gournay. Separation profiles, isoperimetry, growth and compression. *arXiv:1910.11733*.

[2] C. Le Coz. Poincaré profiles of lamplighter diagonal products. *arXiv:2007.04709*.

[3] N. Lazarovich and C. Le Coz. Hyperbolic groups with logarithmic separation profile. *arXiv:2110.13595* .

[4] C. Le Coz, C. Battarbee, R. Flores, T. Koberda, and D. Kahrobaei. Higher dimensional platforms for Tillich-Zémor hash functions. *arXiv:2207.03987*.

Xabier Legaspi

ICMAT and Université de Rennes 1, Spain and France

Growth spectrum in groups à la Gromov

I am interested in growth and combinatorial problems in groups that involve the following ingredients: simple counting arguments; metric inequalities imposing non-positive curvature conditions such that *Gromov's inequality* or *Behrstock's inequality*; and geometric small cancellation theory. Keeping only these ingredients in mind, it turns out that a large part of my work consists on coming up with the precise tree-like picture. The main goal of my research is to understand better some collections of volume entropy that I will introduce below. Using a very general point of view, one can still produce non-trivial results for relatively hyperbolic groups, CAT(0) groups, mapping class groups or graphical small cancellation groups at the same time.

Let G be a group acting properly by isometries on a metric space (X, d). Let $p \in X$. The *metric ball* $B(n)$ of radius $n \in \mathbb{N}_+$ is the set of elements $g \in G$ such that $d(gp, p) \leq n$. The *volume entropy* $\omega(G, X)$ is a non-negative number that measures the exponential growth rate of the volume of metric balls $B(r)$. It is defined as the limit

$$\omega(G, X) = \limsup_{n \to \infty} \frac{1}{n} \log |B(n)|.$$

This number is related to the topological entropy in dynamical systems, to the geodesic flow in Riemannian geometry or to the conformal dimension in Kleinian groups. It is not a number easy to compute and coincides with the critical exponent of the Poincaré series $P(s) = \sum_{g \in G} e^{-sd(p,gp)}$. Lately, I have been studying the following sets:

1. The *subgroup growth spectrum*, i.e. the set of $\omega(H, X)$ such that $H \leq G$.

2. The *quotient growth spectrum*, i.e. the set of $\omega(G/H, X/H)$ such that $H \leq G$. This definition makes sense even if H is not a normal subgroup, thinking about the growth of cosets G/H or the growth in the Schreier graph with respect to H.

3. The *algebraic growth spectrum*, i.e. the set of $\omega(H_S, X_S)$, where S is a finite symmetric subset of G, $H_S = \langle S \rangle$ and X_S is the Cayley graph of H_S with respect to S. We say that G has *uniform growth* if there exist $\omega_G > 0$ such that for every S either H_S is virtually nilpotent or $\omega(H_S, X_S) \geq \omega_G$.

In my first paper [1], I proved that if G contains a contracting isometry and H is an infinite index quasi-convex subgroup of G, then $\omega(H, X) < \omega(G, X)$ provided that G is not virtually cyclic, and $\omega(G/H, X/H) = \omega(G, X)$ with no further restrictions on G. At present, I am studying uniform growth in a joint work with Markus Steenbock.

[1] Constricting elements and the growth of quasi-convex subgroups. arXiv:2206.06749

Elyasheev Leibtag

Weizmann institute of science, Israel

Topological properties of algebraic groups

I research various topological structures on groups, I am particularly interested in algebraic groups over C_p - the completion of the algebraically closed field $\overline{\mathbb{Q}_p}$ which is a complete but not a local field. In the case of algebraic groups over local fields it was shown by H.Omori [1] for $\mathbb{R}$ and later by Bader-Gelander [2] for any local field that semi simple groups have the property that any continuous homomorphism from the group into any other topological group has a closed image, I am currently interested in extending these result for semi-simple algebraic groups over C_p.

Also I am interested in applications of ergodic theory in algebraic groups. In particular Howe-Moore like properties for not necessarily semi-simple algebraic groups over value fields.

[1] Omori, Hideki. "Homomorphic images of Lie groups." Journal of the Mathematical Society of Japan 18.1 (1966): 97-117.

[2] Bader, Uri, and Tsachik Gelander. "Equicontinuous actions of semisimple groups." arXiv preprint arXiv:1408.4217 (2014).

Alex Levine

University of Manchester, UK

Describing solutions to group equations using languages

I am a Research Fellow at the University of Manchester. Some of my work involves describing solutions to equations in groups using formal languages. Formally, an *equation* in a group G is an element $\omega \in G * F_V$, where F_V is the free group on a finite set V, called the set of *variables*. A solution is any homomorphism $\phi \colon G * F_V \to G$ that fixes G pointwise, and such that $(\omega)\phi = 1$. This can be thought of as replacing the variables with elements of G.

In 2016 Ciobanu, Diekert and Elder proved that solutions to systems of equationsin free groups can be expressed using EDT0L languages [1], which also gave bounds on the amount of memory needed to solve certain decision problems. The use of EDT0L languages to describe solutions has been successfully used in a variety of other classes of groups, including hyperbolic groups, RAAGS, virtually abelian groups, and in the case of single equations, the Heisenberg group. Currently, Laura Ciobanu and I are working on showing the central extensions of hyperbolic groups are amongst this class of groups.

[1] L. Ciobanu, V. Diekert, and M. Elder. Solution sets for equations over free groups are EDT0L languages. *Internat. J. Algebra Comput.*, 26(5):843–886, 2016.

[2] A. Evetts and A. Levine. Equations in virtually abelian groups: Languages and growth. *Internat. J. Algebra Comput.*, 32(3):411–442, 2022.

[3] A. Levine Equations in virtually class 2 nilpotent groups. *Groups Complex. Cryptol.*, 14(1):1–17, 2022.

[4] A. Levine. EDT0L solutions to equations in group extensions. *arXiv e-prints*, arXiv:2108.09390, 2021.

[5] A. Levine. Formal languages, quadratic Diophantine equations and the Heisenberg group. *arXiv e-prints*, arXiv:2203.04849, 2022.

Daniel Levitin
University of Wisconsin, Madison

Geometry of Amenable Groups and Metric Spaces

I am interested in the geometry of various amenable metric spaces, especially those related to solvable groups.

My first paper focused on a recently-defined invariant called Scaling Group of a metric space [1]. In it, I leveraged the classification of quasi-isometries of various solvable Lie groups to exhibit spaces achieving every finitely-generated Scaling Group [2].

My current work focuses on quasi-isometries of iterated wreath products, i.e. groups of the form $\mathbb{Z}/m\mathbb{Z} \wr (\mathbb{Z}/n\mathbb{Z} \wr \mathbb{Z})$. The goal is to generalize the work of Eskin-Fisher-Whyte in [3,4] inductively to more complicated metric spaces. I have preliminary results classifying all quasi-isometries that satisfy certain additional assumptions.

[1] Genevois and Tessera, *Measure scaling quasi-isometries*, 2021.

[2] Levitin, *Metric Spaces of Arbitrary Finitely-Generated Scaling Group*, 2022.

[3] Eskin, Fisher, and Whyte, *Coarse differentiation of quasi-isometries I: spaces not quasi-isometries to Cayley graphs*, 2012.

[4] Eskin, Fisher, and Whyte, *Coarse differentiation of quasi-isometries II: Rigidity for Sol and Lamplighter groups*, 2013.

Kevin Li

Universität Regensburg, Germany

Classifying spaces for families of subgroups

I am a postdoc working with Prof. Clara Löh.

I work in the areas of geometric group theory and algebraic topology. My research is loosely focused around classifying spaces for families of subgroups and their equivariant Bredon cohomology.

Group G	Model for $E_{\mathcal{FIN}}G$
graph of finite groups	Bass–Serre tree
right-angled Coxeter group	Davis complex
hyperbolic group	Rips complex
mapping class group	Teichmüller space
$\mathrm{Out}(F_n)$	Culler–Vogtmann outer space

Let G be a group and let $\mathcal{FIN}$ denote the family of finite subgroups of G. The classifying space $E_{\mathcal{FIN}}G$ is a G-CW-complex with finite stabilisers such that for every finite subgroup H of G, the fixed-point set $(E_{\mathcal{FIN}}G)^H$ is contractible. For an arbitrary family $\mathcal{F}$ of subgroups of G, the classifying space $E_{\mathcal{F}}G$ is defined similarly.

I have studied the equivariant bounded cohomology of such classifying spaces [2] and obtained vanishing results for bounded cohomology and ℓ^2-Betti numbers in the presence of amenable open covers [4]. Using constructions for classifying spaces of small dimension, I have computed the amenable category for right-angled Artin groups [3] and Farber's topological complexity for certain relatively hyperbolic groups [1].

Currently, I am thinking about vanishing for gradient invariants of residually finite groups, such as mod p homology growth and torsion homology growth, as well as about different notions of equivariant cohomological dimension for groups G with an action by another group Γ.

[1] K. Li. On the topological complexity of toral relatively hyperbolic groups. *Proc. Amer. Math. Soc.*, 150(3):967–974, 2021.

[2] K. Li. Bounded cohomology of classifying spaces for families of subgroups. To appear in *Algebr. Geom. Topol.*, arXiv:2105.05223, 2021.

[3] K. Li. Amenable covers of right-angled Artin groups. Preprint, arXiv:2204.01162, 2022.

[4] K. Li, C. Löh, and M. Moraschini. Bounded acyclicity and relative simplicial volume. Preprint, arXiv:2202.05606, 2022.

Marco Linton
University of Oxford, United Kingdom

Hyperbolic low dimensional groups

Since the introduction of hyperbolic groups by Gromov in the 80's, a wealth of powerful tools have been developed to study them. Thus, when studying a class of groups, a classification of those that are hyperbolic can be very useful.

My current research focuses on the hyperbolicity of low dimensional groups and, in particular, one-relator groups; that is, groups of the form $F(\Sigma)/\langle\langle w \rangle\rangle$, where $F(\Sigma)$ denotes the free group generated by Σ. One of my recent results in this direction establishes hyperbolicity for all 2-free one-relator groups.

The best possible hyperbolicity result one could hope for in the class of one-relator groups is known as Gersten's conjecture. Motivated by the fact that hyperbolic groups cannot contain certain one-relator subgroups, known as Baumslag–Solitar groups

$$BS(m, n) = \langle a, t \mid ta^m t^{-1} = a^n \rangle,$$

Gersten made the following conjecture:

Conjecture (Gersten's conjecture). *A one-relator group is hyperbolic if and only if it contains no Baumslag–Solitar subgroups.*

Such a dichotomy is known to hold for several classes of groups: Coxeter groups (Moussong), free-by-cyclic groups (Brinkmann), 3-manifold groups (Perelman), virtually special groups (Caprace, Haglund) and, most recently, ascending HNN-extensions of free groups (Mutanguha). However, one cannot hope to go too far in this direction as there exist groups of finite type that are not hyperbolic and do not contain Baumslag–Solitar subgroups (Italiano, Martelli, Migliorini).

Yusen Long
Université Paris-Saclay, France

Topological Tits Alternative of Big Mapping Class Groups

In 1985, McCarthy discovered Tits alternative of mapping class groups of finite type surfaces:

Theorem ([1]). *Let S be a surface of finite type and H be a subgroup of the mapping class group of S. Then H either contains a free subgroup of 2 generators, or it is virtually abelian.*

It is widely known that the same results do not hold for big mapping class groups, *i.e.* the mapping class groups of surface with infinite ends or genus, see [2, 3] for example. But the conterexamples at our disposal are constructed by starting with a countable amenable group for which Tits alternative fails and by embedding this group into a big mapping class group. Yet those construction do not take the topological structure into account and a topological form of Tits alternative that strengthen the dichotomy of amenability and non-amenability will certainly be of one's interest.

On the conference *Mapping class groups and* $\mathrm{Out}(F_n)$, Fanoni suggested that "the entire family of all big mapping class groups might be a too big to be treated with". One may start by considering only a particular subfamily of big mapping class groups, where the surface admits a non-displaceable subsurface, so that it can act non-elementarily and continuously on a Gromov hyperbolic space (see [4]).

In an outcoming paper of the author, one will present the dynamic of amenable groups acting continuously on separable geodesic and Gromov hyperbolic spaces by isometries. The main tools exploited in the paper is the general discussion of groups acting on uniform spaces, ultralimit of metric spaces, and horicompactification of Gromov hyperbolic spaces or metric functionals on a metric space. Broader applications of the result other than big mapping class groups will also be considered.

[1] John McCarthy. A"Tits-alternative" for subgroups of surface mapping class groups. *Trans. Amer. Math. Soc.*, 291(2) :583–612, 1985.

[2] Justin Lanier and Marissa Loving. Centers of subgroups of big mapping class groups and the tits alternative. *Glasnik matematički*, 55(1): 85-91, 2020.

[3] Daniel Allcock. Most big mapping class groups fail the Tits alternative. *Algebraic & Geometric Topology*, 21(7): 3675-3688, 2021.

[4] Camille Horbez, Yulan Qing, & Kasra Rafi. Big mapping class groups with hyperbolic actions: classification and applications. *Journal of the Institute of Mathematics of Jussieu*, (2021): 1-32, 2021.

Marco Lotz
Otto-von-Guericke-University Magdeburg, Germany

Reflection length in non-affine Coxeter groups

The focus of my PhD project is reflection length in non-affine Coxeter groups. Coxeter groups are direct products of finitely generated reflection groups, each acting on a sphere, a Euclidean space or a hyperbolic space. An element in a Coxeter group corresponds to a finite sequence of reflections in the according spaces. For a fixed element, this sequence is not necessarily unique. The minimal number of reflections that suffices to represent a certain group element is called the reflection length of this element.

In the spherical, as well as in the Euclidean case, the reflection length is bounded and formulas exist. On the contrary, by a result of Kamil Duszenko, the reflection length function is unbounded on all Coxeter groups that do not split into a direct product of spherical and Euclidean reflection groups (*non-affine* Coxeter groups). Almost nothing beyond this result is known.
Even though the reflection length is unbounded, it already seems difficult not only to compute it but also to find elements with large reflection length in some non-affine Coxeter groups.

Tianyi Lou

Université Côte d'Azur, France

Finitely generated subgroups in big mapping class groups

The mapping class group of a surface S is the isotopy classes of orientation-preserving homeomorphisms of S. Mapping class groups of surfaces form an important class of groups that has ramifications in several areas of mathematics. The case of compact surfaces is very different from the case where the surface is not compact. In case the surface is compact, the mapping class group of S is finitely generated and has been studied with a variety of tools. In the case of a non-compact surface S, the surface is called *large* and the mapping class group of S is no longer finitely generated, and no longer equal to the group of quasi-conformal homeomorphisms of the surface S. Hence classical tools of geometric group theory and differential geometry are no longer available to study the mapping class group.

However, any group is the inductive limit of its finitely generated subgroups. This viewpoint allows to handle several algebraic questions, hence the importance of understanding infinite families of finitely generated subgroups. In general, nested families of finitely generated subgroups have no geometrical interpretation, but one could start by looking at those who do.

In this project we will be looking at concrete ways to build large classes of finitely generated subgroups. One concrete question could be:

Question: *Can $SL(n,\mathbf{Z})$, Out($\mathbf{F}_n$) or Mod(S_g) be subgroups of Mod(S) for S a big surface?*

First we will start by reading the existing literature on finitely generated subgroups in classical and large mapping class groups, in particular Crisp and Paris' construction embedding Artin groups in mapping class groups [2] or Allcok's construction of Grigorchuck's groups in Mod(S) in case where S has infinitely many ends [1]. While investigating possible finitely generated subgroups, the student will keep in mind the following

Question: *Find obstructions for a group to be a subgroup of the mapping class group of a given large surface.*

Moreover, we will keep testing the finitely generated subgroups for standard geometric group theory properties, like property (T), amenability, hyperbolicity, etc.

[1] D. Allcock: Most large mapping class groups do not satisfy Tits' alternative.

[2] J. Crisp, L. Paris. The solution of a conjecture by Tits on the subgroups generated by squares of the generators of an Artin group.

Alex Loué

UCLouvain, Belgium

Representation theory of chamber systems

I am interested in various aspects of groups acting on discrete structures. Currently, my focus is on the representation theory of groups acting on *chamber systems*, with an eye towards local to global results. In particular, I am investigating conditions for Kazhdan's property (T) to hold. In this context, I have found that there is a subalgebra of the group algebra whose representation theory seems to reflect the representation theory of the group. I call it the *Hecke algebra*.

When the chamber system is a building, it is isomorphic to the usual Iwahori-Hecke algebra, which is a deformation of the underlying Coxeter group. In general, it seems possible to give a similar presentation of the Hecke algebra by studying residues of rank 2 in the chamber system (which are edge-transitive bipartite graphs).

It is a classical result that property (T) is equivalent to the existence of a spectral gap for the Laplace operator. In my setting, it is an element of the Hecke algebra, and it becomes possible to study its spectrum by considering appropriate C^*-completions. In fact, it seems that the existence of a spectral gap can be witnessed within the uncompleted Hecke algebra (see [1] for a similar result in the case of finitely generated groups).

At the moment, my main result is an application of a local criterion for the existence of a spectral gap. It involves estimating spectral quantities called *representation angles* at the level of residues of rank 2. The main question that remains open is whether this criterion is a necessary condition too, which seems to be the case based on a list of examples given in [2]. It would also be interesting to describe to which extent the representation theory of the Hecke algebra reflects the representation theory of the group, as in the case for groups acting sufficiently transitively on semi-regular trees, for instance (see [3]).

Some keywords. Chamber systems, buildings, C^*-algebras, harmonic analysis on tdlc groups, semi-simplicity.

[1] Ozawa, *Noncommutative real algebraic geometry of Kazhdan's property (T)*.

[2] Caprace, Conder, Kaluba, Witzel, *Hyperbolic generalized triangle groups, property (T) and finite simple quotients*.

[3] Matsumoto, *Analyse harmonique dans les systèmes de Tits bornologiques de type affine*.

Rylee Alanza Lyman

Rutgers University–Newark, USA

Groups acting on trees and their outer automorphisms

Many groups of interest to geometric group theorists act on trees; for example free groups, surface groups, Baumslag–Solitar groups, fundamental groups of 3-manifolds, groups with more than one end and so on. Thanks to Bass–Serre theory, not only is the structure of a group acting on a tree well understood, but group actions on trees may be used to construct new examples of groups with interesting properties.

When a group G acts on a tree without global fixed point, it typically acts on many trees in many ways. For example, the free group of rank n acts geometrically on the universal covering tree of any finite connected graph with Euler characteristic $1-n$, but there are stranger actions of F_n on trees as well. For another, any collection of disjoint, essential simple closed curves on a connected surface gives rise to an action of the fundamental group of that surface on a "dual" tree.

Thanks to work of Culler–Morgan, Culler–Vogtmann, Forester, Clay and Guirardel–Levitt, there is a rich theory of *deformations* of actions of groups on trees. This allows us to organize the many actions of a group G on trees into *deformation spaces*. One reason having a deformation space is useful is that you can reframe some questions so that the solution becomes clear: for example, maybe a certain property X varies continuously over the deformation space, so we can conclude the existence of a tree action with value Y for property X by the Intermediate Value Theorem.

Another reason deformation spaces are interesting is the following: Suppose G acts on a tree T and that $\Phi\colon G \to G$ is an automorphism. Then there is a new action of G on T by the rule that g sends a point $p \in T$ to the point $\Phi(g).p$, in other words, in the new action, g acts the way $\Phi(g)$ acted under the old action. This new tree action depends only on the outer class $\varphi \in \mathrm{Out}(G)$, so the group $\mathrm{Out}(G)$ acts on the collection of deformation spaces of tree actions for G. I'd like to advocate for calling the stabilizer of a given deformation space under this action the "modular group" of the deformation space, or the "mapping class group" of an associated graph of groups. These groups are very interesting to me and are just beginning to be understood.

For example, here is an open problem whose solution would be very interesting to me. Let G be a finitely generated virtually free group (so it has a free subgroup of finite index). It has an $\mathrm{Out}(G)$-invariant deformation space of actions on trees with finite stabilizers. Krstić and Vogtmann proved that this space (or really its *spine,* which is a simplicial complex) is contractible, which has the nice corollary that the *virtual cohomological dimension* of the virtually torsion-free group $\mathrm{Out}(G)$ is finite.

Problem. Give invariants associated to G that compute the virtual cohomological dimension of $\mathrm{Out}(G)$. Find a natural action of $\mathrm{Out}(G)$ on a complex of that dimension.

Bianca Marchionna
University of Bielefeld, Germany; University of Milano-Bicocca, Italy

Zeta functions and group actions on trees and buildings

T. Weigel and G. Willis introduced the concept of *double coset Dirichlet series* $\zeta_{G,K}$ of a totally disconnected locally compact group G with respect to a compact open subgroup $K \leq G$. Namely,

$$\zeta_{G,K}(s) = \sum_{KgK \in K\backslash G/K} [KgK : K]^{-s},$$

where $K\backslash G/K$ is the set of K-double cosets of G and $[KgK : K]$ counts the number of (left) translates of K that cover KgK.
The definition originates from BN-pairs: when (G, K) is a BN-pair, each $[KgK : K]$ and $\zeta_{G,K}$ have an explicit interpretation in terms, respectively, of the Bruhat-Tits building and the growth series of the Weyl group associated to the pair.

Inspired by [1], my current research is focused on the study of $\zeta_{G,K}$ when G has a (sufficiently transitive) continuous and proper action on trees or buildings. The broad goal is to read some structural properties back from the analytic ones of the series. E.g., characterizing the vanishing of the abscissa of convergence and deciding whether the value in -1 of the meromorphic continuation of $\zeta_{G,K}$ is the inverse of the Euler-Poincarè characteristic of G. As suggested by some examples, my ultimate goal is to interpret the outcomes in terms of the (topological) simplicity of the group.

The main strategy involved is to reduce the computation of $\zeta_{G,K}$ to counting problems in terms of geometric/combinatorial features of the space. For instance, when G acts on a tree, one reduces to counting geodesics with a common starting vertex.

Recently, I started investigating whether a similar geometric approach can be carried over to a slightly different context such as the submodule zeta functions (cf. [3, §. 2]).

[1] I. Castellano, G. Chinello, and Th. Weigel. "*The Hattori-Stallings rank, the Euler characteristic and the ζ-functions of a totally disconnected locally compact group*". In preparation, (2022)

[2] B. Marchionna. "*The double-coset zeta function for groups acting weakly locally ∞-transitively on locally finite trees*". In preparation, (2022)

[3] T. Rossmann, "*Computing topological zeta functions of groups, algebras and modules, I*". Proc. London Math. Soc. (3) 110, (2015), pp. 1099-1134

Jill Mastrocola

Brandeis University, USA

Intersections of Parabolic Subgroups in Sufficiently Large-Type Artin Groups

---

I am interested in studying Artin groups and their properties. More specifically, I am focused on answering the question of whether or not the set of parabolic subgroups of a particular Artin group is closed under intersection. There have been several results which answer affirmatively for certain classes of Artin groups, but the question is open in general. Classes of Artin groups for which this result is known include spherical type [5], FC-type [3], [4], large-type [1], and $(2,2)$-free two-dimensional [2].

Currently I am studying the case for Artin groups of *sufficiently large type*: Artin groups whose defining graph has the property that any triangle (i.e. any triple of vertices mutually joined by edges) is either labelled by all 2's or whose labels are all 3 or greater.

[1] Martín A. Blufstein. (2022). Parabolic subgroups of two-dimensional Artin groups and systolic-by-function complexes. to appear in Bulletin of the London Mathematical Society, arXiv:2108.04929.

[2] María Cumplido, Volker Gebhardt, Juan González-Meneses, and Bert Wiest. (2019). Advances in Mathematics, 352, 572-610.

[3] María Cumplido, Alexandre Martin, and Nicolas Vaskou. (2022). Parabolic subgroups of large-type Artin groups. Mathematical Proceedings of the Cambridge Philosophical Society, 1-22.

[4] Philip Möller, Luis Paris, and Olga Varghese. On parabolic subgroups of Artin groups. (2022). arXiv:2201.13044

[5] Rose Morris-Wright. (2021). Parabolic subgroups in FC-type Artin groups. Journal of Pure and Applied Algebra, 225(1), 106468.

Ruth Meadow-MacLeod

Temple University, USA

Fibered Face theory for free-by-cyclic groups

Some research interests of mine include Fibered Face theory for free-by-cyclic groups, Bass-Serre theory, mapping class groups, and Culler-Vogtmann outer space.

My current research aims to build on work by Dowdall, Kapovich, and Leininger. They considered the mapping torus $Z_f = \Gamma \times [0,1]/(x,1) \sim (f(x),0)$, where Γ is a finite graph with no valence–1 vertices, and $f\colon \Gamma \to \Gamma$ is an expanding irreducible train track map. This mapping torus has a natural suspension semiflow, ψ, by flowing along the $[0,1]$ direction. The Fried Cone is the cone in $H^1(Z_f, \mathbb{R})$ of classes which are positive on all closed orbits of ψ. Dowdall, Kapovich, and Leininger proved that for any primitive integral class, u, in the Fried cone, there is a corresponding graph embedded in Z_f which is dual to u and transverse to ψ [1].

My goal is to prove that such a dual transverse graph also exists for primitive integral classes in the boundary of the Fried Cone. A similar result has been proven for the mapping torus $M_f = S \times [0,1]/(x,1) \sim (f(x),0)$, where f is a pseudo-Anosov map on a surface, S, of genus g (with $g \geq 2$) [2].

[1] S. Dowdall, I. Kapovich, and C. Leininger, *Dynamics on free-by-cyclic groups*

[2] M. Landry, Y. Minsky, S. Taylor, *A polynomial invariant for Veering Triangulations*

Aaron Messerla
University of Illinois at Chicago, United States

Quasi-isometries of relatively hyperbolic groups with an elementary hierarchy

Quasi-isometry is an equivalence relation on finitely generated groups. Gromov proposed in [1] the problem of classifying all finitely generated groups up to quasi-isometry. Many interesting classes of groups have been shown to be closed under quasi-isometry.

In [2], Sela introduced the class of limit groups. Limit groups play an important role in Sela's work on the Tarski problem about the elementary theory of free groups. Among other results, Sela showed that limit groups are exactly the finitely generated fully residually free groups, and that each limit group has a finite cyclic hierarchy. This hierarchy can be described as a finite rooted tree, where the descendants of a vertex correspond to the vertex groups of a cyclic splitting of the parent.

I have studied a class of relatively hyperbolic groups which come equipped with hierarchies that resemble Sela's hierarchy for a limit group in [3]. This class, which I call $\mathcal{ATEH}$, consists of groups which are hyperbolic relative to virtually abelian subgroups, and has a finite hierarchy over virtually abelian groups. There is one additional condition on how each peripheral subgroup can act on the splittings in the hierarchy. Using tools from the theory of special cube complexes, I have shown that groups in $\mathcal{ATEH}$ share some properties with virtual limit groups, such as being subgroup separable and being virtually torsion free. Additionally, my work shows that $\mathcal{ATEH}$ is closed under quasi-isometry. In particular, if $G \in \mathcal{ATEH}$ and G' is quasi-isometric to G, then $G' \in \mathcal{ATEH}$.

[1] M. Gromov. *Asymptotic invariants of infinite groups*, Geometric Group Theory **2** (1993), 1–295

[2] Z. Sela, *Diophantine geometry over groups I: Makanin-Razborov diagrams*. Publications Mathématiques de l'IHÉS **93** (2001), 31-105

[3] A. Messerla, *Quasi-isometries of relatively hyperbolic groups with an elementary hierarchy*. in preparation.

Greyson Meyer
University of California, Santa Cruz, United States of America

---

Splitting Triangle Artin Groups As Graphs of Free Groups

Having only just embarked on my research journey, the primary focus of my research so far has been triangle Artin groups – specifically in the context of splittings as graphs of free groups. I endeavor to use geometric methods in order to better understand the underlying structure of some of the more mysterious triangle Artin groups, namely Art_{23n} for $n \geq 6$.

Anna Michael

Otto-von-Guericke University Magdeburg, Germany

Algorthmic Properties of Coxeter Shadows

Coxeter shadows were first introduced by Graeber and Schwer in [1], as follows: Choose an orientation of your Coxeter complex, so that for ever hyperplane you get exactly one corresponding positive and negative half space. Consider a minimal gallery $\gamma = (c_f = c_0, p_1, c_1, \ldots, p_n, c_n = x)$ with alcoves c_i and panels p_i connecting the fundamental alcove with the alcove corresponding to the group element x. Now you can *fold* the gallery γ at the panel p_i by reflecting the remaining part of the gallery $(c_i, p_{i+1}, c_{i+1}, \ldots, p_n, c_n)$ on the hyperplane $H_p \supset p_i$ to obtain a new gallery ending in $r_{H_p}(x) \neq x$. Collecting all the group elements you can construct from γ by folding the gallery onto positive half spaces with respect to your chosen orientation, you get the shadow of x (with respect to your orientation).

Folded galleries are related to retractions in Bruhat-Tits buildings and have a variety of applications, see further [3, 4]. Shadows themselfes generalize the Bruhat order of the Coxeter group and also have relations to many algebraic structures like affine Deligne-Lusztig varieties, MV-polytopes and Hall-Littlewood polynomials. They find applications e.g. in represenation theory and arithmetic geometry, see [2]. As my PhD project I am studying algorithmic properties of Coxeter Shadows in finite and affine Coxeter groups, trying to understand how hard it is to decide whether a given group element y is contained in the shadow of another element x with respect to a fixed orientation, and, if possible, provide effective algorithms that do so.

[1] Marius Graeber and Petra Schwer: *Shadows in Coxeter Groups*, Annals of Combinatorics 24 (1), pp. 119-147, Springer Science and Business Media LLC, 2020.

[2] Petra Schwer: *Shadows in the Wild - Folded Galleries and Their Applications*, Jahresbericht der Deutschen Mathematiker-Vereinigung, pp. 3-41, 2022.

[3] Stéphane Gaussent and Peter Littelmann: *LS Galleries, the Path Model, and MV Cycles*, Duke Mathematical Journal 127, 2005.

[4] Arun Ram: *Alcove walks, Hecke algebras, spherical functions, crystals and column strict tableaux*, arXiv, 2006.

Matteo Migliorini
Scuola Normale Superiore, Italy

Hyperbolic manifolds fibering on S^1

In my research I study hyperbolic manifolds and fibrations over the circle (i.e. fiber bundles with S^1 as base space).

While the topic is very well understood in dimension 3, in higher dimension almost nothing is known.

As part of my research, in a joint work with Giovanni Italiano and Bruno Martelli, we tried to construct such fibrations in high dimension. To do so, we used the combinatorial game from [1], which from a hyperbolic right-angled polytope produces a hyperbolic manifold M equipped with a map $f : M \to S^1$. Depending on some conditions, this map may possess some good properties: it might be a fibration if we are really lucky, or at least it may be an *algebraic fibration*, meaning that $f_* : \pi_1(M) \to \pi_1(S^1) = \mathbb{Z}$ is surjective with finitely generated kernel.

Using this, we first proved the following:

Theorem ([2]). *There exist hyperbolic n-manifolds for every $5 \le n \le 8$ which algebraically fiber.*

Later, we managed to refine the result in dimension 5.

Theorem ([3]). *There exist hyperbolic 5-manifolds which fiber over the circle.*

Using this, we could find an answer to a well-known open question.

Corollary. *There exists a hyperbolic group G which has a subgroup $H < G$ which is of finite type but is not hyperbolic.*

Lately, we are trying to understand better this manifold, in order to find a good description for the monodromy.

[1] Kasia Jankiewicz, Sergey Norin, and Daniel T. Wise, *Virtually fibering right-angled Coxeter groups*. Journal of the Institute of Mathematics of Jussieu (2021), 20:957–987

[2] Giovanni Italiano, Bruno Martelli, Matteo Migliorini, *Hyperbolic manifolds that fiber algebraically up to dimension 8*, arXiv:2010.10200

[3] Giovanni Italiano, Bruno Martelli, Matteo Migliorini, *Hyperbolic 5-manifolds that fiber over S^1*. Invent. math. (2022). doi.org/10.1007/s00222-022-01141-w

Francesco Milizia

Scuola Normale Superiore, Pisa, Italy

Simplicial volume of Davis' cube complexes

I am interested in topology, geometry and group theory; during my PhD I am studying some problems about bounded cohomology (of topological spaces or discrete groups) and simplicial volume (of manifolds).

One specific problem I worked on during last year was about two notions of "boundedness" for cohomology classes of a group, one being weaker than the other; in the paper [1] we have constructed the first example of a finitely presented group for which these two notions do not coincide (for cohomology in degree 2). Our construction involved finding a group together with a Lipschitz "projection" onto an infinite cyclic subgroup, satisfying certain conditions.

Another problem I am currently interested in is a particular case of the following famous (at least among people studying simplicial volume) question of Gromov:

Question (Gromov) Consider a connected closed manifold which is aspherical (*i.e.*, its universal cover is contractible) and has vanishing simplicial volume. Is its Euler characteristic equal to 0?

There is a very nice construction, due to Davis, that outputs a cube complex whenever given a simplicial complex as input. If the simplicial complex is homeomorphic to S^{n-1}, then the output is a topological manifold of dimension n. Moreover, if the simplicial complex is also *flag* (this is a combinatorial condition: the complex coincides with the clique-complex of its 1-skeleton), then the output is a locally CAT(0) manifold.

I think that this family of manifolds constitutes an interesting testing ground for Gromov's question, and I am trying to understand when these manifolds have vanishing or positive simplicial volume. Already in dimension $n = 4$, *i.e.*, flag 3-spheres giving rise to 4-manifolds, this seems a quite hard challenge. At least, much harder than in lower dimensions: for $n = 3$ I have found a nice characterization of flag 2-spheres giving positive simplicial volume, and for $n \leq 2$ the problem is trivial.

[1] *Weakly bounded cohomology classes and a counterexample to a conjecture of Gromov*, joint work with Dario Ascari. `arXiv:2207.03972`.

Lawk Mineh

University of Southampton, United Kingdom

Profinite topology and non-positive curvature in groups

My main interest lies in groups satisfying non-positive curvature conditions, such as (relatively) hyperbolic groups and CAT(0) cubulated groups. The focus of my current projects is to try to better understand profinite topologies in such groups. One thing I am particularly interested in is clarifying the somewhat strange relationship between profinite properties and (quasi)convexity in subgroups of non-positively cured groups. In the setting of hyperbolic groups, for example, profinite closure of all quasiconvex subgroups is known to be equivalent to residual finiteness of all hyperbolic groups [1], itself a well-known open problem.

In a recent work [2], we obtained a sort of combination theorem for quasiconvex subgroups in a large class of relatively hyperbolic groups using the profinite topology. We also showed that arbitrary products of quasiconvex subgroups are profinitely closed in such groups, with consequences to limit groups, Kleinian groups, and fundamental groups of certain graphs of free groups.

In future, I'm hoping to be able to better understand the algebraic structure of subgroups given by our combination theorem, generalise our results about products to other classes of non-positively curved groups, and develop new examples of groups of non-positive curvature with interesting profinite properties.

[1] I. Agol, D. Groves, J. Manning, *Residual finiteness, QCERF and fillings of hyperbolic groups*.

[2] A. Minasyan, L. Mineh *Quasiconvexity of virtual joins and separability of products in relatively hyperbolic groups*.

Philip Möller

University of Münster, Germany

Locally compact groups and automatic continuity for groups from GGT

My research consists of multiple projects in different areas of mathematics.

The first project revolves around locally compact groups. In 2017, two papers by Caprace, Reid and Willis (see [1], [2]) opened up a new approach to the study of totally disconnected locally compact groups. In these papers, multiple (boolean) lattices are constructed from these groups and studied. This turns out to be especially powerful if the group in question is (topologically) simple. The general aim of this project is to see if these new methods can be generalized to more general, not totally disconnected locally compact groups and ideally find a suitable definition of a "semi-simple" locally compact group. Since the connected locally compact groups are the other end of the spectrum and are well-studied using the theory of (pro-)lie groups (see [3]), I am currently investigating these groups.
More precisely, for a boolean lattice, one has to find "complements" in a suitable manner for (locally) normal subgroups. In the connected case, we have found multiple notions that seem to be a suitable definition for a "quasi-complement". Furthermore in three cases (compact connected, locally compact abelian and connected locally compact reductive groups), quasi-complements exist for a suitable definition. Since this approach appears to be working in both the connected and the totally disconnected case, it seems to be plausible that a similar approach may be fruitful in a more general setting which is the current state of the project.

The other projects I am working on are almost all related to automatic continuity of some form. The general question is the following: Given two topological groups L, G and an algebraic homomorphism $\varphi\colon L \to G$, can we find algebraic conditions on L, G and φ ensuring that φ is continuous? The first result in this direct is due to Dudley (see [4]) and states that any homomorphism from a locally compact Hausdorff group to a free (abelian) group is continuous. This question has been studied for many groups that are usually studied in GGT. It turns out that often times either the image of the homomorphism is "small" or the homomorphism is continuous.

[1] P.-E.-Caprace, C. Reid, G. Willis. *Locally normal subgroups of totally disconnected groups. Part I: General theory.* Forum Math. Sigma 5 (2017), e11, 76 pp.

[2] P.-E.-Caprace, C. Reid, G. Willis. *Locally normal subgroups of totally disconnected groups. Part II: Compactly generated simple groups.* Forum Math. Sigma 5 (2017), e12, 89 pp.

[3] K. H. Hofmann, S. A. Morris, *The Lie Theory of Connected Pro-Lie Groups.* EMS 2007.

[4] R. M. Dudley, *Continuity of homomorphisms.* Duke Math. J. 28 ,(1961), 587–594.

Lewis Molyneux

Royal Holloway, University of London, United Kingdom

Finiteness Properties of Irrational Slope Thompson Groups

Irrational Slope Thompson Groups are variations on Thompson's Group and the family of Brown-Thompson groups where the interval is divided asymmetrically, allowing for slopes of irrational gradient with irrational breakpoints between. While these groups share many properties with Brown-Thompson groups, the also defy convention in other scenarios, most notably, it has been shown that they are unable to embed into Thompson's Group, which sets them apart from the Brown-Thompson groups.

Finiteness properties are another area in which irrational slope Thompson groups may differ from the Brown-Thompson groups. While Cleary was able to show that the simplest example of an irrational slope Thompson Group, known as F_τ is F_∞, and a subset have known finite presentations, the full landscape of these groups remains unknown with regards to their finiteness properties. In particular, the BNSR invariant, a group invariant which reveals the finiteness properties of certain subgroups, has only been calculated for F_τ.

[1] Brita Nucinkis, Jose Burillo and Lawrence Reeves. "An Irrational-Slope Thompson's Group",

[2] Robert Bieri, Walter Neumann and Ralph Strebel. "A Geometric Invariant of Discrete Groups",

Andrea Egidio Monti
Max Planck Institute for Mathematics - Bonn, Germany

Grafting in (higher) Teichmüller theory

I am interested in Teichmüller theory and in particular I am currently studying an operation called grafting and an action it induces on Teichmüller space.

Given a closed orientable surface S of genus $g \geq 2$, its Teichmüller space $\mathcal{T}(S)$ classifies hyperbolic metrics on S up to isotopy. Such a surface with a hyperbolic metric is always covered metrically by the hyperbolic plane $\mathbb{H}^2$, on which the group $\pi_1(S)$ acts as deck transformations via isometries, where the action depends on the chosen hyperbolic metric on S. Every point in $\mathcal{T}(S)$ is indeed identified by a unique (up to conjugacy) discrete and faithful representation $\rho : \pi_1(S) \to PSL_2(\mathbb{R}) = \text{Isom}^+(\mathbb{H}^2)$.

A path in $\mathcal{T}(S)$ corresponds to a continuous deformation of such a representation, which coincides with a continuous deformation of a hyperbolic metric. A well studied family of deformations are those obtained by *shearing*, that is by cutting the surface along geodesic laminations (suitable sets of disjoint geodesics in S) and letting the different components of the surface slide along these cuts in some suitable way. Thurston first introduced these deformations, which he called *earthquakes*. Shearing deformations are a useful tool, since they generate the whole tangent space of $\mathcal{T}(S)$, they link any two point in $\mathcal{T}(S)$ and also their generalization to higher Teichmüller theory already gave interesting results.

Higher Teichmüller theory arises by replacing $PSL_2(\mathbb{R})$ with another higher rank Lie group G and studying the character variety (i.e. the space of such representations up to conjugacy) and questioning whether it does classify geometric structures on S, as in the case of $PSL_2(\mathbb{R})$. Among the many partial answers to these questions, we have a result of Hitchin that shows that for $G = PSL_n(\mathbb{R})$ there is a connected component $\mathcal{H}_n(S)$, named after him, of the character variety made by discrete and faithful representations. Thus, by seeing $PSL_n(\mathbb{R})$ as group of isometries of the associated symmetric space X_n, one can see Hitchin representations as geometric actions of $\pi_1(S)$ on the space X_n (which in the case $n = 2$ is the hyperbolic plane) and try to give this way a geometric interpretation to shearing deformations generalized to this context.

We can roughly say that in Hitchin components shearing has more degrees of freedom and then its geometric behaviour is better modelled by a combination of the classical shearing and another operation called *grafting*. Grafting is a geometric operation that takes a hyperbolic surface, a closed geodesic γ on it and replaces, via cut-and-paste, the geodesic γ with a flat cylinder $\gamma \times [0, t]$. Currently I am studying the flow induced on $\mathcal{T}(S)$ as follows: given a hyperbolic structure, we do the grafting, produce a geodesic current from the obtained surface and, thanks to recent results of Sapir and Hensel, project it back to another point in $\mathcal{T}(S)$. The hope is that understanding this model may also help us to better understand shearing in the Hitchin component.

Ismael Morales
University of Oxford, United Kingdom

ℓ^2-methods in abstract and profinite group theory

The study of ℓ^2-invariants was initiated by Atiyah in the context of Riemannian manifolds and, in the context of abstract groups, it essentially consists on understanding the homology groups of an abstract group G with coefficients in the module $\ell^2 G$ (i.e. the Hilbert space of real functions on G with bounded ℓ^2-norm). In this abstract setting, it may seem more natural to simply study the homology groups $H_n(G; \mathbb{Z}G)$ (for example for questions about duality). Nevertheless, these are generally harder to compute and the former ones admit analytical tools such as a von Neumann dimension function. Also, possibly surprising, the groups $H_n(G; \ell^2 G)$ contain much algebraic information about G.

My research is mostly focused on the first L^2-Betti number $b_1^{(2)}(G)$, which is the dimension of $H_1(G; \ell^2 G)$. More precisely, I study when $b_1^{(2)}(G)$ can be read off from the profinite completion $\widehat{G}$ (using Lück's approximation-type principles) to infer algebraic properties of G (which will be, in fact, profinite invariants).

For example, in [1] it is seen how in some cases the computation of $b_1^{(2)}(G)$ allows to detect the centre of a group by looking at $\widehat{G}$ (even if, in general, this is not a profinite invariant, as proven by Lubotzky). In [3], $b_1^{(2)}(G)$ turns out to be useful for constructing residually nilpotent groups because they seem to detect if there is a kernel in the natural map of a group $G \to G_{\widehat{p}}$ to its pro-p completion. This idea is due to Jaikin-Zapirain, how had already shown that ICE groups can be densely embedded into free pro-p groups.

If S is a free group (resp. a surface group), then $b_1^{(2)}(S) = b_1(S) - 1$ (resp. $b_1^{(2)}(S) = b_1(S) - 2$), where b_1 is the usual first Betti number. It is studied in [2] how particular and rare are these features and how useful they are to study pro-soluble invariants of S (such as being residually-p), which, as shown by Lubotzky, are not generally invariant.

[1] I. Morales, *On the profinite rigidity of free and surface groups*. Preprint available at `https://arxiv.org/abs/2211.12390`

[2] Appendix to *Characterising surface groups by their virtual second betti number*, by J. Fruchter. Preprint available at `https://arxiv.org/abs/2209.14925`

[3] A. Jaikin-Zapirain and I. Morales, *Parafree fundamental groups of graphs of free groups*. Preprint available at `https://arxiv.org/abs/2110.11655`

Chiranjib Mukherjee
University of Münster, Germany

I am interested in understanding geometry of (hyperbolic) groups using and combining tools from probability theory, ergodic theory and C* algebras. For instance, in a recent work with my PhD student (K. Recke) we provided a characterization of amenability using group-invariant percolation on Cayley graphs and decomposition of Schur multipliers of Roe Algebras via a group-invariant compactification of probability measures. Currently we are working on extending this result to explore further geometric properties of groups using percolation tools.

Zachary Munro
McGill University, Canada

Random Group Actions on CAT(0) *Cube Complexes*

A cube complex X is a complex formed from a collection of Euclidean cubes by identifying faces of cubes via isometries. The *link* of a 0-cell $x \in X^0$ is a complex with a simplex for each corner of a cube containing x, and inclusion of simplices corresponds to inclusion of the associated corners. A simplicial complex is *flag* if all sets of pairwise adjacent 0-cells in the 1-skeleton span simplices. A simply-connected cube complex is $\mathrm{CAT}(0)$ if links are flag simplicial complexes. In this situation, the standard Euclidean metrics on each cube induce a $\mathrm{CAT}(0)$ metric on the complex.

A *midcube* of a cube $[-1,1]^n$ is the subspace defined by setting a single coordinate to 0. A *hyperplane* in a $\mathrm{CAT}(0)$ cube complex is a connected, nonempty subspace which intersects each cube in a midcube. Complements of hyperplanes have two connected components. The partitions induced by hyperplanes encode the structure of a $\mathrm{CAT}(0)$ cube complex. If a group action preserves a fixed collection of partitions, then one recovers an action on a "dual" $\mathrm{CAT}(0)$ cube complex using a construction of Sageev.

Given a finite set $S = \{s_1, \dots, s_n\}$ of generators, there are $\approx (2n-1)^L$ reduced words of length L over $S \cup S^{-1}$. For some $d \in (0,1)$, a *random group at density d and length L* is given by a presentation $\langle S \mid R \rangle$, where R is a collection of $(2n-1)^{dL}$ reduced words of length L chosen uniformly at random. One says a random group at a particular density satisfies property Q if the probability of Q approaches 1 as $L \to \infty$. Although many random groups act without a global fixed point on finite dimensional cube complexes, it is conjectured that the dimension of these cube complexes must grow with L. Working in this direction, I proved the following theorem.

Theorem. *Let $G = \langle S \mid R \rangle$ be a random group at any density with $|S| \geq 7$. Any action of G on a $\mathrm{CAT}(0)$ square complex has a global fixed point.*

Proving the above theorem involved defining a notion of *progression* for a regular language over S given an action of a free group $F_S \to \mathrm{Aut}(X)$ on a $\mathrm{CAT}(0)$ cube complex X. I believe these ideas can generalize to higher dimensions at the cost of a stronger bound on the number of generators. I am interested in the following:

Problem. Generalize the Theorem to high dimensional cube complexes.

[1] *Random group actions on* CAT(0) *square complexes*; Zachary Munro; submitted for publication; arXiv:2210.06378

[2] *Random groups do not split*, Francois Dahmani and Vincent Guirardel and Piotr Przytycki, Mathematische Annalen 349 no.3 (2011), 657-673.

Raquel Murat García
Westfälische Wilhelms-Universität Münster (WWU), Germany

Master's student application

As I am still a Master student, I have not yet begun my own research. However, I have been attending courses and seminars on GGT since I started my studies at Münster and plan on applying for PhD positions related to this field, and so it would be really helpful for me to attend this conference, both to learn more about what I already know and to discover current research topics on the area.

In my first year at WWU I attended two courses on GGT covering topics like free groups, Serre graphs and trees, group presentations, amalgamation, HNN-extensions, CAT(0) spaces and cube complexes, Right angled Artin groups, hyperplanes and halfspaces and cubulation of Coxeter groups. Besides, in the present semester I am taking part on a seminar on topics on GGT, where I already gave a talk on Small Cancellation Theory.

Ever since I started attending the first course on it, I enjoyed GGT very much, as it covers many of the areas I have always found most interesting, such as Topology, Group Theory and even Logic. Among the topics I have already come across, what I found most intriguing were graphs and trees, Coxeter groups, Right angled Arting groups and Small Cancellation Theory. Moreover, I would like to learn more about the relation that GGT has with Geometric Topology, in particular about topics like mapping class groups, braid groups or knot theory.

Jake Murphy

Louisiana State University, United States of America

Subgroups of Fundamental Groups of Non-positively Curved Cube Complexes

My research has been generalizing the results of Kapovich and Myasnikov in *Stallings Foldings and Subgroups of Free Groups* to fundamental groups of non-positively curved cube complexes. In particular, I have been using the idea of completions found in *Subgroups of Right-Angled Coxeter Groups Via Stallings-Like Techniques* by Dani and Levcovitz and *Folding-like Techniques for CAT(0) Cube Complexes* by Ben-Zvi, Kropholler, and Lyman. In the case of a non-positively curved cube complex X, the completion of a subgroup $H \leq \pi_1(X)$ creates a map $f : M \mapsto X$, such that the induced homomorphism of fundamental groups is an inclusion with $f_*(\pi_1(M)) \cong H$. We can then study the cube complex M to determine various properties of H.

I have also been interested in techniques in *Coxeter Groups, 2-Completion, Perimeter Reduction and Subgroup Separability* by Schupp and how they can be applied to a larger class of groups.

Jean Pierre Mutanguha

Princeton University, USA

Dynamics of Free Group Automorphisms

The mapping torus of a free group automorphism is also known as a free-by-cyclic group. I am particularly interested in how the dynamics of an automorphism determine the geometry of its mapping torus and vice-versa. For instance, Peter Brinkmann proved that the automorphism is *atoroidal* — i.e. has no periodic conjugacy class of nontrivial elements — if and only if the mapping torus is hyperbolic.

I would like to give a similar geometric characterization of an automorphism that is atoroidal and *fully irreducible* — i.e. has no periodic conjugacy class of nontrivial proper free factors. To this end, I am developing canonical forms for free group automorphisms that might lead to canonical geometric decompositions of free-by-cyclic groups.

Patrick Nairne

University of Oxford, United Kingdom

Quasiisometries and regular languages

I am interested in finding connections between geometric (i.e. invariant under quasi-isometry) and computational properties of finitely generated groups. A nice result of this form is this: a finitely generated group G is hyperbolic if and only if the (λ, ϵ)-quasigeodesics form a regular language for all $\lambda \in \mathbb{Q}_{\geq 1}$ and $\epsilon \in \mathbb{R}_{\geq 0}$. This follows from work of Holt and Rees [1] and Hughes, Nairne and Spriano [2].

I also like all things related to coarse/quasisometric/bilipschitz/rough-isometric embeddings between groups and metric spaces. In the paper [3], I looked at quasiisometric embeddings between solvable Baumslag-Solitar groups, and more generally between treebolic spaces.

Recently, inspired by the results of Buyalo, Dranishnikov and Schroeder [4], I have been wondering about when you can quasiisometrically embed groups into products of binary trees.

[1] “Regularity of quasigeodesics in a hyperbolic group”, Derek F. Holt and Sarah Rees, International Journal of Algebra and Computation 13.5 (2003)

[2] “Regularity of quasigeodesics characterises hyperbolicity”, Sam Hughes, Patrick S. Nairne and Davide Spriano, https://arxiv.org/abs/2205.08573 (2022)

[3] “Embeddings of Trees, Cantor Sets and Solvable Baumslag-Solitar Groups”, Patrick S. Nairne, https://arxiv.org/abs/2204.03983 (2022)

[4] “Embedding of hyperbolic groups into products of binary trees”, Sergei Buyalo, Alexander Dranishnikov and Viktor Schroeder, Inventiones Mathematicae 169 (2007)

Merik Niemeyer

Ruprecht-Karls-Universität Heidelberg, Germany

Fock-Goncharov coordinates for representations into $\mathrm{SO}(p, q)$

I am a second-year PhD student in Heidelberg where my advisor is Anna Wienhard. Broadly speaking, my research falls into the area of Higher Teichmüller Theory. The (classical) Teichmüller space can be viewed as the space of all marked hyperbolic structures on a given topological surface S (of negative Euler characteristic). From a more algebraic point of view, it is a connected component of the representation variety

$$\mathrm{Hom}(\pi_1(S), \mathrm{PSL}(2, \mathbb{R}))/\mathrm{PSL}(2, \mathbb{R})$$

which consists entirely of discrete and faithful representations. You can imagine that endowing the surface S with a hyperbolic structure means that its Riemannian universal cover is isometric to the hyperbolic plane $\mathbb{H}^2$ and its fundamental group acts by isometries. Higher Teichmüller Theory is the following generalisation of this: A higher Teichmüller space is a connected component of the representation variety

$$\mathrm{Hom}(\pi_1(S), G)/G$$

for some (usually higher rank, real) Lie group G which contains only discrete and faithful representations. There are several examples of these, for instance they are known to exist when G is a so-called split real group.

In the case where S has some punctures (or boundary) and G is split real, Fock and Goncharov [1] introduced a framework for coordinates on the representation variety and gave a complete construction of these for $G = \mathrm{SL}(n, \mathbb{R})$. Interestingly, these coordinates carry a cluster structure, meaning that different coordinate systems can be obtained from each other by moves called cluster mutations. These preserve positivity of the coordinates, meaning that the set of positive representations is well-defined. In fact, this is a higher Teichmüller space.

All known examples of higher Teichmüller spaces can be understood using the notion of Θ-positivity introduced by Guichard and Wienhard [2], which also predicts new classes of examples.

One of these is the case $G = \mathrm{SO}(p, q)$ and I am currently part of a project in which we are constructing Fock-Gonachrov coordinates for these representations. An interesting aspect of this is that it should provide us with a new class of non-commutative cluster algebras, which we are currently investigating.

[1] V. Fock & A. Goncharov, *Moduli Spaces of Local Systems and Higher Teichmüller Theory*, Publ. Math. Inst. Hautes Études Sci. **103**, 2006.

[2] O. Guichard & A. Wienhard, *Positivity and Higher Teichmüller Theory*, Proceedings of the 7th European Congress of Mathematics, 2016.

Martin Nitsche
Karlsruhe Institute of Technology, Germany

Human property (T) proofs based on the SDP-approach

A discrete group Γ with finite generating set $S = S^{-1} \subset \Gamma$ has Kazhdan's property (T) if for any group representation $\rho\colon \Gamma \to \mathcal{B}(H)$ the operator $\sum_{s\in S} \mathbf{1} - \rho(s)$ has vanishing spectrum in the interval $(0, \varepsilon)$, for some $\varepsilon > 0$ not depending on ρ. This is a much studied rigidity property, but proving it for a given group is typically difficult.

In 2016, Ozawa [1] established a new method to prove property (T) via semidefinite programming (SDP), by finding with the computer a decomposition of the group Laplacian $\Delta := \sum_{s\in S} \mathbf{1} - s \in \mathbb{R}[\Gamma]$ as a sum $\sum_i x_i{}^* x_i$, $x_i \in \mathbb{R}[\Gamma]$. Starting with the case of $n = 5$ [2], this new method was successfully applied to prove property (T) for the automorphism groups of the free groups $\mathrm{Aut}(F_n)$, $n \geq 4$. To date, this is the only known property (T) proof for $\mathrm{Aut}(F_n)$.

The computer generated decomposition is huge and does not exhibit any recognizable pattern. I am currently interested in extracting information from the computer proof, specifically for the groups $\mathrm{Aut}(F_n)$ as $n \to \infty$. Ideally, I hope to obtain a decomposition of Δ as above that is fully motivated by human understanding of the group instead of a brute force or numerical computer search.

One way to approach this task is to study the dual problem associated to the property (T) SDP, which is to prove that non-trivial 1-cocycles on Γ cannot be harmonic. With this dual perspective the algebraic decomposition problem becomes amenable to geometric intuition [3]. The restriction to the asymptotic case $n \to \infty$ leads to simplifications that can be more easily expressed in the dual.

Aside from group Laplacians, I am also interested in applying similar methods to study the spectral properties of other operators that can be expressed in terms of group algebras.

[1] N. Ozawa. Noncommutative real algebraic geometry of Kazhdan's property (T). *J. Inst. Math. Jussieu*, 15:85–90, 2016.

[2] M. Kaluba, P. Nowak and N. Ozawa. $\mathrm{Aut}(\mathbb{F}_5)$ has property (T). *Math. Ann.*, 375:1169–1191, 2019.

[3] M. Nitsche. Computer proofs for Property (T), and SDP duality. ArXiv:2009.05134.

Eduardo Oregón-Reyes
University of California, Berkeley, United States

An extension of Teichmüller/Outer space for hyperbolic groups

I like studying discrete groups acting isometrically on non-positively curved spaces. This last year, I've been thinking of a version of Teichmüller/Outer space for arbitrary hyperbolic groups.

Given a non-elementary hyperbolic group Γ, we consider the set of all the left-invariant, hyperbolic pseudo metrics on Γ that are quasi-isometric to a word metric. These pseudo metrics are exactly the ones induced by (looking at orbits of) proper and cobounded isometric actions of Γ on geodesic hyperbolic metric space. The *space of metric structures* is $\mathfrak{D}_\Gamma$, the quotient of this set under the equivalence relation of rough similarity, which we equip with a metric inspired by Thurston's distance on Teichmüller space.

We can see $\mathfrak{D}_\Gamma$ as a (thick) version of Teichmüller/Outer space valid for an arbitrary hyperbolic group. Indeed, if Γ is a hyperbolic surface group, then $\mathfrak{D}_\Gamma$ contains a copy of Teichmüller space, and similarly, $\mathfrak{D}_\Gamma$ contains the Outer space when Γ is free. This space encodes many other interesting actions and constructions, such as word metrics, Green metrics, marked negatively curved Riemannian metrics, geometric cubulations, geodesic currents, and Anosov representations. There are lots of interesting questions about how all these sets interact inside $\mathfrak{D}_\Gamma$, and I'd be happy to discuss them!

All the actions mentioned above have different flavors, but they fit together nicely since $\mathfrak{D}_\Gamma$ is contractible, separable, and unbounded (even when $\mathsf{Out}(\Gamma)$ is finite!). Moreover, with Stephen Cantrell, we showed that $\mathfrak{D}_\Gamma$ is geodesic and that there are plenty of bi-infinite geodesics, which we used to define a boundary $\partial\mathfrak{D}_\Gamma$ for $\mathfrak{D}_\Gamma$. There are some natural questions about $\mathfrak{D}_\Gamma$ that one may ask: is it uniquely geodesic? is it convex? is it non-positively curved somehow?

I'm also interested in studying the action of $\mathsf{Out}(\Gamma)$ on $\mathfrak{D}_\Gamma$ induced by pullback, which is isometric and proper. I presume there is a Nielsen-Thurston classification for elements of $\mathsf{Out}(\Gamma)$ in terms of the dynamics of the action on $\mathfrak{D}_\Gamma \cup \partial\mathfrak{D}_\Gamma$, and I wonder to what extent we can extend this classification to arbitrary subgroups of $\mathsf{Out}(\Gamma)$.

[1] S. Cantrell, E. Oregón-Reyes. Manhattan geodesics and the boundary of the space of metric structures on hyperbolic groups. Preprint (2022), arXiv:2210.07136.

[2] E. Oregón-Reyes. The space of metric structures on hyperbolic groups. Journal of the LMS, to appear. arXiv:2204.12545.

Panagiotis Papadopoulos
LMU München, Germany

Handlebody Groups and Disk Complexes

The *handlebody group* $\mathcal{H}_g$ is defined as the mapping class group of a three-dimensional handlebody V_g of genus $g \geq 0$. These groups have been shown to have similarities and differences, both strong, with mapping class groups of surfaces and Out(F_n) (see [5] for a survey of such results). In an attempt to transfer ideas from the successful study of mapping class groups of surfaces via their action on curve complexes, an analogous complex, called the disk complex, has been defined for the handlebody group. The *disk complex* $\mathcal{D}(V_g)$ is defined as the flag complex whose vertices correspond to isotopy classes of *meridians*, i.e. essential simple closed curves on ∂V_g that bound a disk embedded in V_g, with two vertices connected by an edge, if there are disjoint representatives in the respective isotopy classes. The disk complexes $\mathcal{D}(V_g)$ have been shown to have infinite diameter and to be Gromov hyperbolic for $g \geq 2$ [4, 7]. In the case of curve complexes, an even stronger statement has been proven, namely that the curve complexes $\mathcal{C}(S_g)$ are uniformly hyperbolic for $g \geq 2$, in the sense that the hyperbolicity constant is independent of the genus g of the surface [1–3, 6, 8]. Uniform hyperbolicity of the disk complexes, however, remains unknown, and is the problem I am currently most interested in.

[1] Aougab, T. (2013). Uniform hyperbolicity of the graphs of curves. *Geometry & Topology, 17*(5), 2855-2875.

[2] Bowditch, B. (2014). Uniform hyperbolicity of the curve graphs. *Pacific journal of mathematics* 269.2, 269-280.

[3] Clay, M., Rafi, K., & Schleimer, S. (2015). Uniform hyperbolicity of the curve graph via surgery sequences. *Algebraic & Geometric Topology, 14*(6), 3325-3344.

[4] Hamenstädt, U. (2016). Hyperbolic relatively hyperbolic graphs and disk graphs. *Groups, Geometry, and Dynamics, 10*(1), 365-405.

[5] Hensel, S. (2018). A primer on handlebody groups. *Handbook of Group Actions, 14,* 143-177.

[6] Hensel, S., Przytycki, P., & Webb, R. C. (2015). Slim unicorns and uniform hyperbolicity for arc graphs and curve graphs. *Journal of the European Mathematical Society, 17*(4), 755–762.

[7] Masur, H., & Schleimer, S. (2013). The geometry of the disk complex. *Journal of the American Mathematical Society, 26*(1), 1-62.

[8] Przytycki, P., & Sisto, A. (2015). A note on acylindrical hyperbolicity of mapping class groups. *arXiv preprint arXiv:1502.02176.*

Irene Pasquinelli
University of Bristol, UK

Complex hyperbolic lattices acting on spaces

My current projects are in several areas: low-dimensional topology, hyperbolic geometry, dynamics on translation surfaces and geometric group theory. The common underlying topic is hyperbolic geometry and group actions. While I am not so early career, only this year, after moving to Bristol, I have been learning about several tools in geometric group theory and looking for new project which can use both my expertise in complex hyperbolic geometry and the topics I have learnt about.

I. *Complex hyperbolic geometry:* I study lattices in $PU(n, 1)$, the space of holomorphic isometries of complex hyperbolic space. I have built fundamental domains for some known lattices and look at representations of them in $PGL(3, \mathbb{C})$. I am now working on ways to build new lattices using either complex hybridisation or moduli spaces of flat surfaces of low genus. Moreover, together with Mark Hagen and Thomas Ng, I have started investigating what interesting questions can arise by looking at complex hyperbolic reflection groups acting on, for example, hierarchically hyperbolic spaces or $CAT(0)$ cube complexes.

II. *Asymptotic growth of closed geodesics on non-orientable surfaces:* On an orientable surface, the number of closed curves with bounded length grows exponentially when the length increases. A celebrated result of Mirzakhani tells us that if we restrict to curves of a given mapping class group orbit, the growth is polynomial. Very little is known about the non-orientable case. With Gendulphe, Erlandsson and Souto we explored some of the tools from the orientable case that fail, namely the action of the mapping class group on the space of (projective) measured laminations. We are now working towards an asymptotic growth theorem in the spirit of Mirzakhani.

III. *Symbolic coding of geodesic flow on translation surfaces:* Starting from a polygonal representation for the translation surface, it is possible to code linear trajectories using labels of sides of the polygons. It is then natural to ask how to characterise the set of sequences obtained in this way from trajectories and whether one can recover the angle of the original trajectory. This has been studied in the case of regular polygons and in a joint work with Davis and Ulcigrai, I characterised the sequences in a class of translation surfaces called Bouw-Möller surfaces. This also has an interpretation in terms of the Teichmüller space of deformations of the surface. I am now working with Corinna Ulcigrai to generalise this to a generic Veech surface.

Ludovic Pedro de Lemos
Westfälische Wilhelms-Universität Münster, Germany

Master student registration

As I am currently doing my Master's Degree, I am not yet conducting any research. I have, however, attended 2 courses on GGT so far and have been taking part on a GGT seminar this semester, and would very much enjoy seeing what higher level research in the area looks like. My main interest in GGT is its intersection with the field of geometric topology.

Leon Pernak
Universität des Saarlandes, Germany

Computational properties of Self-similar groups

In 1980 Rostislav I. Grigorchuk constructed in [1] a group which served as an interesting example in several ways: The group has intermediate growth, i.e. grows stronger than polynomially but less than exponentially, answering a question by Milnor. It also is an infinte finitely generated torsion group, which is a counter-example to a problem famously proposed by Burnside in 1902. Finally, the group is amenable, but not elementary amenable.

One construction of the Grigorchuk group is as a group acting on a regular rooted tree. More precisely, let T be the infinite n-ary tree with distinguished root vertex v. The group of graph automorphisms of $\mathcal{T}$ that fix v is denoted by $Aut(\mathcal{T})$. There are several ways to describe elements and subgroups of $Aut(\mathcal{T})$, for example through input/output automata or their actions on words over an alphabet preserving prefixes. Another way is a recursive definition like the following:

Fix $n = 2$ and define the graph automorphism $a : \mathcal{T} \to \mathcal{T}$ to be the permutation of the children of v and their respective subtrees. Further, define recursively

$$b = (a, c), c = (a, d), d = (id, b)$$

where $x = (y, z)$ defines an automorphism on $\mathcal{T}$ which acts on the subtree below the left child of v as y and on the subtree below the right child as z. The group generated by a, b, c, d is called the Grigorchuk group and denoted by $\mathfrak{G}$.

The recursive structure of above definition naturally leads to a computational perspective. It turns out that for example, both the word and the conjugacy problem are decidable over $\mathfrak{G}$. Even further, Lysenok, Miasnikov and Ushakov showed in [3] that there is an algorithm that solves any quadratic equation in an arbitrary number of variables over $\mathfrak{G}$.

I am interested in applying the methods which turned out to be successful in studying algorithmic problems of the Grigorchuk group to a wider array of groups acting on rooted trees, among them branch groups, weakly branch groups and groups of functionally recursive automorphisms. Besides, I am generally curious about using geometric methods for solving algorithmic problems over groups.

[1] Grigorčuk, R.I.: “On Burnside’s problem on periodic groups”, (Russian) Funktsional. Anal. i Prilozhen. 14 (1980), no. 1, 53–54.

[2] Lysenok, I., Miasnikov, A., Ushakov, A.: “Quadratic equations in the Grigorchuk group”, arXiv:1304.5579

Felix Physiker
Universität Greifswald, Germany

Macroscopic invariants of manifolds

A Riemannian manifold (M, g) has macroscopic dimension at most n ($\dim_{mc} M \leq n$) if there is a continuous map $g\colon M \to K^n$ to a n-dimensional simplicial complex K^n with a uniform upper bound b on the size of preimages : $diam(g^{-1}(y)) < b$ for all $y \in K^n$. This concept was originally introduced by Gromov [1] to phrase the conjecture: *a closed n-manifold M which admits a Riemanninan metric with positive scalar curvature satisfies* $\dim_{mc} \tilde{M} \leq n-2$.

I am interested in homological descriptions of macroscopic dimension, examples, non-examples and other related discussion in this area. For example, Dranishnikov [2] proved that for a closed oriented n-manifold with fundamental group G and classifying map $f\colon M \to BG$, $\dim_{mc} \tilde{M} \leq n-1$ is equivalent to the condition that the image of the fundamental class $f_*([M])$ lies in a particular subgroup $H_n^{\mathrm{sm}}(BG)$ of $H_n(BG)$. [3] In this case, the lift of f to the universal covers $\tilde{f}\colon \tilde{M} \to EG$ can be deformed to a map with image in the $n-1$ skeleton of EG by a bounded homotopy for any choice of proper geodesic metric on BG and the image of $[\tilde{M}]$ in locally finite homology $\tilde{f}_*([\tilde{M}]) \in H_n^{lf}(EG; \mathbb{Z})$ is zero.

With this characterisation of "smallness" one can investigate how macroscopic dimension of submanifolds of codimension $q \geq 1$ relate to the macroscopic dimension of M. Engel proved in [4] that under certain conditions there is a "wrong way map" $H_*(M) \to H_{*-q}(N)$ which induces a map $H_*(BG) \to H_{*-q}(B\pi_1 N)$ sending $H_n^{\mathrm{sm}}(BG)$ to $H_{n-q}^{\mathrm{sm}}(B\pi_1 N)$. I would like to see where these maps send higher codimensional obstruction classes as well as generalise the notion to other spaces (for example, CAT(0) spaces or systolic complexes).

[1] M. Gromov, *Positive curvature, macroscopic dimension, spectral gaps and higher signatures*, Functional analysis on the eve of the 21st century, Vol II (New Brunswick, NJ, 1993), Progress in Mathematics 132 Birkhäuser Boston, 1996, pp. 1-213

[2] A. N. Dranishnikov *On macroscopic dimension of universal coverings of closed manifolds*, Tr. Mosk. Mat. Obs. **74** (2013), no. 2, 279-296

[3] M. Brunnbauer, B. Hanke, *Large and small group homology*, J. Topol. **3** (2010), no. 2, 463-486

[4] A. Engel, *Wrong way maps in uniformly finite homology and homology of groups*, J. Homotopy Relat. Struct. **13** (2018), no. 2, 423-441

Amethyst Price
University of California Santa Cruz, USA

Quaternionic Triangle Groups and Quadratic Forms

My research is at the intersection of group theory, geometry, and arithmetic. Specifically, my thesis is related to arithmetic triangle groups - groups which on one hand are generated by reflections in the sides of a triangle in the spherical or hyperbolic plane, and on the other hand are isomorphic to an arithmetic group. For each arithmetic triangle group, I hope to (1) connect the geometry of a triangular tiling to the arithmetic of quaternions, and (2) apply this to classical number theory, e.g., variations on quadratic forms.

The prototype is found in the work of J.H. Conway (Ch. 1, *Sensual Quadratic Form*). In his case, the arithmetic group is $PGL_2(\mathbb{Z})$, which is isomorphic to the $(2,3,\infty)$ triangle group. The quaternion algebra is the "split" one, i.e., the matrix algebra $M_2(\mathbb{Z})$. The triangular tiling of the hyperbolic plane has an arithmetic connection, with vertices, edges, faces connected to what Conway calls primitive lax vectors, lax bases, and lax superbases in $\mathbb{Z}^2$. In this way, the shape of the tiling tells us about $\mathbb{Z}$-bases of $\mathbb{Z}^2$ and how they overlap. Conway applies this to largely replace the challenging work of Gauss (*Disquisitiones*, 1801) on binary quadratic forms with a straightforward and geometric approach.

My current work seeks analogues for the (ℓ, m, n) triangle group,

$$\Delta(\ell,m,n) = \langle e,f,g | e^2 = f^2 = g^2 = (ef)^\ell = (fg)^m = (ge)^n = -1 \rangle.$$

When such a triangle groups is arithmetic, Takeuchi connects it to a number field F, and a quaternion algebra B/F. I seek to go deeper, identifying a specific quaternion order $A \subset B$ over the ring of integers $\mathcal{O} \subset F$, a specific kind of $\mathcal{O}$-basis of A such that $A = \mathcal{O} + \mathcal{O}\alpha + \mathcal{O}\beta + \mathcal{O}\gamma$, and a specific kind of isomorphism $PA^\times \cong \Delta^+(\ell, m, m)$ (the even triangle group). This stage of my research is nearing completion in the spherical case and the hyperbolic case is next.

With this connection established, I pursue the analogue of Conway's lax vectors, bases, and superbases. This is more difficult, as the vertices, edges, and triangles seem related to certain roots of unity in the quaternion order A, and at the same time to quadratic (over $\mathcal{O}$) subalgebras of the quaternion order A. This should elucidate the precise connection between the geometry of the triangle group and the arithmetic of the quaternion algebra, and is my goal for the next year of research.

Finally, just as Conway's "topograph" (the tiling of type $(2,3,\infty)$) was useful in the theory of binary quadratic forms, I hope that the other triangular tilings shed light on a different sort of binary quadratic form. Just as Conway's topograph relates to the classification of binary quadratic forms under $GL_2(\mathbb{Z})$-equivalence, I expect the triangular tilings to relate to the classification of trace-zero quaternions in the order A, under a twisted conjugation action ($\alpha \mapsto g\alpha\bar{g}$).

José Pedro Quintanilha
Universität Bielefeld, Germany

Extending Sigma-invariants to locally compact topological groups

The features of a group being finitely generated or finitely presented are well-known to be, respectively, the $n = 1$ and $n = 2$ cases of the property F_n: a group G is said to be of type F_n if it admits a classifying space $\mathrm{K}(G, 1)$ with finite n-skeleton. Towards the end of the last century, the question of when such finiteness conditions descend to subgroups of G led to the discovery of the homotopical Sigma-invariants $\Sigma^n(G)$ and their homological counterparts $\Sigma^n(G; A)$ (for A a $\mathbb{Z}G$-module). Intuitively, $\Sigma^n(G)$ can be thought of as the set of group homomorphims $\chi\colon G \to \mathbb{R}$ for which G is "of type F_n in the direction of χ", with the classical property F_n being equivalent to $0 \in \Sigma^n(G)$. In the literature, Sigma-invariants are often called BNSR-invariants due to Bieri, Neumann, Strebel and Renz [1–3], who pioneered the theory.

If G is equipped with a locally compact topology, one can consider the "compactness properties" C_n introduced by Abels and Tiemeyer [4], which specialize to F_n in the discrete case. Kouchloukova has generalized the properties $\mathrm{C}_1, \mathrm{C}_2$ to invariants $\Sigma^1_{\mathrm{top}}, \Sigma^2_{\mathrm{top}}$ [5], and one of my current interests is in studying a uniform definition of Σ^n_{top}, and extending classical results of Sigma-theory to these more general invariants.

I am also interested in geometric topology, and have worked on questions related to knot theory [6] and spatial graphs [7].

[1] R. Bieri, W. Neumann, R. Strebel, *A geometric invariant of discrete groups*, Invent. Math. 90, 451–477 (1987)

[2] R. Bieri, B. Renz, *Valuations on free resolutions and higher geometric invariants of groups.* Comment. Math. Helv. 63, No. 3, 464–497 (1988)

[3] B. Renz, *Geometrische Invarianten und Endlichkeitseigenschaften von Gruppen*, doctoral dissertation available on the author's website: `https://esb-dev.github.io/publ.html` (1988)

[4] H. Abels, A. Tiemeyer, *Compactness properties of locally compact groups*, Transform. Groups 2, No. 2, 119–135 (1997)

[5] D. Kochloukova, *On the topological invariants Σ^1_{top} and Σ^2_{top} for extensions of (Lie groups over a p-adic field)-by-abelian groups*, J. Algebra 282, No. 2, 538–574 (2004)

[6] S. Friedl, C. Kausik, J. P. Quintanilha, *An algorithm to calculate generalized Seifert matrices*, J. Knot Theory Ramifications 31, No. 11, Article ID 2250068 (2022)

[7] S. Friedl, L. Munser, J. P. Quintanilha, Y. Santos-Rego, *Canonical decompositions and algorithmic recognition of spatial graphs*, arXiv:2105.06905 [math.GT]

Karthika Rajeev

University of Bielefeld, Germany

Asymptotic group theory and groups acting on rooted trees

My research focuses on asymptotic group theory and geometric group theory. I find interest in investigating problems in asymptotic group theory, particularly in understanding arithmetic and analytic properties of zeta functions encoding group theoretical data, which led to my work on the emerging field of representation growth of groups acting on infinite rooted trees. My PhD thesis contains a study of the asymptotic distribution of finite-dimensional irreducible complex representations of groups acting which acts faithfully on infinite rooted trees and exhibit strong self-similarity features.

My primary expertise revolves around groups acting on rooted trees. Inspired by the Basilica group, together with Petschick, in [1], we introduce a general construction called the *Basilica operation* that produces an infinite family of *Basilica groups* from a given group of automorphisms of a rooted tree. We investigate which properties of groups of automorphisms of rooted trees are preserved under the Basilica operation. For groups that display strong self-similarity features, we develop new techniques for computing their Hausdorff dimension, which is generally difficult to calculate. Furthermore, we investigate an analogue of the classical congruence subgroup problem, which is studied in the context of arithmetic groups. In [2], together with Thillaisundaram, we study maximal subgroups of certain Basilica groups and prove that they are of finite index in the corresponding Basilica groups

[1] J. M. Petschick and K. Rajeev, On the Basilica operation, Groups Geom. Dyn., to appear, available at arXiv:2103.05452.

[2] K. Rajeev and A. Thillaisundaram, Maximal subgroups of Basilica groups, available at arXiv:2109.12503.

Emmanuel Rauzy
Saarland University, Germany

Decision problems for word problem algorithms and other descriptions of groups

Global decision problems for groups take the form: is there an algorithm that, given a certain description of a group, determines if this group has such and such properties?
The study of decision problems for groups goes back to Max Dehn, who was motivated by the study of manifolds via their fundamental group. However, as the fundamental group of a manifold is most often computed via a finite presentation, Dehn introduced decision problems not in full generality, but only for finite presentations.
My work deals with decision problems for other descriptions of groups, and in particular for groups described by word problem algorithms. This is equivalent to considering groups described by their labelled Cayley graph (i.e. with edges marked by generators), and thus one tries to answer the problem:

- What can we say about a group given by an algorithm that produces its labelled Cayley graph?

This problem turns out to be very interesting for several reasons. In particular, it must be studied thanks to the topology of the space of marked groups, as in practice semi-decidable sets correspond to open subsets of the space of marked groups. There are several theories that should be able to help us understand the link between computability on the space of marked groups and topology: one could hope to apply results of *effective descriptive set theory* ([2]), or continuity results of *computable analysis*, for example from [3].

However, the space of marked groups is a Polish space that is not an "effectively Polish space", and my work has consisted in showing that many results one could have hoped to apply to the space of marked groups in fact do not apply there.
A question of particular importance is the continuity problem on the space of marked groups: must the computable functions defined on the space of marked groups be continuous?

[1] Emmanuel Rauzy, Computable analysis on the space of marked groups, 2021.

[2] Emmanuel Rauzy, Remarks and problems about algorithmic descriptions of groups, 2021.

[3] Yiannis N. Moschovakis, Descriptive set theory. Studies in Logic and the Foundations of Mathematics, 1980

[4] Ceitin, G.S. Algorithmic operators in constructive metric spaces. 1962

Rebecca Rechkin

University of Utah, United States

Big Mapping Class Groups

I am a third year PhD student at the University of Utah. I am broadly interested in big mapping class groups. I have studied topics from [1] including curves, surfaces, hyperbolic geometry, the Milnor-Schwarz lemma, mapping class group basics, generation of the mapping class group by Dehn twists, lantern relations and first homology of Map(S), and the Dehn-Nielson-Baer theorem under the guidance of my advisor Priyam Patel. During this last year I have been working on a expository paper *Thurston's Theorem: Entropy in Dimension One*[2], which is available on the archive at [5]. I have also read *The First Integral Cohomology of Pure Mapping Class Groups* [3] earlier this semester and am currently reading *Large Scale Geometry of Big Mapping Class Groups* [4] with graduate student peers. This semester I have also been reading about Outer Space under the guidance of Mladen Bestvina. By the end of the summer I hope to be working on a research problem.

[1] Farb B Margalit D. *A Primer on Mapping Class Groups.* Princeton: Princeton University Press; 2012. doi:10.1515/9781400839049

[2] Thurston, William P. “Entropy in Dimension One.” arXiv: Dynamical Systems (2014): n. pag.

[3] Javier Aramayona, Priyam Patel, Nicholas G Vlamis, The First Integral Cohomology of Pure Mapping Class Groups, International Mathematics Research Notices, Volume 2020, Issue 22, November 2020, Pages 8973–8996, https://doi.org/10.1093/imrn/rnaa229

[4] Mann, Kathryn, and Kasra Rafi. “Large scale geometry of big mapping class groups." arXiv preprint arXiv:1912.10914 (2019).

[5] Dickmann, R., Domat, G., Hill, T., Kwak, S., Ospina, C., Patel, P., & Rechkin, R. (2022). Thurston's Theorem: Entropy in Dimension One. arXiv preprint arXiv:2209.15102.

Ulysse Remfort-Aurat
Aix-Marseille University, France

Discontinuity domains on $PSL_2(\mathbb{C})$ character variety

Let Γ be a hyperbolic group, $G = PSL_2(\mathbb{C})$ the isometry group of the 3-dimensional hyperbolic space $\mathbb{H}^3$ and $X(\Gamma, G)$ be the space of conjugacy classes of homomorphism form Γ to G. There is an action of the outer automorphism group $Out(\Gamma)$ on $X(\Gamma, G)$ and it is well-known that $Out(\Gamma)$ preserves the open set $CC(\Gamma, G)$ of convex-cocompact representations and acts on it properly discontinuously.

A natural question that arises from this fact is (Q) : can we find an open set of $X(\Gamma, G)$, which is $Out(\Gamma)$-invariant, contains strictly the set of convex-cocompact representations, and on which the action of $Out(\Gamma)$ is properly discontinuous ?

On one hand, when Γ is the fundamental group of a closed, connected and orientable surface of genus $g > 2$, Goldman [1] conjectured that the action of $Out(\Gamma)$ on $X(\Gamma, G) \setminus CC(\Gamma, G)$ is ergodic. Hence, if this conjecture happens to be true, we would have a negative answer to (Q).
On the other hand, if Γ is a free group, Minsky [2] introduced the set $PS(\Gamma, G)$ of *primitive-stable representations* which has all the wanted properties to give a positive answer to (Q).

I am currently trying to give a positive answer to (Q) for a large class of torsion-free, one ended hyperbolic group using the canonical JSJ decomposition and the ideas from [3].

[1] W. Goldman, "Topological components of spaces of representations,"

[2] Y. Minsky, "On dynamics of $Out(F_n)$ on $PSL_2(\mathbb{C})$ characters"

[3] R. Canary, M. Lee, M. Stover "Amalgam Anosov representations"

Anna Ribelles Pérez

LMU Munich, Germany

Subgroup distortion in the mapping class group

Let G be a finitely generated group, with some generating set S_G. If G is a subgroup of another finitely generated group H, and we extend S_G to a finite generating set S_H of H, we can view $\mathrm{Cay}(G)$ as a subset of $\mathrm{Cay}(H)$. Therefore $\|g\|_G \geq \|g\|_H$ for every $g \in G$, since the ambient graph contains more edges and, potentially, some shortcuts between vertices in G. More generally, for arbitrary finite generating sets, this inequality will hold up to a multiplicative constant, i.e. $\|g\|_G \geq C \cdot \|g\|_H$.

The converse, however, is not always true; for certain pairs of groups $G < H$, the shortcuts we take in the ambient graph can be very efficient, so the embedding of G in H is not quasi-isometric. In that case, we say that G is *distorted* in H. For example, G is at least exponentially distorted in H if there exists a sequence $(g_n) \subseteq G$ such that $\|g_n\|_G$ grows exponentially in n while $\|g_n\|_H$ grows only linearly.

Note that distortion is a form of worst-case behavior. A group can have distorted subsequences and yet the subgroup norm of other elements can be comparable to the ambient group norm. Given some element g of an e.g. exponentially distorted G in H, one can ask what its H-norm is, and hope to obtain a finer estimate than the bounds known from the exponential distortion.

I am interested in studying the norm of point-pushing maps in the mapping class group of a punctured surface S with $\chi(S) < 0$: in the Birman exact sequence [1]

$$1 \to \pi_1(S, x_0) \overset{\mathrm{Push}}{\to} \mathrm{MCG}(S, x_0) \to \mathrm{MCG}(S) \to 1$$

the image of the fundamental group is exponentially distorted in $\mathrm{MCG}(S, x_0)$ [2]. I would like to understand $\|\mathrm{Push}(\gamma)\|_{\mathrm{MCG}(S,x_0)}$ in terms of the information that can be extracted from the curve γ.

[1] Birman, J.S. (1969), Mapping class groups and their relationship to braid groups. Comm. Pure Appl. Math., 22: 213-238. https://doi.org/10.1002/cpa.3160220206

[2] Nathan Broaddus, Benson Farb, Andrew Putman, Irreducible Sp-representations and subgroup distortion in the mapping class group. Comment. Math. Helv. 86 (2011), no. 3, pp. 537–556 DOI 10.4171/CMH/233

José Andrés Rodríguez Migueles

LMU Munich, Germany

Geodesics on hyperbolic 2-orbifolds and knots complements

My research interests are in the areas of low dimensional topology, hyperbolic geometry and geometric group theory. Most of my research has been focused on problems relating the geometry of 3-dimensional manifolds and closed curves on surfaces.

Due to the Hyperbolization Theorem, we know precisely when does a given compact 3-manifold admits a hyperbolic metric, that is a metric with constant curvature -1. Moreover, by the Mostow's Rigidity Theorem this geometric structure is unique, meaning that any geometric invariant is also a topological invariant. However, finding effective and computable connections between the geometry and topology is a challenging problem. Most of the results fit into the theme of making the geometrization program more concrete and effective.

One of my current research project is to better understand the geometry of some link complements in the projective unit tangent bundle of a given hyperbolic surface. These links come from canonical lifts of closed geodesics, corresponding to periodic orbits of the geodesic flow. This is joint work with Tommaso Cremaschi, Andrw Yarmola and Dídac Martínez.

A second project is understanding the action of the homeomorphism group of a surface on the fine curve graph. We wish to create a dictionary linking dynamical properties of homeomorphisms acting on the surface to the geometry of the action on the fine curve graph. This is joint work with Jonathan Bowden and Richard Webb.

A third project focus on giving a normal form for free homotopy classes closed curves on triangular hyperbolic 2-orbifolds with at least one cusp, in such a way the corresponding cutting sequence has no ambiguity (not passing through a conical point), minimizes the word length and the self-intersection among all representants. This is joint work with Max Neumann.

[1] A. Rodríguez . *A lower bound for the volumes of complements of periodic geodesics* J. London Math. Soc. (2) 00 (2020) 1-27.

[2] T. Cremaschi, and A. Rodríguez. *Hyperbolicity of link complements in Seifert fibered spaces* Algebraic & Geometric Topology 20 (2020) 3561–3588.

[3] T. Cremaschi, A. Rodríguez, and A. Yarmola. *On volumes and filling collections of multicurves*, arXiv:1911.02732, 2020.

[4] A. Rodríguez . *Periods of continued fractions and volumes of modular knots complements*, arXiv:2008.12436, 2021.

[5] J. Bowden, S. Hensel, and R. Webb. *Quasi-morphisms on surface diffeomorphism groups* arXiv:1909.07164, 2019. To appear in J. Amer. Math. Soc.

Jacob Russell

Rice University, United States

Generalized hyperbolicity and the mapping class group

A central theme of my work building bridges between Gromov's hyperbolic groups and the mapping class group of a surface. While the mapping class group is not hyperbolic, it exhibits many features reminiscent of hyperbolic geometry. Thus, several generalizations of hyperbolic geometry (e.g. hierarchical hyperbolicity, relative hyperbolicity, and Morse geodesics) give powerful tools for understanding the mapping class group. For example, my collaborators and I have used hierarchical hyperbolicity to elucidate the geometry of complexes that the mapping class group acts [1, 2] and Morse geodesics to understand the convex cocompact subgroups of the mapping class group [3, 4].

Some questions/topics that I am interested in:

- Does there exists a convex cocompact closed surface group? Is every finitely generated and purely pseudo-Anosov subgroup convex cocompact? These open problems have strong implications for the geometry of surface bundles and surface group extensions as convex cocompact subgroups essentially characterize the Gromov hyperbolicity of these extensions.
- Developing a robust notion of "geometric finiteness" in the mapping class group. There are several naturally occurring examples of subgroups that ought to be considered geometrically finite (e.g. Veech groups, curve stabilizers), but no satisfying definition has been established.
- What is the geometry of the "framed mapping class groups" studied by Calderon, Salter, and Hamenstädt? Are they hierarchically hyperbolic?
- Which groups are hierarchically hyperbolic? Artin groups and free-by-cyclic groups seem to be promising places to find new examples.
- The boundary of hierarchically hyperbolic groups.

[1] *Thickness and relative hyperbolicity for graphs of multicurves*, with Kate Vokes. Journal of Topology

[2] *From hierarchical to relative hyperbolicity*, International Mathematics Research Notices.

[3] *Regularity of Morse geodesics and growth of stable subgroups*, with Matthew Cordes, Davide Spriano, and Abdul Zalloum. Journal of Topology

[4] *The local-to-global property for Morse quasi-geodesics*, with Davide Spriano and Hung C. Tran. Mathematische Zeitschrift

Yuri Santos Rego

Otto-von-Guericke-Universität Magdeburg, Germany

Groups, spaces, and invariants

In my work I aim at getting a better understanding of groups and spaces of arithmetic, Lie-theoretic, combinatorial, or low-dimensional nature. Here are some examples of groups that I like: S-arithmetic groups such as the lattice $\mathrm{SL}_n(\mathbb{Z}) \leq \mathrm{SL}_n(\mathbb{R})$, the Baumslag–Solitar group $BS(1,p) \leq \mathrm{GL}_2(\mathbb{Z}[\frac{1}{p}])$, and the lamplighter group $C_p \wr \mathbb{Z} \leq \mathrm{GL}_2(\mathbb{F}_p[t,t^{-1}])$; Thompson groups such as the braided variant of R. Thompson's V or the golden-ratio group F_τ; Coxeter groups, such as the hyperbolic triangle group $\Delta(2,3,7) \leq \mathrm{Isom}(\mathbb{H}^2)$; locally compact groups such $\mathrm{GL}_n(\mathbb{R})$, $\mathrm{SL}_n(\mathbb{Q}_p)$, or profinite and pro-$p$ groups. On the geometric front, spaces that I enjoy include, but are not limited to: symmetric spaces, buildings (spherical and affine), and simplicial complexes resembling them — such as coset complexes; low-dimensional structures, such as Haken manifolds and spatial graphs; and Cantor sets.

I thus like the interplay between algebra, geometry, and topology, using tools from one area to solve a problem originating from — or motivated by — the other. Consequently, interesting invariants — such as (co)homology rings, BNSR Σ-invariants, or homotopical finiteness properties — and questions revolving around (twisted) conjugacy or decision problems play a role in my research.

Let me briefly mention some recent projects. In the article [1], Petra Schwer and I introduced the *Coxeter galaxy*, a space encoding isomorphism types of Coxeter systems. The goal of this space is to give a new perspective on the (still open) isomorphism problem for Coxeter groups, making it more tractable and allowing for some natural refinements and new questions. In the realm of S-arithmetic groups, Benjamin Brück, Robin Sroka and I showed that the top-dimensional rational *cohomology* of many classical arithmetic groups *vanishes* [2], extending the well-known case of $\mathrm{SL}_n(\mathbb{Z})$. And in an ongoing recent project jointly with Brita Nucinkis and Lewis Molyneux, we are investigating the *Σ-invariants* of the *golden-ratio Thompson group F_τ*. Along the way we address a question of Strebel about computing, under mild assumptions, Σ-invariants of an overgroup from the Σ-invariants of well-behaved subgroups of finite index.

[1] Santos Rego, Y. and Schwer, P., *The Galaxy of Coxeter groups*, Preprint (2022), `arXiv:2211.17038`, pp. 30.

[2] Brück, B., Santos Rego, Y. and Sroka, R. J., *On the top-dimensional cohomology of arithmetic Chevalley groups*, Preprint (2022), `arXiv:2210.12784`, pp. 8.

Bakul Sathaye

University of Münster, Germany

Nonpositively curved manifolds

I am interested in understanding nonpositively curved manifolds and groups using visual boundaries and large scale geometry. The classical notion of curvature is the sectional curvature of Riemannian manifolds. A generalized notion of nonpositive curvature for the bigger class of geodesic spaces is given by CAT(0) spaces. A natural question to ask is: how are these two notions of non-positive curvature related?

In low dimensions, the two classes of closed manifolds are the same: (1) manifolds supporting a Riemannian metric of non-positive sectional curvature, and (2) manifolds supporting a locally CAT(0) metric. For dimensions ≥ 4, there are examples of closed locally CAT(0) manifolds that do not support a smooth Riemannian structure with non-positive sectional curvature. For dim ≥ 5, Davis-Januskiewicz [1] showed that for each $n \geq 5$, there is a piecewise flat, non-positively curved closed manifold M^n whose universal cover $\tilde{M}^n$ is not simply connected at infinity. This, in particular, means that $\tilde{M}^n$ is not homeomorphic to $\mathbb{R}^n$, and hence M cannot have a smooth non-positively curved Riemannian metric.

The techniques for dim = 4 involve constructing a "knottedness" or "linking" in the boundary at infinity of $\tilde{M}$ which gives the obstruction to non-positively curved Riemannian smoothing [2,3]. I am currently extending the knotting obstruction to dimensions ≥ 5. For higher dimensions we consider knotting and linking spheres of co-dimension 2. Unfortunately the same methods do no work and we need to develop new tools to detect the obstruction to Riemannian smoothing within the manifold instead of the boundary at infinity. These consist of large scale coarse homotopy and homology invariants of pairs of spaces.

[1] M. Davis and T. Januszkiewicz, *Hyperbolization of polyhedra*, J. Differential Geometry 34 (1991), 347-388.

[2] M. Davis, T. Januszkiewicz and J.-F. Lafont, *4-dimensional locally CAT(0)-manifolds with no Riemannian smoothings*, Duke Math. Journal 161 (2012), 1-28.

[3] B. Sathaye, *Link obstruction to Riemannian smoothings of locally CAT(0) 4-manifolds*, arXiv:1707.03433

Anschel Schaffer-Cohen

UNAM, Mexico

Big Mapping Class Groups: Geometry and Randomness

I study big mapping class groups, which are the mapping class groups of infinite-type surfaces. In particular, I've discovered a family of surfaces whose mapping class groups' geometries are captured up to quasi-isometry by their actions on a particular curve graph [1]. The graph is easy to describe, but beyond "it's definitely not hyperbolic" and "there is a natural cubical structure" I haven't been able to discover many of its features. Come talk to me about this if you like thinking about the large-scale geometry of infinite-valence simplicial graphs.

One of my more recent projects has been trying to answer the following question: what does (the mapping class group of) a "typical" infinite-type surface look like? That is, if we draw a surface randomly out of a hat, what properties does it('s mapping class group) have? This question is natural because a lot of results in big mapping class groups depend on some assumption about the underlying surface (e.g. infinite genus, a tame end space, the existence of a non-displaceable subsurface) and we don't currently have a good way to describe how exceptional or generic these results actually are.

[1] Anschel Schaffer-Cohen. Graphs of curves and arcs quasi-isometric to big mapping class groups. To appear in *Groups, Geometry, and Dynamics*. `https://arxiv.org/abs/2006.14760`

Lancelot Semal
UCLouvain, Belgium

Harmonic analysis of locally compact groups.

I am studying the tight interactions existing between the algebraic and geometrical properties of locally compact groups and the regularity of their unitary representations. For instance an old conjecture of C.Nebbia states that a group of automorphisms of a semi-regular tree is 2-transitive on the boundary of the tree if and only if its representations are all CCR (the associated $*$-representation of the maximal C^*-algebra range inside the compact operators).

Most of my research so far concerns totally disconnected locally compact (t.d.l.c.) groups like groups of automorphsisms of trees or buildings. These t.d.l.c. groups are characterized amoung locally compact groups by the property of admitting a basis of neighbourhood of the identity consisting of compact open subgroups (b.o.n.c.o.). The existence of such a basis has various direct consequences on the representation theory of these groups. For isntance, every representaiton of a t.d.l.c. group admits non-zero invariant vectors for at least one of its compact open subgroups. It natural to ask whether additional properties of the b.o.n.c.o.'s of the group provides additional information on the representation theory. Inspired by the work of G.I. Ol'shanskii who completely classified the equivalence classes of irreducible unitary representations of the full group of automorphisms of a thick regular tree, my work provide a positive answer to this question. Amoung other things I have enlightened a particular kind of factorization properties under-which I can describe the irreducible representations of the group admitting invariant vectors for a small enough compact open subgroup [1]. This work has various applications on automorphisms groups of trees and buildings. One of these is the complete classification of the irreducibles of the automorphsisms groups of trees whose local action at every vertex contains the alternating group, leading to the conclusion that these groups are all CCR and hence providing a a significant contribution to Nebbia's CCR conjecture on trees [2].

[1] L.Semal, Unitary representations of totally disconnected locally compact groups satisfying Ol'shanskii's factorization, arXiv preprint 2021

[2] L.Semal, Radu groups acting on trees are CCR arXiv preprint 2022.

Jiayi Shen
University of Notre Dame, United States

Normal generators of Out(F_n)

Let S_g denote a connected, closed, orientable surface of genus g. The mapping class group Mod(S_g) is the group of homotopy classes of orientation-preserving homeomorphisms of S_g. If an element of Mod(S_g) that has normal closure equal to the whole group, we say that the element *normally generates* Mod$((S_g)$ and we call this element, the *normal generator*. In 2018, Justin Lanier and Dan Margalit [1] have proved the following theorem.

Theorem. *For $g \geq 3$, every nontrivial periodic mapping class that is not a hyperelliptc involution normally generates Mod(S_g).*

The outer automorphism group Out(F_n) of the free group F_n is in many ways analogous to Mod(S_g). See [2]. Marc Culler has showed that any finite subgroup of Out(F_n) can be realized as a group of automotphisms of a graph with fundamental group F_n[3]. My current work is on finding the normal closure of a periodic element in Out(F_n) by studying the automorphism of a graph. I have proved that for the graph which is a wedge of n circles, we do have a very similar result as Lanier and Margalit have given. Notice that the graph automorphism group of a wedge of n circles is the signed permutation group $S_n^{\pm} \cong S_n \rtimes \mathbb{Z}_2^n$.

Theorem. *The normal closure of any nontrivial permutation element in $S_n^{\pm}$ is the whole group.*

I am working on the finite order automorphism of a general graph with fundamental group F_n, which the graph of a wedge of n circles is only a special case.

[1] Justin Lanier, Dan Margalit, Normal generators for mapping class groups are abundant. Comment. Math. Helv. 97 (2022), no. 1, pp. 1–59

[2] Mladen Bestvina, and Michael Handel, Train tracks and automorphisms of free groups. Annals of Mathematics (2), vol. 135 (1992), no. 1, pp. 1–51.

[3] M. Culler, Finite groups of outer automorphisms of a free group, Contribu- tions to group theory, 197–207, Contemp. Math., 33, Amer. Math. Soc., Providence, R.I., 1984.

Jiajun Shi
Heidelberg University, Germany

Gerneralization of Thurston's Lipschitz metric

Consider a connected oriented surface S of negative Euler characteristic. Thurston defined in [1] an asymmetric metric on Teichmüller space:

$$L(x,y) := \log \sup_{\alpha \in \mathcal{S}} \frac{\ell_y(\alpha)}{\ell_x(\alpha)}, \qquad \text{for } x,y \in \mathcal{T}(S)$$

where $\mathcal{S}$ is the set of isotopy classes of simple closed curves on S.

Walsh managed to prove in [2] that the horofunction boundary of Teichmüller space with this Lipschitz metric is the same as the Thurston boudary, namely the space of projective measured laminations.

Now there are several different kinds of generalization of Lipschitz metric, for example, Anosov representation [3] and the space of projective filling geodesic currents [4]. There might be other possible generalization, for example, to the space of flat cone metrics.

I am interested in computing the horoboundary for the generalized Lipschitz metric and how it reflects the geometry of the corresponding space.

[1] Thurston, W. Minimal stretch maps between hyperbolic surfaces. *ArXiv Preprint Math/9801039*. (1998)

[2] Walsh, C. The horoboundary and isometry group of Thurston's Lipschitz metric. *ArXiv Preprint ArXiv:1006.2158*. (2010)

[3] Carvajales, L., Dai, X., Pozzetti, B. & Wienhard, A. Thurston's asymmetric metrics for Anosov representations. *ArXiv Preprint ArXiv:2210.05292*. (2022)

[4] Sapir, J. An extension of the Thurston metric to projective filling currents. *ArXiv Preprint ArXiv:2210.08130*. (2022)

Emily Shinkle
University of Illinois Urbana-Champaign, USA

Mapping class group related complexes, pseudo-Anosovs, and more!

Broadly, I study the mapping class group, which is the group of homeomorphisms of a surface, up to homeomorphism. The mapping class group is important for understanding 3-manifolds, many of which can be constructed by crossing a surface with an interval, and then gluing the "ends" via a homeomorphism. You can determine geometric information about 3-manifolds constructed this way from purely topological data about the gluing homeomorphism. Mapping class groups and their actions on other spaces also provide a rich field of examples of results in geometric group theory.

I've done work involving the arc complex and flip graph, similar to the better known curve complex. My work involved finding a sort of "finite basis" for automorphisms of these complexes, and these automorphisms are in correspondence with mapping class group elements.

Recently, I've been working on a project involving stretch factors of pseudo-Anosov homeomorphisms, which are the generic elements of the mapping class group. These stretch factors have certain algebraic properties that can be used to understand the structure of the Veech group associated to that homeomorphism. The Veech group helps us understand the geometric structure induced on the surface by the pseudo-Anosov homeomorphism.

I've also taken an interest in computational topology, and computational mathematics more generally. I'm particularly interested in machine learning applications and topological data analysis.

If you're interested in any of the topics described above, I'd love to talk!

Rachel Skipper

École Normale Supérieure, France

Groups acting on Cantor sets

My research focuses primarily on groups acting on infinite trees and on boundaries of infinite trees, which can be identified with Cantor spaces. This is a large class of groups; for instance it contains all residually finite groups but also many of the known examples of infinite simple groups.

Groups acting on rooted trees often exhibit unusual properties while still being tractable. A well-studied example of such a group is the group introduced by Grigorchuk. The Grigorchuk group was the first group known to have intermediate growth (answering a question of Milnor) and to be amenable but not elementary amenable (answering a question of Day). In addition, it is a finitely generated, infinite, torsion group, and has many other interesting properties. In the wake of Grigorchuk's discovery, entire families of suprising and accessible groups that act on rooted trees have been discovered. Another set of groups which appears prominently in my research is the Thompson's groups, in particular the ones known as T and V and their generalizations due to Higman. These are groups of homeomorphisms of the Cantor set and provided the first examples of finitely presented, infinite simple groups.

This area of research overlaps with many fields within mathematics including group theory, geometry, topology, ring theory, dynamics, graph theory, descriptive set theory, logic and universal algebra. Many of my results have come by combining different areas of mathematics. For instance with Stefan Witzel and Matthew C. B. Zaremsky, we combined self-similar groups, i.e. groups like the Grigorchuk group, with Higman-Thompson groups to construct the first simple groups with the prescribed topological finiteness properties of being type F_{n-1} but not F_n (generalizations of being finitely generated and finitely presented) for all $n > 2$, answering a question due to Rémy which gave a new infinite family of quasi-isometry classes of finitely presented simple groups. Likewise in my work with Xiaolei Wu, we considered the labeled braided Higman-Thompson groups. By proving that particular examples of these could be identified with the so-called asymptotic mapping class groups, we were able to apply tools from the well studied area of mapping class groups to prove the ribbon Higman-Thompson groups statisfy homological stability. This provided the first such result for dense subgroups of a big mapping class group. A third example of this was my joint work with Benjamin Steinberg where we produced most lamplighter groups of the form $A \wr \mathbb{Z}$ for a finite abelian group A as bireversible automata groups. In order to understand exactly which groups we had produced, we studied local rings and classified exactly which abelian groups can be the additive group of a finite commutative ring having two units whose difference is a unit.

Mireille Soergel
ETH Zürich, Switzerland

Artin groups and spaces of non-positive curvature

I am interested in actions of Artin groups on spaces of non-positive curvature. For simplicial complexes Januszkiewicz and Swiatkowski introduced the notion of systolic complexes as a combinatorial form of non-positive curvature.

In [1], I investigate systolicity for Garside groups. Garside groups were introduced by Dehornoy and Paris as a generalization of spherical Artin groups. The Garside structure on a group naturally gives a presentation leading to a simplicial Cayley graph, we call this the Garside presentation of a Garside group. In [1] I give a classification of the Garside groups for which the flag complex of the Cayley graph, with respect to the Garside presentation is systolic. For $n, m \in \mathbb{N}$, the group

$$G_{n,m} = \langle x_1, \dots, x_n \mid \mathrm{prod}(x_1, \dots, x_n; m) = \mathrm{prod}(x_2, \dots, x_n, x_1; m) = \dots \\ = \mathrm{prod}(x_n, x_1, \dots, x_{n-1}; m) \rangle,$$

where $\mathrm{prod}(x_1, \dots, x_p; 0) = \mathbf{e}$ and $\mathrm{prod}(x_1, \dots, x_p; m) = \underbrace{x_1 x_2 \dots x_p x_1 x_2 \dots}_{m}$, is systolic.

More recently I have been studying the following class of groups: consider a simplicial graph Γ with maps $f : V(\Gamma) \to \mathbb{N}_{\geq 2} \cup \{\infty\}$ and $m : E(\Gamma) \to \mathbb{N}_{\geq 2}$ such that for every edge $e = \{v, w\}$ with $f(v) \geq 3$ we have $m(e) = 2$. The Dyer groups associated to such a graph is given by the following presentation:

$$D = \langle x_v, v \in V \mid x_v^{f(v)} = \mathbf{e} \text{ if } f(v) \neq \infty, \\ [x_v, x_u]_{m(e)} = [x_u, x_v]_{m(e)} \text{ for all } e = \{u, v\} \in E \rangle,$$

where $[a, b]_k = \underbrace{aba \dots}_{k}$ for any $a, b \in D$, $k \in \mathbb{N}$ and we denote the identity with $\mathbf{e}$. Coxeter groups and right angled Artin groups are examples of Dyer groups. I recently showed that Dyer groups are finite index subgroups of Coxeter groups. Moreover I constructed a generalization of the Davis-Moussong complex better suited to Dyer groups.

[1] Mireille Soergel *Systolic complexes and group presentations*, Accepted in *Groups, Geometry and Dynamics*, 2022

Henrique Augusto Souza
Universidad Autónoma de Madrid, Spain

The Atiyah Conjecture for pro-p groups

If G is a countable group and $\ell^2(G)$ is the complex Hilbert space with orthonormal basis G, one can view every $n \times m$ matrix A over the group ring $\mathbb{C}[G]$ as a bounded linear operator from $\ell^2(G)^n$ to $\ell^2(G)^m$ by left multiplication. It's *von Neumann rank* $\mathrm{rk}_G(A)$ is defined as the von Neumann trace of the orthogonal projection onto $\overline{\ell^2(G)^n A}$, the closure of it's image. Let $\mathrm{lcm}(G)$ denote the supremum of the orders of the finite subgroups of G.

The strong Atiyah conjecture predicts that, if $\mathrm{lcm}(G)$ is finite, then $\mathrm{rk}(A)$ is an integral multiple of $\frac{1}{\mathrm{lcm}(G)}$ for every such matrix A. This is a very strong conjecture, as for instance it implies Kaplansky's zero divisor conjecture for $\mathbb{C}[G]$ and characterizes the division closure of $\mathbb{C}[G]$ inside the von Neumann algebra of affiliated operators of $\ell^2(G)$ as it's universal division ring of fractions: the "largest possible" division ring containing $\mathbb{C}[G]$. It is known to hold for many classes of groups, such as finite groups, elementary amenable groups, free groups, free-by-cyclic groups and fundamental groups of hyperbolic 3-manifolds with empty or toroidal boundary. Moreover, if $G_i \trianglelefteq G$ is a chain of finite index normal subgroups with trivial intersection, the Lück approximation theorem tells us that $\mathrm{rk}_{G/G_i}(A_i)$ converges to $\mathrm{rk}_G(A)$, where A_i is the reduction of the matrix A modulo G_i. This motivates the following variant of the Atiyah conjecture:

If G is a finite group and A is an $n \times m$ matrix over the group algebra $\mathbb{F}_p[G]$, we can define the normalized rank

$$\mathrm{rk}_G(A) = \frac{\mathrm{rk}_{\mathbb{F}_p} A}{|G|}\,.$$

If G is a countably based pro-p group and $G_i \trianglelefteq_o G$ is a chain of open normal subgroups of G with trivial intersection, we define the rank of a matrix A over the completed group algebra $\mathbb{F}_p[\![G]\!] = \varprojlim \mathbb{F}_p[G/G_i]$ as the limit

$$\mathrm{rk}_G(A) = \lim_{i\to\infty} \mathrm{rk}_{G/G_i}(A_i)\,,$$

in analogy with the Lück approximation theorem in the discrete case. If $\mathrm{lcm}(G)$ is finite, then the *strong Atiyah conjecture for pro-p groups* says that $\mathrm{rk}_G(A)$ should be an integral multiple of $\frac{1}{\mathrm{lcm}(G)}$.

As in the discrete case, this is a strong conjecture with analogous implications for the completed group algebra $\mathbb{F}_p[\![G]\!]$. However, it's validity is established for far fewer groups: so far it is only known for torsion-free p-adic analytic pro-p groups and subdirect products of those (such as free pro-p groups and pro-p completions of surface groups). I'm currently studying this conjecture and related themes for certain classes of pro-p groups that are the pro-p analogues of classes for which we know the conjecture to hold in the discrete case, such as free-by-cyclic pro-p groups.

Davide Spriano
University of Oxford, UK

Hyperbolic models for CAT(0) spaces

My main research focus are various notion of non-positive curvature in group theory and their interplay. For instance, I care about groups that are hyperbolic, (acylindrically, relatively, hierarchically) hyperbolic, CAT(0), cubical...

A recent project I am excited about [1] is joint work with H. Petyt and A. Zalloum on hyperbolic models for CAT(0) spaces. Two of the most well-studied topics in geometric group theory are CAT(0) cube complexes and mapping class groups. This is in part because they both admit powerful combinatorial-like structures that encode interesting aspects of their geometry: hyperplanes for the former and curve graphs for the latter, and recent years, analogies between the two theories have become more and more apparent. However, the considerably larger class of CAT(0) *spaces* is left out of this analogy, as the lack of a combinatorial-like structure presents a difficulty in importing techniques from those areas. With Petyt and Zalloum, we develop versions of hyperplanes and curve graphs for them, and prove results about rank-one elements, asymptotic cones, visual boundary and WPD actions.

[1] Harry Petyt, Davide Spriano and Abdul Zalloum, *Hyperbolic models for CAT(0) spaces*, arXiv2207.14127.

Stephan Stadler

Max Planck Institute for Mathematics Bonn, Germany

Geometry of spaces with upper curvature bounds

Setup

I am interested in the geometry of CAT(κ) spaces. These are synthetic generalizations of Riemannian manifolds with sectional curvature at most κ and injectivity radius at least $\frac{\pi}{\sqrt{\kappa}}$ where the curvature bound is expressed by triangle comparison.

Rank Rigidity

It has been conjectured by Werner Ballmann that CAT(0) spaces with enough symmetries are subject to the following dichotomy. Let X be a locally compact geodesically complete CAT(0) space with a geometric group action $\Gamma \curvearrowright X$. Then either X contains a Γ-periodic axis which does not bound a flat half-plane, or X is a Riemannian symmetric space of higher rank, a Euclidean building of higher rank, or X splits non-trivially as a metric product. I would like to confirm this statement.

Optimal isoperimetric inequalities

A metric space X satisfies the Euclidean isoperimetric inequality for curves, if every closed curve c of finite length bounds a Sobolev disc $u \in W^{1,2}(D, X)$ with

$$\mathrm{Area}(u) \leq \frac{1}{4\pi} \cdot \mathrm{Length}(c)^2.$$

In a landmark paper, Lytchak-Wenger showed that a locally compact geodesic space is CAT(0) if and only if it satisfies the Euclidean isoperimetric inequality for curves. Together with Stefan Wenger we are currently trying to remove the local compactness assumption.

Embeddedness of minimal surfaces

With Paul Creutz we are proving the following theorem. Let Z be a CAT(κ) space and $\Gamma \subset Z$ a Jordan curve. If the total curvature τ satisfies

$$\tau(\Gamma) < 4\pi - \kappa \cdot \mathrm{Fillarea}(\Gamma),$$

then any least area disc filling Γ is embedded.

Bogdan Stankov

Université Claude Bernard Lyon 1, France

Random walks on Cayley and Schreier graphs

Throughout my PhD, I have explored several somewhat varying questions related to amenability. My first topic concerns a specific group. In his article [1], Monod defines a class of groups $H(A)$ of piecewise projective homeomorphisms on the real line, depending on a subring A of $\mathbb{R}$. They never have a free subgroup, and if A is dense Monod proves them to be non-amenable by comparing the orbit equivalence relation with that of $PSL_2(A)$. I studied $H(\mathbb{Z})$, the amenability of which is an open question. It is known that a group is non-amenable if and only if every non-degenerate measure on it has non-trivial Poisson boundary. I have obtained in [2] the non-triviality of the Poisson boundary for certain classes of measures. Specifically, I have shown that for a finitely generated subgroup of $H(\mathbb{Z})$, either it is solvable or any strictly non-degenerate measure on it with finite first moment has non-trivial Poisson boundary. Further developing from that, I have also shown in [3] similar results for induced random walks on Schreier graphs. In particular, those results extend (slightly) what was previously known about random walks on Thompson's group F (which is a subgroup of $H(\mathbb{Z})$).

Afterwards, I worked on a different subject - studying the exact values of Følner functions for a given group and generating set. In [4] I have obtained those values as well as the sets where they are obtained for the wreath product $\mathbb{Z} \wr \mathbb{Z}/n\mathbb{Z}$ and a generating set containing the translation on the index, as well as a unit function for every element of $\mathbb{Z}/n\mathbb{Z}$.

Currently, I am working on whether similar results to those in [2] can be obtained on groups of affine integer exchange transformations. My result can be extended without much difficulty to the claim that finitely generated subgroup of $H(\mathbb{Z})$ is either solvable or does not have property $\overline{FW}$. Results have been obtained by Juschenko, Matte Bon, Monod et de la Salle on AIET groups that satisfy a stronger property than property $\overline{FW}$. I am interested in describing IET groups with property $\overline{FW}$.

[1] N. Monod, Groups of piecewise projective homeomorphisms. *Proc. Natl Acad. Sci. USA* 110(12):4524–4527, mar 2013. doi:10.1073/pnas.1218426110

[2] B. Stankov, Non-triviality of the Poisson boundary of random walks on the group $H(\mathbb{Z})$ of Monod. *Ergodic Theory and Dynamical Systems*, 41(4):1160-1189, apr 2021. doi:10.1017/etds.2019.76

[3] B. Stankov, Convergence towards the end space for random walks on Schreier graphs. *Journal of Theoretical Probability*, may 2021. doi:10.1007/s10959-021-01104-6.

[4] B. Stankov. Exact descriptions of Følner functions and sets on wreath products and Baumslag-Solitar groups. *preprint*, nov 2021. arXiv:2111.09158.

Christian Steinhart
Universität des Saarlandes, Germany

Geometry of Outer Space

Outer Space CV_n was introduced by Marc Culler and Karen Vogtmann in [1] to study the outer automorphism group $\mathrm{Out}(F_n)$. It can be seen as the analogue of Teichmüller space for graphs, namely points in Outer Space CV_n correspond to equivalence classes of finite metric graphs without leaves together with a marking, that is a homotopy equivalence with the rose R_n with n petals. Similar to Thurston's metric in Teichmüller space, Stefano Francaviglia and Armando Martino introduced in [2] an asymmetric metric called the *Lipschitz metric* for Outer Space by the supremal stretching of curves. As this supremal stretching is realised by one of finitely many candidates, it can be explicitly computed e.g. with the algorithm implemented in [4].

Typically geodesics in Outer Space are far from being unique. This leads to the notion of *envelopes*, that is the set of all points lying on a geodesic between two given points. Using envelopes we proved in [5], that the isometry group of reduced Outer Space is the isometry group of Outer Space, which is by [3] $\mathrm{Out}(F_n)$ for $n > 2$ and $\mathrm{PGL}_2(\mathbb{Z})$ for $n = 2$. Furthermore envelopes yield a construction of piecewise unique geodesics between any two given points and a local geodesic which is dense in CV_n. One future goal is to use envelopes to give a simplified proof that Outer Space is contractible.

In [5] we further studied isometric embeddings between Outer Spaces. We introduced two families of isometric embeddings coming from finite index subgroups and free factors of free groups. While the isometric embeddings coming from finite index subgroups of F_n for $n > 2$ exhibit some sort of rigidity, the embeddings coming from finite index subgroups of F_2 and free factors yield continuous families of isometric embeddings. It is still unknown, if there exist isometric embeddings which do not come from these algebraic constructions.

[1] Culler, M. & Vogtmann, K. Moduli of graphs and automorphisms of free groups. *Inventiones Mathematicae*. **84**, 91-119 (1986), https://doi.org/10.1007/BF01388734

[2] Francaviglia, S. & Martino, A. Metric properties of Outer Space. *Publicacions Matemàtiques*. **55**, 433-473 (2011), https://doi.org/10.5565/PUBLMAT_55211_09

[3] Francaviglia, S. & Martino, A. The isometry group of Outer Space. *Advances In Mathematics*. **231**, 1940-1973 (2012), https://dx.doi.org/10.1016/j.aim.2012.07.011

[4] Steinhart, C. CV-Lipschitz-Calculator. (2018), https://gitlab.com/ctst/CV-Lipschitz-Calculator, https://gitlab.com/ctst/CV-Lipschitz-Calculator

[5] Steinhart, C. Isometric maps between Outer Spaces. (2021), https://dx.doi.org/10.22028/D291-35505

Chaitanya Tappu
Cornell University, United States

A Marked Moduli Space for Infinite Type Surfaces

I am a fifth-year PhD student. Currently, I am interested in understanding mapping class groups of infinite type surfaces via action on a space of marked hyperbolic structures. In a forthcoming paper, we define the space of marked, complete, Nielsen-convex hyperbolic structures on a surface S of negative (but not necessarily finite) Euler characteristic. The emphasis is on infinite type surfaces. As a set, this is defined in exact analogy with the Teichmüller space of compact or finite type surfaces:

Definition (Set of marked, complete, Nielsen-convex hyperbolic structures)**.**

$$\mathcal{T}(S) = \left\{ (X, f) \middle| \begin{array}{l} X \text{ is a complete Nielsen-convex hyperbolic surface} \\ f : S \to X \text{ is a homeomorphism} \end{array} \right\} \Big/ \sim \qquad (1)$$

where $(X_1, f_1) \sim (X_2, f_2)$ *if there is an isometry* $\varphi : X_1 \to X_2$ *isotopic to* $f_2 \circ f_1^{-1}$.

Here we have adopted the term *Nielsen-convex* from work of Alessandrini, Liu and others ([1, Definition 4.3]). A complete hyperbolic surface X is *Nielsen-convex* if the convex core of X equals X.

We topologise $\mathcal{T}(S)$ in a natural way by considering the injection into $\mathrm{Hom}(\pi_1(S), \mathrm{PSL}(2, \mathbb{R}))/\mathrm{PSL}(2, \mathbb{R})$. The main theorem of our paper is that the mapping class group of S, which is a non-discrete topological group if S is an infinite type surface, acts continuously on this marked moduli space.

Theorem. *The change of marking action* $A : \mathrm{MCG}(S) \times \mathcal{T}(S) \to \mathcal{T}(S)$ *given by*

$$A([\psi], [X, f]) = [X, f \circ \psi^{-1}] \qquad (2)$$

is a continuous map.

[1] Daniele Alessandrini, Lixin Liu, Athanase Papadopoulos, Weixu Su, and Zongliang Sun. On Fenchel-Nielsen coordinates on Teichmüller spaces of surfaces of infinite type. *Annales Academiae Scientiarum Fennicae Mathematica*, 36:621–659, Aug 2011.

[2] John Hamal Hubbard. *Teichmüller theory and applications to geometry, topology, and dynamics. Vol. 1.* Matrix Editions, Ithaca, NY, 2006. Teichmüller theory, With contributions by Adrien Douady, William Dunbar, Roland Roeder, Sylvain Bonnot, David Brown, Allen Hatcher, Chris Hruska and Sudeb Mitra,With forewords by William Thurston and Clifford Earle.

[3] William P. Thurston. Earthquakes in two-dimensional hyperbolic geometry, volume 112 of London Math. Soc. Lecture Note Ser., pages 91–112. Cambridge Univ. Press, Cambridge, 1986.

Matteo Tarocchi
Università di Milano-Bicocca / Università degli Studi di Pavia, Italy

Rearrangement Groups of Fractals and Invariable Generation

I am studying the family of *rearrangement groups*, defined by J. Belk and B. Forrest in [2]. These groups are a natural generalization of *Thompson groups* F, T and V, which are groups of certain piecewise linear hoemomorphisms of the unit interval, the unit circle and the Cantor set, respectively, and can also be built as automata groups ([3] is widely regarded as a standard introduction to Thompson groups). Each rearrangement group essentially acts on a limit space X, built as the limit of a sequence of graphs, by homeomorphisms that "rearrange" the self-similar pieces of X. Rearrangements can be represented as isomorphisms of graphs or as forest pair diagrams, which naturally generalize the concept of tree pair diagram commonly used for Thompson groups.

In [5] I studied the rearrangement group T_A of the Airplane Julia set, proving that, just like the rearrangement group T_B of the Basilica Julia set (originally studied in [1] by Belk and Forrest), it is finitely generated and its commutator subgroup $[T_A, T_A]$ is simple and finitely generated. I also showed that, differently from T_B, the index of the commutator subgroup of T_A is infinite.

Davide Perego and I recently proved that every sufficiently transitive rearrangement group is not Invariably Generated ([4]), where by *Invariable Generation* of a group G we mean that there exists an $S \subseteq G$ such that, for every choice $g_s \in G$ for $s \in S$, the group G is generated by $\{s^{g_s} \mid s \in S\}$.

I am currently working on decision problems in this family of groups.

[1] J. Belk and B. Forrest. "A Thompson group for the basilica". In: *Groups Geom. Dyn.* 9.4 (2015), pp. 975–1000. DOI: 10.4171/GGD/333.

[2] J. Belk and B. Forrest. "Rearrangement groups of fractals". In: *Trans. Amer. Math. Soc.* 372.7 (2019), pp. 4509–4552. DOI: 10.1090/tran/7386.

[3] J. W. Cannon, W. J. Floyd, and W. R. Parry. "Introductory notes on Richard Thompson's groups". In: *Enseign. Math.* (2) 42.3-4 (1996), pp. 215–256.

[4] D. Perego and M. Tarocchi. *A Class of Rearrangement Groups that are not Invariably Generated*. 2022. DOI: 10.48550/ ARXIV.2207.04235. URL: https://arxiv.org/abs/2207.04235.

[5] M. Tarocchi. *Generation and Simplicity in the Airplane Rearrangement Group*. 2021. DOI: 10.48550/ARXIV.2107.08744. URL: https://arxiv.org/abs/2107.08744.

Thomas Titz Mite
JLU Giessen, Germany

$\tilde{C}_2$-buildings admitting panel-regular lattices

In April 2021, I started my PhD project under supervision of Stefan Witzel. We investigate a class of Euclidean Buildings admitting a lattice, which is roughly speaking a pair consisting of a certain contractible simplicial complex and a group that acts properly and co-compactly on it. By construction, these buildings are identical locally but the global structure can differ slightly. Currently, we try to understand this difference.
To be precise, the buildings are of type $\tilde{C}_2$ and the lattices act regularly on two types of panels (respectively one type of panel). They were introduced, besides a class of $\tilde{A}_2$-buildings, by Essert in [1]. The latter class has been studied by Witzel [2], who could determine several properties of the buildings, such as their automorphisms groups and isomorphism class, from purely combinatorial data. We aim for a similar understanding of the $\tilde{C}_2$-buildings.

[1] Essert, Jan: A geometric construction of panel-regular lattices for buildings of types $\tilde{A}_2$ and $\tilde{C}_2$. Algebr.

[2] Witzel, Stefan: On panel-regular Ã2 lattices, Geometriae Dedicata, 191 (2017), 85-135, arXiv:1608.07141.

Marie Trin

Université de Rennes 1, France

Geodesic currents for hyperbolic surfaces

I am a second year Ph.D student in Rennes. My advisor is Juan Souto. My research is about hyperbolic surfaces, curves on surfaces and geodesic currents.

If S is a surface of negative Euler characteristic then a *geodesic current* of S is a $\pi_1(S)$-invariant Radon measure on the set, for some hyperbolic metric, of bi-infinite unoriented geodesics of the universal cover. The set $\mathcal{C}(S)$ of geodesic currents has been introduced by Bonahon in [2] and since have find use in the study of 3-manifolds, in Teichmüller theory, in dynamic, and in geometric group theory.

In [3], Bonahon recovered Thurston's compactification of *Teichmüller space* using currents however, for some technical reasons (continuity of the intersection form, non-existence of the Liouville current..), his argument only applies for closed surfaces. During the first year of my PhD I extended his proof to the case of non-compact finite area surfaces. In a nutshell, it is to replace the Liouville current by *sequences of random geodesics*. A preprint for that work is available on Arxiv [1] and has been submitted.

Currently, I am working on two other projects linked to *Mirzakhani's curve counting*, that is counting the curves of S in some *mapping class group orbit*. On the one side, I want to obtain a generalized version of [6]: while Bell obtains a counting theorem for arcs, I want to prove, along the lines of [5], a measure convergence result instead. This would for example allow to count arcs with respect to more general notions of length. Second, I am working on counting curves in the orbit under certain *subgroups of the mapping class group* rather than by the whole mapping class group. More precisely, I am interested in the action of centralizers of elements and subgroups of the mapping class group.

[1] N. Bell, "Counting arcs on hyperbolic surfaces", *to be published in GGD*, arXiv:2011.13969

[2] F. Bonahon, "Bouts des Variétés Hyperboliques de Dimension 3", *Annals of Mathematics* 124 (1986), no. 1, p. 71-158

[3] F. Bonahon, "The geometry of Teichmüller space via geodesic currents", *Inventiones mathematicae* (1988), p. 139-162

[4] V. Erlandsson and J.Souto, "Mirzakhani's curve counting and geodesic currents", *Birkhäuser, Prog. in Math.* vol.345 (2022)

[5] M. Trin, "Compactification of Teichmüller space via geodesic currents: the case of non-compact finite area surfaces", arXiv:2208.10763

Alexander Trost
University of Wrocław, Poland

Conjugation-invariant norms on groups

Many questions in group theory can be recast as questions concerning conjugation-invariant norms on said groups. To give an elementary example, consider a perfect group G. Then each element $g \in G$ can be written as a product of commutators. Assuming $\|g\|_c \in \mathbb{N}_0$ is the minimal number of commutators needed, one easily sees that the function $\|\cdot\|_c : G \to [0, +\infty)$ is invariant under conjugation by group elements. This approach was used to study for example the commutator width of diffeomorphism groups [1]. Other algebraic question concerning the groups can also be recast in terms of conjugation-invariant norms for example the width of the group in terms of its conjugacy classes. More geometrically, an important subgroup of the "isometry group" of a symplectic manifold (M, ω), the hamiltonian diffeomorphism group $Ham(M, \omega)$, also admits a conjugation-invariant norm, the so-called Hofer norm $\|\cdot\|_{Hofer}$ that has become one of the principal objects of study in symplectic topology.

However, assuming the group G in question is sufficiently non-abelian, the inner automorphism group of the group is quite massive, which heavily restricts the possible conjugation-invariant norms on the group. For example, any non-discrete conjugation-invariant norm on arithmetic matrix groups like $\mathrm{SL}_{n\geq 3}(\mathbb{Z})$ is necessarily profinite. In any case, I am currently interested in two problems in particular. The first one is quite classical and concerns conjugation-invariant norms on more general arithmetic groups than $\mathrm{SL}_n(\mathbb{Z})$: Much of the previous results by myself and other researchers [2] on conjugation-invariant norms on arithmetic groups seem strongly connected to a very strong version of finite generation, called bounded generation. However, it is very unclear which arithmetic groups actually satisfy this property and my interest is in understanding this property for other arithmetic groups as well as gaining a more precise quantitative understanding of the known instances of bounded generation. Secondly, I am interested in whether the restricted topologies the Hofer norm can induce on a subgroup of $Ham(M, \omega)$ can be used to exclude the possibility that certain arithmetic groups can act on symplectic manifolds at all. Tantalizingly (to me at least), this problem seems to be unconnected to bounded generation of arithmetic groups and instead related to topological rigidity of certain profinite groups.

[1] Burago, Dmitri and Ivanov, Sergei and Polterovich, Leonid; *Conjugation-invariant norms on groups of geometric origin*, https://arxiv.org/abs/0710.1412

[2] Alexander Trost; *Bounded generation by root elements for Chevalley groups defined over rings of integers of function fields with an application in strong boundedness*, https://arxiv.org/abs/2108.12254

Konstantinos Tsouvalas

Institut des Hautes Études Scientifiques, France

Anosov representations and restricted linearity of Gromov hyperbolic groups

---

My research concerns Anosov representations of Gromov hyperbolic groups, whose images comprise a particular class of discrete subgroups of Lie groups playing an important role in higher Teichmüller theory and geometric group theory. Anosov representations were introduced by Labourie [Lab] in his seminal work on the Hitchin component and further generalized by Guichard–Wienhard in [GW] to the class of all Gromov hyperbolic groups.

In the past years, I have mainly focused on imposing topological and geometric restrictions on the class of Anosov representations [CT], constructing pathological examples of linear hyperbolic groups with restricted linearity properties [DT, TT] and exhibiting examples of non-linear hyperbolic groups with certain finiteness properties.

[CT] R. Canary and K. Tsouvalas, *Topological restrictions on Anosov representations*, J. Topol. **13** (2020) 1497-1520

[DT] S. Douba and K. Tsouvalas, *Anosov groups that are indiscrete in rank one*, preprint, arXiv:2210.17549, 2022.

[GW] O. Guichard and A. Wienhard, *Anosov representations: Domains of discontinuity and applications*, Invent. Math. **190** (2012), 357–438.

[Lab] F. Labourie, *Anosov flows, surface groups and curves in projective space*, Invent. Math. **165** (2006), 51–114.

[TT] N. Tholozan and K. Tsouvalas, *Linearity and indiscreteness of amalgamated products of hyperbolic groups*, I.M.R.N., 2022.

Matthias Uschold
Universität Regensburg, Germany

ℓ^2-Betti numbers and other homology growth invariants

In Algebraic Topology, a classical invariant is given by Betti numbers. Intuitively, they measure how many 'holes' a space has. More formally, we define the n-th Betti number of a space X as $b_n(X, \mathbb{Q}) := \dim_{\mathbb{Q}} H_n(X, \mathbb{Q})$.

Atiyah defined an equivariant sibling of the classical Betti numbers, the so-called ℓ^2-Betti numbers $b_n^{(2)}(G \curvearrowright X)$. Originally defined analytically, Lück's approximation theorem provides a different viewpoint to these numbers, viewing them as gradients, i.e.

Theorem (Lück approximation theorem)**.** *Let X be a finite type connected CW-complex. Let $G := \pi_1(X)$ be residually finite, i.e. there exists a residual chain $(G_i)_{i\in\mathbb{N}}$ (i.e. a sequence of nested, normal, finite index subgroups whose intersection is trivial). Then, for all $n \in \mathbb{N}$, we have*

$$b_n^{(2)}(G \curvearrowright \widetilde{X}) = \lim_{i\to\infty} \frac{b_n(G_i\backslash\widetilde{X}, \mathbb{Q})}{[G : G_i]}.$$

One advantage of this viewpoint is that it is easy to generalise to different coefficient fields and to construct different homology gradients. Similarly to ℓ^2-Betti numbers, also these gradient invariants vanish for classifying spaces of amenable groups.

Theorem ([3, Theorem 1])**.** *Let G be an finitely generated, infinite, amenable, residually finite group, let $(G_i)_{i\in\mathbb{N}}$ be a residual chain and let $G \curvearrowright EG$ denote its classifying space. Then, for all $n \in \mathbb{N}$ and fields $\mathbb{K}$, we have*

$$\lim_{i\to\infty} \frac{b_n(G_i\backslash EG, \mathbb{K})}{[G : G_i]} = 0 \quad \textit{and} \quad \lim_{i\to\infty} \frac{\log|\operatorname{tors} H_n(G_i\backslash EG, \mathbb{Z})|}{[G : G_i]} = 0,$$

where $|\operatorname{tors}\cdot|$ is the size of the torsion subgroup.

I'm interested in the behaviour of these invariants in general and especially, what their vanishing behaviour is. In a recent paper, Abert, Bergeron, Fraczyk and Gaboriau proved that vanishing of these two invariants follows follows from the *cheap n-rebuilding* property [1, Theorem 10.20], providing a possible strategy for showing such vanishing statements.

[1] *M. Abert, N. Bergeron, M. Fraczyk, and D. Gaboriau*. On homology torsion growth. arXiv.2106.13051 (2021).

[2] *H. Kammeyer*, Introduction to ℓ^2-invariants. Cham: Springer (2019).

[3] *A. Kar, P. Kropholler, and N. Nikolov*, Math. Proc. Camb. Philos. Soc. 162, No. 2, 337–351 (2017).

Inga Valentiner-Branth

Ghent University, Belgium

High-dimensional expanders and Kac–Moody–Steinberg groups

I just started my PhD in September 2022 under the supervision of Tom De Medts, as part of a larger research project together with Pierre-Emmanuel Caprace and Timothée Marquis (UCLouvain) with the goal to construct high-dimensional expanders using Kac-Moody-Steinberg groups.

Expander graphs — graphs which are somewhat sparse but nevertheless sufficiently connected — proved to have many applications in computer science and mathematics. The notion of expander graphs has been extended to higher dimensions, giving rise to (families of) simplicial complexes called high-dimensional expanders. Unlike in the graph case, not all notions of high-dimensional expanders (e.g. spectral, topological, coboundary) are equivalent, see [1] for a survey.

Probably the first construction of bounded degree high-dimensional expanders were the Ramanujan complexes due to Lubotzky, Samuels and Vishne [2]. The construction uses Bruhat–Tits buildings of algebraic groups over local fields, the expanders being obtained as suitable quotients of such buildings by cocompact arithmetic lattices.

On the other hand, Kac-Moody-Steinberg (KMS) groups are fundamental groups of certain complexes of finite p-groups, where p is a fixed prime.

Our goal is to connect the theory of high-dimensional expanders with KMS-groups and related geometric and algebraic objects.

I am still at the stage of reading more about the topic and understanding all the notions and underlying theories, while also participating in a reading group on expander graphs and various seminars in Ghent and Louvain-la-Neuve.

My master's thesis, which I wrote at the University of Innsbruck, supervised by Tim Netzer and Gemma de las Cuevas, looked at quantum generalizations of Latin squares and magic squares using tools from free semi-algebraic geometry. This resulted in a paper that we submitted for publication; see [4].

[1] Alexander Lubotzky. High dimensional expanders. In Proceedings of the International Congress of Mathematicians — Rio de Janeiro 2018. Vol. I. Plenary lectures, page 705–730. World Sci. Publ., Hackensack, NJ, 2018.

[2] Alexander Lubotzky, Beth Samuels, and Uzi Vishne. Explicit constructions of Ramanujan complexes of type $\tilde{A}_d$. European J. Combin., 26(6):965–993, 2005.

[3] Gemma De las Cuevas, Tim Netzer, and Inga Valentiner-Branth. Magic squares: Latin, Semiclassical and Quantum. arXiv:2209.10230, 2022.

Vladimir Vankov

University of Bristol, UK

Finiteness properties

I am interested in geometric, algebraic/homological, as well as more classical finiteness properties of groups. The latter includes properties such as residual finiteness and even being virtually torsion-free [6], whereas the former involves group actions on certain spaces or exact sequences of group modules.

An attractive feature of this topic is the way in which it draws on various aspects of geometry and group theory to create exotic combinations. For example, Morse theory [1] and cube complex geometry [7].

Finiteness properties often play a key role in the assumptions of many theorems in geometric group theory; such as in the remarkable result of Gersten [2], where the required condition for a subgroup of a low-dimensional hyperbolic group to be itself hyperbolic, is for it to be almost finitely presented. Hyperbolic geometry in general provides a rich playground for finiteness properties [5]. Indeed, the properties of subgroups of hyperbolic groups (only very recently has a subgroup of type F which is non-hyperbolic emerged [3]), as well as their own classical finiteness properties [4], remain mysterious.

[1] M. Bestvina and N. Brady. Morse theory and finiteness properties of groups. *Inventiones mathematicae*, 129:445-470, 1997.

[2] S. Gersten. Subgroups of word hyperbolic groups in dimension 2. *Journal of the London Mathematical Society*, 54(2):261–283, 1996.

[3] G. Italiano, B. Martelli and M. Migliorini. Hyperbolic 5-manifolds that fiber over S^1. *Inventiones mathematicae,* 2022.

[4] I. Kapovich and D. Wise. The equivalence of some residual properties of word-hyperbolic groups. *Journal of Algebra*, 223(2):562–583, 2000.

[5] R. Kropholler and V. Vankov. Finitely generated groups acting uniformly properly on hyperbolic space. To appear in: *Groups, Geometry and Dynamics*, arXiv:2007.13880.

[6] I. Leary and V. Vankov. On the virtual and residual properties of a generalization of Bestvina-Brady groups. *Mathematische Zeitschrift*, 302:2353-2373, 2022.

[7] V. Vankov. Virtually special non-finitely presented groups via linear characters. *Geometriae Dedicata*, 216:44, 2022.

Ignacio Vergara

Universidad de Santiago de Chile, Chile

Weaker forms of amenability: affine actions and approximation properties

My research lies at the intersection of functional analysis and group theory. I am particularly interested in generalisations of amenability arising from group actions on Banach spaces and finite-dimensional approximation properties of operator algebras. Two key concepts for my research can be traced back to Haagerup's highly influential work [1] on the reduced C*-algebra of the free group: weak amenability and the Haagerup property. Although these properties are analytic in nature, their connections with the geometry of groups have been extremely fruitful from the very beginning.

One of the problems I am interested in is understanding which classes of groups admit proper uniformly Lipschitz affine actions on subspaces of L^1, which can be thought of as a weaker form of the Haagerup property. In [2], I showed that this holds for groups acting properly on products of quasi-trees and for a subclass of weakly amenable groups. Later, in [3], I was able to give a proof for all hyperbolic groups. Among the many questions that still remain is whether $\mathrm{SL}(3,\mathbb{Z})$ admits such an action, which is known not to hold for subspaces of L^p with $p \in (1,\infty)$.

Another topic I am interested in is the Dixmier problem, which connects unitarisability of uniformly bounded representations with amenability. In [4], I studied an approximation property of groups, called M_d-AP, whose origins lie in harmonic analysis and operator algebras. I showed that unitarisable groups satisfying this new property are amenable. Furthermore, the class of groups with M_d-AP is much larger than the class of amenable groups, and it includes all the groups of the form $\Gamma \wr \Lambda$, where Γ is amenable and Λ acts properly on a finite-dimensional $\mathrm{CAT}(0)$ cube complex.

[1] Uffe Haagerup. An example of a nonnuclear C^*-algebra, which has the metric approximation property. *Invent. Math.*, 50(3):279–293, 1978/79.

[2] Ignacio Vergara. Proper cocycles for uniformly bounded representations on subspaces of L^1. *arXiv:2109.12949*, 2021.

[3] Ignacio Vergara. Hyperbolicity and uniformly Lipschitz affine actions on subspaces of L^1. *arXiv:2207.14691*, 2022.

[4] Ignacio Vergara. The M_d-approximation property and unitarisability. *Proc. Amer. Math. Soc.*, 151(3):1209-1220, 2023.

Maya Verma

University of Oklahoma, USA

Research Statement

I am graduate student at University of Oklahoma. I am working under the supervision of Dr. Max Forester in Geometric group theory. Before joining as a graduate student at OU, I completed my bachelors and masters degree in mathematics from Indian Institute of Science Education and Research, Bhopal. For my final year project during masters, I worked in the field of Topological K-theory.

I developed my interest in Geometric Group Theory after reading the notes by Brian H. Bowditch titled 'A course on geometric group theory' which introduces the basic concepts of GGT, like model space for a finitely generated group (Caley graph), quasi isometry, commensurabiity, Svarc-Milnor lemma, hyperbolic group and Gromov boundary. I explored other topics like CAT(0) cube complex and its geometry, special cube complex and, Marshall Hall's theorem which states that every finitely generated group of a finitely generated free group is virtually a free factor.
Currently, I am working on lattice of Aut$(X_{m,n})$; the set of combinatorial automorphism of combinatorial model of Baumsslag-solitar group BS(m, n) ($i.e.$ a locally finite CW complex on which BS(m, n) acts freely and cocompactly). I am also doing a reading course on Mapping Class Groups.
I aspire this workshop will give an insight on other interesting as well as cardinal topics in GGT. I look forward to meet various people working in the field of GGT through this workshop.

Elliott Vest

University of California, Riverside, USA

Characterizing Sublinearly Morse Geodesics

I am a fourth year PhD student at University of California, Riverside, and my advisor is Matthew Durham. My research involves characterizing sublinearly Morse geodesics in new lights in order to gain more information about sublinear Morse boundaries of CAT(0) spaces.

Definition. *Let X be a proper geodesic metric space. A geodesic α in X is said to be* ***N-Morse*** *if there exists a function $N = N(K, C)$ such that for any (K, C)-quasi-geodesic ϕ with endpoints on α, we have that ϕ is within the $N-$neighborhood of α.*

Such a definition was inspired by the Morse Lemma for hyperbolic spaces. That is, all geodesics in a Gromov hyperbolic space are $N-$Morse for some N. Thus, Morse geodesics in any proper geodesic metric space can be interpreted as the "hyperbolic directions" of that space. It is known that, when regarding hyperbolic spaces, a quasi-isometry between hyperbolic spaces induces a homeomorphism on their visual boundaries. Such a statement isn't true for non-Gromov hyperbolic spaces. However, when one looks only at Morse geodesic rays emanating from a basepoint (which we define as the ***Morse boundary***), we get the following:

Theorem (Cordes 2016)**.** *Any quasi-isometry between proper geodesic metric spaces will induce a homeomorphism on their Morse boundaries.*

This above result can also be found when regarding the $\kappa-$Morse boundary as well - where κ is some sublinear function. $\kappa-$**Morse geodesics** are defined similarly to Morse geodesics except that the neighborhood of a $\kappa-$Morse geodesic that bounds quasi-geodesics is allowed to grow in a sublinear fashion. This is a looser definition than the original Morse geodesic definition.

When working in a CAT(0) space, one can characterize sublinearly Morse geodesics using a new tool called ***curtains***. For any geodesic $\alpha : I \to X$ in a CAT(0) space, for any interval $[r - \frac{1}{2}, r + \frac{1}{2}] \subset I$ of length 1, the ***curtain dual*** to α at r is $h = \pi_\alpha^{-1}(\alpha[r - \frac{1}{2}, r + \frac{1}{2}])$ where π_α is the closest point projection of the space to α. A curtain in a CAT(0) space is meant to have the same feel as a hyperplane in a CAT(0) cube complex. Such a tool allows us to describe $\kappa-$Morse geodesics in a combinatorial fashion.

[1] Cordes, 2016, *Morse Boundaries of Proper Geodesic Metric Spaces*. arXiv:1502.04376

[2] Petyt, Spriano, Zalloum, 2022, *Hyperbolic Models for CAT(0) spaces*. arXiv:2207.14127

Christian Vock

Christian-Albrechts-Universität zu Kiel, Germany

Kac–Moody Symmetric Spaces

I am a PhD student of Ralf Köhl and I am interested in Kac–Moody groups and their symmetric spaces. As a starting point, one can consider Kac–Moody algebras, which are a generalization of Lie algebras to infinite dimensional Lie algebras with a generalized Cartan matrix. This matrix provides all the important information to construct such an algebra.

As in the theory of finite-dimensional semisimple Lie algebras, one can try to connect a group to a Kac–Moody algebra which behave like Lie groups. Therefore, one needs the Steinberg functor, which leads to a group containing mainly root groups (integrated roots). To complete this group one needs to add the whole Cartan integrated subalgebra, so one defines the split torus scheme and finally the group is given by the free product of the Steinberg functor together with the torus modulo of some relations that arise on the way to construct this group.

My PhD project starts at this point. In [1], my thesis advisor and his colleagues established a theory of symmetric Kac–Moody spaces that is similar to Riemannian symmetric spaces. The main difference is that in the Riemannian case one has all the tools of differential geometry at one's disposal, but for symmetric Kac-Moody spaces one does not. One powerful tool that remains, just like for Riemannian symmetric spaces of noncompact type, is the combinatorial structure of a building at infinity. Thus, one can use this machinery to study Kac–Moody symmetric spaces. The article [1] treats the theory only for split real Kac–Moody symmetric spaces and only for a special type of generalized Cartan matrices. So the first part of my project was to establish this theory for all types of generalized Cartan matrices and to apply it also to the domain of complex numbers.

The second part, which I am working on at the moment, is to apply the methods developed by Rèmy in [2] to symmetric Kac–Moody spaces. He developed a theory of almost split Kac–Moody groups via Galois descent. Galois descent is well known, for example, in the context of algebraic groups and vector spaces. In particular, for real vector spaces, one can obtain a complex vector space by expanding the scalar field; moreover, one can make a real basis of the vector space a complex basis. The idea of Galois descent is that one finds a real subvector space of the complex vector space such that the real basis of the subspace is just together with the complex scalars the basis of the whole vector space.

[1] Walter Freyn, Tobias Hartnick, Max Horn, and Ralf Köhl. *Kac–Moody symmetric spaces*. Münster J. Math. 13 (2020), 1–114.

[2] Bertrand Rémy. *Groupes de Kac–Moody déployés et presque déployés*. Astérisque, (277):viii+348, 2002.

Katie Vokes

University of Luxembourg, Luxembourg

Geometry of graphs associated to surfaces

My research so far has been around various **combinatorial objects** we can associate to a (topological) **surface** S which have applications to the study of mapping class groups, Teichmüller spaces and hyperbolic 3-manifolds. A first example is the **curve graph**, which has a vertex for every isotopy class of homotopically non-trivial simple closed curves in S. Masur and Minsky proved that the curve graph is Gromov hyperbolic, and in a later paper gave a **distance formula** for the word metric on the **mapping class group** MCG(S) in terms of projections to curve graphs of subsurfaces [1].

A generalisation of the ideas behind Masur–Minsky's distance formula for MCG(S) is the notion of a **hierarchically hyperbolic space** (HHS) defined by Behrstock, Hagen and Sisto. Such a metric space is equipped with a family of projection maps to Gromov hyperbolic spaces, satisfying a list of conditions which result in an analogous structure to that of the mapping class group. In particular, every hierarchically hyperbolic space has a distance formula like that for MCG(S). In [1], I proved that a family of graphs associated to surfaces are hierarchically hyperbolic spaces. Jacob Russell (Rice University) used my result to prove that some of these graphs are **relatively hyperbolic spaces**, and in joint work, we completed a classification of hyperbolicity and relative hyperbolicity for this family of graphs [2].

I also really like combinatorial objects associated to **3-manifolds**. The **disc graph** of a handlebody V is the subgraph of the curve graph of ∂V (∂V is a surface) which is spanned by those curves which bound discs in V. In [3], I gave an elementary proof of a result of Hamenstädt that the disc graphs are uniformly quasiconvex in the curve graphs. The disc graph has links to the study of **Heegaard splittings** and of **handlebody groups**.

I'm interested too by connections of curve graphs and similar graphs to other areas. For example, analogues of curve graphs for Artin groups, or sphere complexes used to study $\mathrm{Out}(F_n)$.

[1] Howard A. Masur, Yair N. Minsky, *Geometry of the complex of curves II: Hierarchical structure* : Geom. and Funct. Anal. **10** (2000) 902–974

[2] Kate M. Vokes, *Hierarchical hyperbolicity of graphs of multicurves* : Algebr. Geom. Topol. 22 (2022) 113–151

[3] Jacob Russell, Kate M. Vokes, *Thickness and relative hyperbolicity for graphs of multicurves* : preprint (2020) arXiv:2010.06464.

[4] Kate M. Vokes, *Uniform quasiconvexity of the disc graphs in the curve graphs* : in "Beyond Hyperbolicity", London Math. Soc. Lecture Note Ser. **454** (2019) 223–231

Noam von Rotberg
Otto von Guericke University Magdeburg, Germany

Conjugacy classes and reflection length in affine Coxeter groups

Deciding whether two given group elements are conjugate is a classical problem in group theory. In the setting of affine Coxeter groups conjugacy classes carry a geometric structure. Currently, in my masters thesis, I aim to understand this geometric meaning and connect it with the description of conjugacy classes given by Marquis in [1].

In any Coxeter group W the elements conjugate to the standard generators are called reflections. For a given group element $w \in W$ the minimal length of w written in this expanded generating set is called reflection length of w. Petra Schwer conjectured a new formula to compute the reflection length in affine Coxeter groups. As part of my bachelors thesis I provided a proof to this formula for affine Coxeter groups of rank one and two and showed one inequality in arbitrary rank [3].

In another project we computed connected components on unstructured regular triangle grids [2]. We used the method of cubulating the regular triangle tiling of the plane yielding a tiling by regular cubes. Using this one-to-one translation from triangles to cubes efficient algorithms on cubical grids can be used to compute connected components thereby solving the overall problem time-efficiently for large grids.

[1] T. Marquis, *Structure of conjugacy classes in Coxeter groups*, arXiv:2012.11015

[2] A. Voigt, P. Schwer, N. von Rotberg, N. Knopf, *TriCCo – a cubulation-based method for computing connected components on triangular grids*, arXiv:2111.13761

[3] N. von Rotberg, *Reflection length in affine Coxeter groups*, bachelors thesis, `https://www.mathi.uni-heidelberg.de/~geodyn/teaching/Theses/BA_2020_Noam%20von%20Rotberg%20(Pozzetti).pdf`

Thomas Witdouck

KU Leuven, Belgium

Which closed manifolds admit an Anosov diffeomorphism?

Anosov diffeomorphisms were introduced by D. Anosov as important examples of differentiable dynamical systems. They combine both chaotic behaviour with structural stability. For an exact definition, we refer to S. Smale's survey on differentiable dynamical systems [1]. The simplest example of an Anosov diffeomorphism is the map on the torus $\mathbb{R}^2/\mathbb{Z}^2$ induced by the matrix $\begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$, which is called Arnold's cat map. As of this moment, it is still unknown which closed manifolds can admit an Anosov diffeomorphism. The only known examples are on infra-nilmanifolds, i.e. manifolds finitely covered by a compact nilmanifold, and it is conjectured these are the only ones. One possible direction towards solving this conjecture is to look at the induced automorphism of an Anosov diffeomorphism on the fundamental group of the manifold. One would expect some sort of hyperbolic behaviour with respect to the word metric on the group. One of the objectives of my PhD is to come up with a proper definition of a hyperbolic automorphism on a finitely generated group and investigate how much the existence of such an automorphism restricts the structure of the group.

This methodology is based on the related problem of determining the closed manifolds which admit an expanding map. In this case, the induced morphism on the fundamental group is expanding with respect to the word metric and has an image of finite index. It is not hard to show that a finitely generated group admitting such a morphism must have polynomial growth and thus using Gromov's celebrated theorem on groups of polynomial growth [2], that such a group is virtually nilpotent. This, together with the fact that the universal cover of a closed manifold admitting an expanding map is contractible, lead to a proof that every expanding map on a closed manifold is topologically conjugate to an affine map on an infra-nilmanifold.

[1] S. Smale. Differentiable dynamical systems. Bull. Amer. Math. Soc., 73,:pp. 747-817, 1967.

[2] Gromov, M. (1981). Groups of polynomial-growth and expanding maps. Publications Mathématiques. Institut Des Hautes Études Scientifiques, 53, 53–78.

Yandi Wu
University of Wisconsin, Madison, USA

Rigidity and Right-Angled Coxeter Groups

I am a fifth year PhD student interested in rigidity results on quotients of model geometries of Right-Angled Coxeter Groups (RACGs). For example, closed surfaces glued along simple closed curves, or surface amalgams, are branched covers of some of these quotients (see [1]). Most recently, I have been studying marked length spectrum rigidity of surface amalgams, which one can think of as a generalization of the classical result that the lengths of $9g - 9$ simple closed curves on a hyperbolic closed surface completely determine the metric.

In the past, I have also studied topological rigidity on Davis orbicomplexes, $K(G, 1)$ spaces of RACGs (a collection of topological spaces is *topologically rigid* if an isomorphism on the level of fundamental groups induces a homeomorphism). While the class of Davis orbicomplexes of 1-ended RACGs is known to be far from topologically rigid, I found an infinite subcollection of orbicomplexes that is (see [2]).

While the abstract commensurability classification of RACGs rends the QI rigidity problem incredibly nuanced and messy, I am also happy to discuss any results in that direction, as that has also been an interest of mine.

[1] Pallavi Dani, Emily Stark, and Anne Thomas. Commensurability for certain right-angled Coxeter groups and geometric amalgams of free groups. *Groups, Geometry, and Dynamics*, 12(4):1273-1341, 2018.

[2] Yandi Wu. A topologically rigid set of quotients of the Davis complex. `https://arxiv.org/abs/2204.12028`, 2022.

David Xu

Institut Elie Cartan de Lorraine, Nancy, France

Topology and geometry of $\mathrm{Isom}(\mathbf{H}^\infty)$

Similarly to Euclidean spaces, there is an infinite dimensional analog of the finite dimensional hyperbolic spaces.

Let $\mathcal{H}$ be a real separable Hilbert space and let Q be the quadratic form defined by $Q(x) = -x_0^2 + \sum_{i\geqslant 1} x_i^2$ where the x_i are the coordinates of $x \in \mathcal{H}$ in some fixed Hilbert basis. The separable infinite dimensional hyperbolic space is

$$\mathbf{H}^\infty = \{x \in \mathcal{H} \mid Q(x) = -1, x_0 > 0\}.$$

As in finite dimension, there is a hyperbolic distance on $\mathbf{H}^\infty$ which makes it a complete geodesic $\mathrm{CAT}(-1)$ space. My main objects of study are the discrete subgroups of its group of isometries $\mathrm{Isom}(\mathbf{H}^\infty) = PO(\infty, 1)$.

To define what a discrete subgroup $\Gamma < \mathrm{Isom}(\mathbf{H}^\infty)$ is, we can

- either put a topology on $\mathrm{Isom}(\mathbf{H}^\infty)$ (for example the pointwise convergence topology as studied in [3] by Duchesne);
- or use the action $\mathrm{Isom}(\mathbf{H}^\infty) \curvearrowright \mathbf{H}^\infty$ (for example by calling Γ discrete if it acts on $\mathbf{H}^\infty$ with discrete orbits).

Some issues already appear here, since these points of view are not equivalent contrary to the finite dimensional setting, as described in [1].

I am interested in finding and studying new examples of subgroups of $\mathrm{Isom}(\mathbf{H}^\infty)$ that are (as) discrete (as possible) by generalizing constructions coming from the standard case, such as Coxeter groups and groups generated by reflections. Another idea to produce subgroups acting irreducibly on $\mathbf{H}^\infty$ is to deform convex-cocompact representations coming from the exotic representations $\rho : \mathrm{Isom}(\mathbf{H}^n) \to \mathrm{Isom}(\mathbf{H}^\infty)$ studied by Delzant, Monod and Py in [2] and [4].

[1] T. Das, D. Simmons and M. Urbański. Geometry and dynamics in Gromov hyperbolic metric spaces. *Mathematical Surveys and Monographs 218* (2017).

[2] T. Delzant and P. Py. Kähler groups, real hyperbolic spaces and the Cremona group. *Compos. Math. 148*, No. 1, 153-184 (2012).

[3] B. Duchesne. The Polish topology of the isometry group of the infinite dimensional hyperbolic space. ArXiv:2005.12204.

[4] N. Monod and P. Py. An exotic deformation of the hyperbolic space. *Am. J. Math. 136*, No. 5, 1249-1299 (2014).

Sofiya Yatsyna

Royal Holloway, University of London, United Kingdom

Totally disconnected locally compact groups

I first studied the cohomology of groups using [2] during my masters. I wrote my dissertation focused on crystallographic groups and an introduction to cohomology. It focused on low dimensions and properties of the cohomology of finite and cyclic groups. In particular, I interpreted definitions of crystallographic groups presented in [3] topologically and algebraically. Algebraically, using cohomology, I proved Bieberbach's three theorems and the Main Theorem of Mathematical Crystallography. I provided examples of how crystals may be distinguished in low dimensions.

Currently, I am in the first year of my PhD under the supervision of Professor Nucinkis at Royal Holloway. I'm interested in totally disconnected locally compact groups. Following my reading of [1], I hope to look at the finiteness conditions of totally disconnected locally compact groups over the next few years

[1] K. Brown, *Cohomology of groups*. Springer-Verlag, New York.

[2] I. Castellano and T. Weigel, *Rational discrete cohomology for totally disconnected locally compact groups.* **J. Algebra**, 453 (2016), 101–159.

[3] R. L. E. Schwarzenberger, *N-dimensional crystallography* Pitman (Advanced Publishing Program), Boston, Mass.- London, 1980.

Paul Zellhofer
Christian-Albrechts-Universität zu Kiel, Germany

Kostant's convexity theorem for split real Kac-Moody groups

Kostant's convexity theorem is a classical result in the theory of Lie groups & Lie algebras. It generalizes the famous *Schur-Horn theorem* from linear algebra that completely determines the possible diagonal vectors of Hermitian matrices with a prescribed set of eigenvalues. In this form, the theorem reduces to the following result. Let $n \in \mathbb{N}_{\geq 1}$, $d = (d_1, \ldots, d_n) \in \mathbb{R}^n$ be a point and denote by D the $(n \times n)$-matrix whose diagonal vector is d and whose off-diagonal entries are all equal to 0. For any complex unitary $(n \times n)$-matrix A, the conjugate ADA^{-1} is a Hermitian matrix with eigenvalues $d_1, \ldots, d_n$ and every such matrix is of this form. Moreover, its diagonal vector has real entries, so the set $\mathcal{S}$ of all diagonal vectors of matrices of the form ADA^{-1}, where $A \in U(n)$, is a subset of $\mathbb{R}^n$. The Schur-Horn theorem asserts that $\mathcal{S}$ coincides with the convex hull of the points $\{(d_{\pi(1)}, \ldots, d_{\pi(n)}) \mid \pi \in S_n\}$ obtained from all permutations of d.

In the 1970's, B. Kostant realized that the Schur-Horn theorem was a particular instance of a far more general result in Lie theory. If G is a semisimple Lie group and $G = KAN$ an Iwasawa decomposition, every element $g \in G$ can be uniquely written as $g = kan$ with $k \in K$, $a \in A$ and $n \in N$ and we denote the A-component by $A(g) = A(kan) := a$. We then have $A(kg) = A(g)$ for every $k \in K$, but there is no obvious way to compute $A(gk)$. Starting with an element in A, the subset of A obtained in this way from right multiplication by elements of K has an explicit description. To formulate it, let $\mathfrak{g}$ and $\mathfrak{a}$ be the Lie algebras of G and A, then the exponential map of G restricts to a bijection $\exp : \mathfrak{a} \to A$ whose inverse is denoted by $\log : A \to \mathfrak{a}$. Kostant's convexity theorem [1, Theorem 4.1] states that if $a \in A$ is arbitrary, then

$$\{A(ak) \mid k \in K\} = \exp \operatorname{conv}(W \cdot \log(a)),$$

where W is the Weyl group of $\mathfrak{g}$ with respect to $\mathfrak{a}$ and $\operatorname{conv}(W \cdot \log(a))$ denotes the convex hull of the Weyl group orbit of $\log(a) \in \mathfrak{a}$.

Kac-Moody groups are a generalization of semisimple Lie groups and share many of their properties. A weaker version of Kostant's theorem is also true in that setting [2, Theorem 2], but a much closer analogue of Kostant's statement also makes sense for Kac-Moody groups. Its validity is a matter of ongoing research which has recently also been linked to certain billiard models in quantum gravity.

[1] Kostant, Bertram. *On convexity, the Weyl group and the Iwasawa decomposition.* Annales scientifiques de l'École Normale Supérieure, Série 4, Tome 6 (1973) no. 4, pp. 413-455.

[2] Kac, Victor G. and Peterson, Dale. *Unitary structure in representations of infinite-dimensional groups and a convexity theorem.* Inventiones mathematicae 76 (1984): 1-14.

Felix Zhang
FernUniversität in Hagen, Germany

Zeta functions of induced representations

A representation ϱ of a profinite group G is called *strongly admissible*, if it decomposes into a direct sum of irreducible representations and if for each natural number n, the number $r_n(\varrho)$ of irreducible consistuents with dimension n is finite. The field of representation growth is concerned with determining the dimensions of these constituents, because we believe it will give us insight into the group structure of G.

You can assign a strongly admissible representation the *zeta function*

$$\zeta_\varrho(s) = \sum_{n=1}^{\infty} \frac{r_n(\varrho)}{n^s}$$

Representations for which the zeta function has a finite abscissa of convergence are called *polynomially strongly admissible*.

The General Linear Group G of degree n over the p-adic integers $\mathbb{Z}_p$ acts on the set of free submodules of $\mathbb{Z}_p^n$ with rank $m \leq n/2$. This gives rise to an induced representation $\varrho = \mathrm{Ind}_S^G(1_G)$, where 1_G is the trivial representation of G and S is the stabilizer subgroup of G with respect to an arbitrary free submodule. Kionke co-authored a paper, in which they determined the zeta function for the case $m = 1$ [1].

I am currently trying to determine the representation zeta function of the induced representations for the cases $m \geq 2$.

[1] S. Kionke, B. Klopsch, *Zeta functions associated to admissible representations of compact p-adic Lie groups*, Trans. Amer. Math. Soc. 372 (2019), no. 11, 7677–7733.

Michael Zshornack

UC Santa Barbara, United States of America

Arithmetics of higher Teichmüller spaces and thin groups

My research is centered around the interactions between number theory and low-dimensional topology. A broad goal of mine is to understand discrete subgroups of lattices in semisimple Lie groups which are Zariski dense. Such groups are known as *thin groups* (cf. [3]) and have lead to rich areas of research in number theory in recent years. For instance, tools such as Super Approximation and expander families have been developed to study the various interesting properties of these groups (see [1]).

Among the thin groups, those which are free are relatively well understood compared to those which are not, and so it is of interest to understand those thin groups which are *freely indecomposable*, i.e. ones which do not decompose as a free product of two smaller groups.

My work has been focused on studying the *Hitchin component*, an example of a *higher Teichmüller space* (cf. [4]), to produce new examples of Zariski dense surface subgroups of $\mathrm{SL}(n, \mathbf{R})$. This perspective was taken by Long and Thistlethwaite in [2] where the authors use *bending*, a construction of Thurston's, to produce Zariski dense surface groups inside $\mathrm{SL}(2k+1, \mathbf{Z})$. Their result provided the first examples of freely indecomposable isomorphism types of thin groups in $\mathrm{SL}(n, \mathbf{Z})$ for infinitely many n.

In [5], I generalize the methods of [2] to show that any representation of a closed surface group into $\mathrm{SL}(n, \mathbf{R})$ on the Hitchin component may be deformed using this same bending construction to being Zariski dense, while preserving integrality properties of the original representation. This reduces the problem of finding thin surface groups in $\mathrm{SL}(n, \mathbf{R})$ to finding integral ones.

Many interesting questions along these lines remain open for future research. For instance, it still seems largely difficult to understand for which n, $\mathrm{SL}(n, \mathbf{Z})$ contains a surface subgroup. In general, studying the arithmetics of higher Teichmüller spaces through methods of low-dimensional topology and geometric group theory provides one possible path towards some answers to these and other related questions.

References

[1] J. Bourgain, A. Gamburd and P. Sarnak, *Affine linear sieve, expanders, and sum-product,* Inventiones mathematicae **179** (2010), no. 3, $559-644$.

[2] D. D. Long and M. B. Thistlethwaite, *Zariski dense surface groups in* $\mathrm{SL}(2k+1, \mathbf{Z})$*,* Geometry and Topology, to appear.

[3] P. Sarnak, *Notes on thin matrix groups,* 2012.

[4] A. Wienhard, *An invitation to higher Teichmüller theory,* 2018.

[5] M. Zshornack, *Integral Zariski dense surface groups in* $\mathrm{SL}(n, \mathbf{R})$*,* in preparation.