# Workshop: <br> Random discrete structures 

March 20-24, 2023
Münster

## Organizing Committee:

Gerold Alsmeyer Zakhar Kabluchko
Steffen Dereich Matthias Löwe
Anna Gusakova

## General information

Venue. Registration: Second floor of the SRZ (Orléans-Ring 12, cf. Map A) on Monday.
Lecture room: SRZ 217.
You can find the latest information on the homepage (tinyurl.com/RaDiS23).

Wi-Fi access. If you are part of the eduroam community, you may connect to the network "eduroam" as usual. Otherwise you can connect to the SSID "GuestOnCampus" and start any web browser. You will automatically be redirected to the login page. Confirm the terms of use and click on "log in for free". 1 GB data volume is available per device and day. Please note that the connection is not encrypted.

Talks. Please send your presentation to uzie.kupffer@uni-muenster.de.

Coffee break/Lunch. We provide coffee and snacks during the coffee breaks. There are a couple of restaurants for lunch in the vicinity:

- Canteen - Mensa am Ring, Domagkstraße 61 (most convenient option, even if not the most idyllic place)
- Ristorante Milano (Italian), Wilhelmstraße 26 (closed on Monday)
- II Gondoliere (Italian), Von-Esmarch-Straße 28 (closed on Monday)
- Buddha Palace (Indian), Von-Esmarch-Straße 18
- La Gondola D’oro (Italian), Hüfferstraße 34
- A2 am See (German), Annette-Allee 3
- Gustav Grün (Green Fast Food), Wilhelmstraße 1
- Áro (Green Fast Food), Neutor 3
- Royals and Rice (Vietnamese Fusion), Frauenstraße 51

Conference dinner. The conference dinner starts on Wednesday at 18.30 at the Schlossgarten (Schlossgarten 4).

Public transportation. You can check the bus schedule on the website of Stadtwerke-Münster (in German and English), or use Google maps.

Free afternoon on Wednesday/City tour. There will be a free afternoon on Wednesday. The city tour starts in front of the City Hall at 15:00 (selfpaid, $10 €$ per participant). You may walk together to the City Hall from the conference venue at 14:30 (meeting point: in front of the conference building). Further suggestions: You may want to go and see the castle, its surroundings and the botanical garden which is right next to it. You can also visit a museum, e.g. the LWL Museum of Art and Cultural History, or the Picasso-Museum. You may also enjoy a walk around the lake " Aasee" or visit the City Hall, a centerpiece of European history, where the "Westphalian peace" terminating the Thirty Years' War was signed in 1648. Further information will be provided during the conference.

Emergency contact. If you need help, please contact Morris Kopelke:
Email: m.kopelke@uni-muenster.de
Phone: +49 2518333784

## Acknowledgements

The conference is funded by the Cluster of Excellence "Mathematics Münster" and part of the focus programme "Effective and dynamic behaviour of random structures".

## Schedule

## Monday, March 20

08:30-09:00 Registration
09:00-09:50 Nina Gantert: Branching random walks with annihilation
09:50-10:40 Jean-Francois Delmas: SIS model on dense graphs: probabilistic approach and remarks on (deterministic) optimal vaccinations

## 10:40-11:10 Coffee

11:10-12:00 Cornelia Pokalyuk: Invasion of cooperative parasites in structured host populations

12:00-14:00 Lunch

14:00-14:50 Daniel Valesin: Scaling limit of an adaptive contact process
14:50-15:40 Maria Deijfen: Superconcentration, chaos and multiple valleys in first passage percolation

15:40-16:10 Coffee

16:10-17:00 Pierre Calka: Fluctuation theory for random polytopes
17:00-17:50 Mathias Sonnleitner: A probabilistic approach to Lorentz balls
18:30 Poster session \& Wine and cheese reception, open end

## Tuesday, March 21

09:00-09:50 Zhan Shi (Zoom): Some questions on random recursive models 09:50-10:40 Bastien Mallein: Extremal process of multidimensional branching Brownian motion
10:40-11:10 Coffee
11:10-12:00 Noemi Kurt: Particle systems with coordination, and the seedbank coalescent with simultaneous switching
12:00-14:00 Lunch
14:00-14:50 Peter Mörters: Metastability of the contact process on evolving scale-free networks
14:50-15:40 Oleksandr Marynych: High-dimensional limits for random walks 15:40-16:10 Coffee
16:10-17:00 Nelly Litvak: Detecting hyperbolic geometry in networks17:00-17:50 Andrea Montanari (Zoom): Local algorithms for Maximum Cutand Minimum Bisection on locally treelike graphs
Wednesday, March 22
09:00-09:50 Mathew Penrose: Components of dense random geometric graphs
09:50-10:40 Daniel Hug: Poisson flats in constant curvature spaces
10:40-11:10 Coffee
11:10-12:00 Yueyun Hu: Capacity of the range of a critical branching random walk
12:00-12:50 Bénédicte Haas: Tail asymptotics for exponential functionals of subordinators and extinction times of self-similar fragmentations
12:50-14:00 Lunch
15:00 City tour (Start: in front of the City Hall)
18:30 Conference dinner (Schlossgarten Café)

## Thursday, March 23

| 09:00-09:50 | Julien Berestycki: The F-KPP equation in the half-plane |
| :--- | :--- |
| 09:50-10:40 | Pascal Maillard: Discrete-time Hawkes process with inhibition |
| 10:40-11:00 | Coffee |
| 11:00-11:50 | Dariusz Buraczewski: Random walks in a sparse random |
| environment |  |

Friday, March 24
09:00-09:50 Debleena Thacker (Zoom): Continuous-time digital search tree and a border aggregation model09:50-10:40 Tobias Müller: Cycles in Mallows random permutations10:40-11:00 Coffee
11:00-11:50 Oleksandr Iksanov: FLTs for nested Karlin's occupancy scheme generated by light-tailed distributions
11:50-12:40 Christoph Thäle: Random (Beta) Polytopes

## Maps and locations



Map A: Lecture building, canteen, SRZ, MM building, parking lot.


Map B: Route to Agora Gästehaus am Aasee.


Map C: Route to Hotel Jellentrup.


Map D: Route to Stadthotel Münster.

## Book of abstracts

# The mutation process on the ancestral line in a model of population genetics 

Ellen Baake (Bielefeld)

We consider the Moran model of population genetics with two types, mutation, and selection, and investigate the line of descent of a randomlysampled individual from a contemporary population. We trace this ancestral line back into the distant past, far beyond the most recent common ancestor of the population (thus connecting population genetics to phylogeny) and analyse the mutation process along this line.

To this end, we use the pruned lookdown ancestral selection graph, a random graph that consists of the set of potential ancestors of the sampled individual at any given time. A crucial observation is that a mutation on the ancestral line requires that the ancestral line occupy the top position in the graph just 'before' the event (in forward time). The type process on the ancestral line is not Markov, but can be turned into a Markov process by adding the number of lines in the graph and thus enlarging the state space. We show that, relative to the case without selection, the average rate of beneficial (deleterious) mutations increases (decreases) with the selection intensity. This is in line with the fact that the ancestral line consists of individuals that have descendants surviving until the present; we therefore expect a bias towards the beneficial type. The proof relies on an intriguing connection between the sampling recursions (for the types in a sample from the present population) and Fearnhead's recursions (for the tail probabilities of the stationary distribution of the line-counting process of the pruned lookdown ancestral selection graph).

This talk is based on joint work with Fernando Cordero and Enrico Di Gaspero.

## The F-KPP equation in the half-plane

## Julien Berestycki (Oxford)

It has been shown by H. Berestycki and G. Cole (2022) that the F-KPP equation $\partial_{t} u=\frac{1}{2} \Delta u+u(1-u)$ in the half-plane with Dirichlet boundary conditions admits travelling wave solutions for all speed $c \geq c^{*}=\sqrt{2}$.

We show that the minmal speed travelling wave $\Phi$ is in fact unique (up to shift) and give a probabilistic representation as the Laplace transform of a certain martingale limit associated to the branching Brownian motion with absorption. This representation allows us to study the asymptotic behaviour of $\Phi$ away from the boundary of the domain, proving that

$$
\lim _{y \rightarrow \infty} \Phi\left(x+\frac{1}{\sqrt{2}} \log y, y\right)=w(x)
$$

where $w$ is the usual one-dimensional critical travelling wave.
We are able to extend our result to the case of the half-space $\mathbb{H}^{d}=\{x \in$ $\left.R^{d}: x_{1} \geq 0\right\}$. Finally, if time allows, I will also mention some results regarding the convergence towards the critical travelling wave.

This talk is based on joint work with Graham Cole, Yujin Kim and Bastien Mallein.

## Random walks in a sparse random environment

## Dariusz Buraczewski (Wroclaw)

Random walks in random environment (RWRE) were introduced in the 1970s to model a random motion of a particle in presence of some kind of obstacles. The behavior of any RWRE is affected by both randomness of the environment and randomness of the walker. During the talk we introduce random walks in a sparse random environment. The integer points of the real line are marked by the positions of a standard random walk with positive integer jumps. We consider a nearest neighbor random walk on the set of integers having jumps $+/-1$ with probability $1 / 2$ at every nonmarked site, whereas a random drift is imposed at every marked site. We will present some new limit theorems for the so defined random process.

This talk is based on joint work with Piotr Dyszewski, Alexander Iksanov, Alicja Kołodziejska and Alexander Marynych.

## References

[1] D. Buraczewski, P. Dyszewski, A. Iksanov, A. Marynych and A. Roitershtein, Random walks in a moderately sparse random environment, Electronic Journal of Probability, 24(69), 1-44, 2019.
[2] D. Buraczewski, P. Dyszewski, A. Iksanov and A. Marynych, Random walks in a strongly sparse random environment, Stochastic Processes and their Applications, 130, 3990-4027, 2020.
[3] D. Buraczewski, P. Dyszewski and A. Kołodziejska, Weak quenched limit theorems for a random walk in a sparse random environment, https://arxiv.org/abs/2301.00478.

## Fluctuation theory for random polytopes

Pierre Calka (Rouen)

We consider the random polytope generated by $n$ independent and uniformly distributed points in a smooth convex body called the mother body. We are in particular interested in its fluctuations with respect to the limit shape of the mother body when the size $n$ of the random input goes to infinity. In a work that complements our previous study of extremal convergences, we concentrate on the typical distributions of several characteristics of the facets of the polytope. We show that the typical heights, typical facet volume and typical facet diameter have explicit limit distributions in two different regimes. In the particular case of dimension two and when the mother body is the ball of radius $n$, we obtain that the limit distribution of the typical height resembles a Tracy-Widom distribution. Moreover, the corresponding height process satisfies a so-called 1:2:3 scaling property which makes it similar to the growth processes from the infamous KPZ universality class, even though the limit process, known as the Burgers' festoon, does not involve any Airy process. Interestingly, the analogy with the KPZ equation seems to disappear in higher dimension.

This talk is based on joint work with J. E. Yukich.

# Superconcentration, chaos and multiple valleys in first passage percolation 

Maria Deijfen (Stockholm)

We consider a dynamical version of first-passage percolation on the $d$ dimensional integer lattice with i.i.d. edge weights, where edge weights are resampled independently in time. Let $T(n)$ denote the first-passage time from the origin to the site $n$ steps along the first coordinate axis at time $t=0$, and let $\mu(t)$ denote the expected overlap between the time minimizing paths at time 0 and $t>0$. We show that a subdiffusive behaviour of $T(n)$ is equivalent to a chaotic behavior of the time minimizing paths, manifested in that $\mu(t)=o(n)$. Known bounds for $\operatorname{Var}(T(n))$ thus imply that indeed $\mu(t)=o(n)$ for $t>0$. As a consequence, we show that there are many almost disjoint paths with almost optimal passage time. This gives evidence that earlier work by Sourav Chatterjee for certain Gaussian disordered systems reflects a more general principle.

This talk is based on joint work with Daniel Ahlberg and Matteo Sfragara.

## References

[1] D. Ahlberg, M. Deijfen and M. Sfragara: Chaos, concentration and multiple valleys in first passage percolation, submitted (2023), arxiv.org/abs/2302.11367.
[2] D. Ahlberg, M. Deijfen and M. Sfragara: From stability to chaos in lastpassage percolation, submitted (2023), arxiv.org/abs/2302.11379.

SIS model on dense graphs: probabilistic approach and remarks on (deterministic) optimal vaccinations

## Jean-Francois Delmas (Paris)

We consider an individual based model on a random dense graph for a SIS epidemic in an heterogeneous population of size $n$ :

- Each individual $i$ has a feature (age, localization, ...), say $x_{i} \in X$.
- Individuals $i$ and $j$ are connected with probability $w_{E}^{(n)}\left(x_{i}, x_{j}\right) \in[0,1]$.
- Individual $i$ is either Susceptible (S) or Infectious (I), and denote by $E_{t}^{i}$ its state at time $t \geq 0$.
- Individual $j$ infects individual $i$ at rate $w_{I}^{(n)}\left(x_{i}, x_{j}\right) \geq 0$ (provided that $i$ is $\mathrm{S}, j$ is I and $i, j$ are connected).
- Individual $i$ recover at rate $\gamma\left(x_{i}\right)>0$ (provided that $i$ is I$)$.

The infected population at time $t$ is described by the random measure:

$$
\rho_{t}^{(n)}(\mathrm{d} x)=\frac{1}{n} \sum_{i=1}^{n} \mathbf{1}_{\left\{E_{t}^{i}=I\right\}} \delta_{x_{i}}(\mathrm{~d} x) .
$$

Under suitable conditions (see [1]), the random measure $\rho_{t}^{(n)}$ converges to a deterministic limit $\rho_{t}(\mathrm{~d} x)=u(t, x) \mu(\mathrm{d} x)$, with $u(t, x)$, the infected fraction at time $t$ of individuals with feature $x$, unique solution to:

$$
\partial_{t} u(t, x)=(1-u(t, x)) \int_{x^{\prime} \in X} k\left(x, x^{\prime}\right) u\left(t, x^{\prime}\right) \mu\left(\mathrm{d} x^{\prime}\right)-\gamma(x) u(t, x),
$$

where $\mu\left(\mathbf{d} x^{\prime}\right)$ is the (asymptotic) distribution of the feature $x^{\prime}$ in the population and:

$$
k=\lim _{n \rightarrow \infty} n w_{E}^{(n)} w_{I}^{(n)}
$$

The reproduction number $R_{0}$ is then the spectral radius of the integral operator with kernel $k\left(x, x^{\prime}\right) / \gamma\left(x^{\prime}\right)$, see [2]. If $R_{0} \leq 1,0$ is the only equilibrium of the ODE and $\lim _{t \rightarrow \infty} u(t, \cdot)=0$; if $R_{0}>1$ then there exists a non-zero maximal equilibrium, say $g^{*}$.

The use of a perfect vaccine at time $t=0$, can be interpreted as replacing the population distribution $\mu(\mathrm{d} x)$ by the effective population distribution $\eta(x) \mu(\mathrm{d} x)$, where $\eta(x)$ is the fraction of un-vaccinated population with feature $x$, see $[2,3]$. The corresponding effective reproduction number $R_{e}(\eta)$ is the spectral radius of the integral operator with kernel $k\left(x, x^{\prime}\right) \eta\left(x^{\prime}\right) / \gamma\left(x^{\prime}\right)$. In particular, vaccinating uniformly the population with probability $1-1 / R_{0}$ (that is $\left.\eta^{\text {unif }}=1 / R_{0}\right)$ is critical as $R_{e}\left(\eta^{\text {unif }}\right)=1$ (so that the epidemic vanishes asymptotically).

We prove in [4] that vaccinating a fraction $g^{*}(x)$ of the population with feature $x$, that is $\eta=1-g^{*}$, is critical:

$$
R_{e}\left(1-g^{*}\right)=1
$$

This talk is based on joint work with D. Dronnier, P. Frasca, F. Garin, V. C. Tran, A. Velleret and P.-A. Zitt.

## References

[1] J.-F. Delmas, P. Frasca, F. Garin, V. C. Tran, A. Velleret, and P.-A. Zitt. Individual based SIS models on (not so) dense large random networks. Arxiv (2023)
[2] J.-F. Delmas, D. Dronnier, and P.-A. Zitt. An infinite-dimensional metapopulation SIS model. Journal of Differential Equations (2022).
[3] J.-F. Delmas, D. Dronnier, and P.-A. Zitt. Targeted vaccination strategies for an infinite-dimensional SIS model. ArXiv (2022).
[4] J.-F. Delmas, D. Dronnier, and P.-A. Zitt. Vaccinating according to the maximal endemic equilibrium achieves herd immunity. ArXiv (2022).

# Branching random walks with annihilation 

## Nina Gantert (Munich)

We study a branching annihilating random walk in which particles move on the $d$-dimensional lattice and evolve in discrete generations. Each particle produces a poissonian number of offspring with mean $\mu$ which independently move to a uniformly chosen site within a fixed distance $R$ from their parent's position. Whenever a site is occupied by at least two particles, all the particles at that site are annihilated. We prove that for any $\mu>1$ the process survives when $R$ is sufficiently large. For fixed $R$ we show that the process dies out if $\mu$ is too small or too large. Furthermore, for fixed (but large) $R$ and $1<\mu<e^{2}$ we exhibit an interval of $\mu$-values for which the process survives. For such $\mu$ 's we can also show that the process has a unique non-trivial ergodic equilibrium and prove complete convergence starting from arbitrary initial conditions. The main techniques involve comparison with oriented percolation and coupling arguments.

This talk is based on joint work with Alice Callegaro, Matthias Birkner, Jirí Černý and Pascal Oswald.

# Tail asymptotics for exponential functionals of subordinators and extinction times of self-similar fragmentations 

## Bénédicte Haas (Paris)

Exponential functionals of subordinators have been thoroughly investigated since they are involved in the des- cription of various processes ranging from the analysis of algorithms to coagulation or fragmentation processes. In this talk we will provide the exact large-time equivalents of the density and upper tail distribution of the exponential functional of a subordinator in terms of its Laplace exponent. This improves previous results on the logarithmic asymptotic behaviour of this tail.

We will then see how this result can be used to determine the large-time behavior of the tail distribution of the extinction time of a self-similar fragmentation process with a negative index of self-similarity. The extinction time of a typical fragment in such a process is an exponential functional of a subordinator. But the tail of the extinction time of the whole fragmentation process decreases much more slowly in general. We will quantify this difference by determining the asymptotic ratio of the two tails.

## Critical percolation on scale-free random graphs

## Remco van der Hofstad (Eindhoven)

Empirical findings have shown that many real-world networks are scalefree, in the sense that there is a high variability in the number of connections of the elements of the networks. Spurred by these empirical findings, models have been proposed for such networks.

Percolation on networks is one of the simplest models for network functionality. It can be interpreted as describing the effect of random attacks on the network, where edges are removed independently with a fixed probability, or the result of a simple epidemic on the network.

We investigate the percolation critical behavior for a popular models of complex networks, the Poisson random graph, which can be interpreted as a model with multi-edges, or single edges by collapsing the multi-edges. We identify what the critical values are, and how they scale with the graph size. Interestingly, this scaling turns out to be rather different for the multi-edge case compared to the single-edge case in the scale-free regime. This clears up part of the confusion in the physics literature. Furthermore, the singleedge case has an unexpected phase transition at the appropriate scale of the percolation parameter, where the size of the largest component jumps from a random value to a much larger almost deterministic value that is proportional to the root of the graph size.

## Capacity of the range of a critical branching random walk

## Yueyun Hu (Paris)

Let $R_{n}$ be the range of a critical branching random walk with $n$ particles on $\mathbb{Z}^{d}$, which is the set of sites visited by a random walk indexed by a critical Galton-Watson tree conditioned on having exactly $n$ vertices. For $d \in\{3,4,5\}$, we show that $n^{-\frac{d-2}{4}} \operatorname{Cap}\left(R_{n}\right)$, the renormalized capacity of $R_{n}$, converges in law to the capacity of the support of the integrated superBrownian excursion. The proof relies on a study of the intersection probabilities between the critical branching random walk and an independent simple random walk on $\mathbb{Z}^{d}$.

This talk is based on joint work with Tianyi Bai.

## References

[1] Bai, T. and Hu. Y. (2022+) Convergence in law for the capacity of the range of a critical branching random walk. Ann. Appl. Probab.
[2] Bai, T. and Wan. Y. (2022). Capacity of the range of tree-indexed random walk. Ann. Appl. Probab.
[3] Le Gall, J.F. and Lin, S. (2015). The range of tree-indexed random walk in low dimensions. Ann. Probab.
[4] Le Gall, J.F. and Lin, S. (2016). The range of tree-indexed random walk. J. Inst. Math. Jussieu.

# Poisson flats in constant curvature spaces 

## Daniel Hug (Karlsruhe)

Poisson hyperplane processes in Euclidean, spherical and more recently also in hyperbolic space have been investigated. It has been found that certain mean values of Poisson functionals in these spaces do not differ too much, but the asymptotic behaviour of variances and the asymptotic fluctuations may depend on the signature of the constant curvature $\kappa \in$ $\{-1,0,1\}$ in an essential way, depending on the specific asymptotic setup that is considered. For instance, for fixed intensity and a growing spherical observation window we have previously shown [1] that a central limit theorem holds in hyperbolic space $\mathbb{H}^{d}$, for the volume of intersection processes of Poisson $(d-1)$-flats within the observation window only if $d \in\{2,3\}$, which is in stark contrast to the corresponding behaviour in Euclidean spaces or to the situation for growing intensity. Several questions arise naturally:

1) What is so special about dimension 4 in hyperbolic space?
2) What is the limit behaviour if no CLT holds?

We present some answers to these questions and also discuss asymptotic and extremal properties of related variances.

This talk is based on joint work with Carina Betken and Christoph Thäle.

## References

[1] F. Herold, D. Hug and Ch. Thäle. Does a central limit theorem hold for the $k$-skeleton of Poisson hyperplanes in hyperbolic space? Probab. Theory and Relat. Fields 179(3) (2021), 889-968.
[2] C. Betken, D. Hug., Ch. Thäle. Intersections of Poisson $k$-flats in constant curvature spaces. Preprint (2023) ArXiv:2302.09524

# FLTs for nested Karlin's occupancy scheme generated by light-tailed distributions 

## Oleksandr Iksanov (Kyiv)

Let $\left(p_{k}\right)_{k \in \mathbb{N}}$ be a discrete probability distribution for which the counting function $x \mapsto \#\left\{k \in \mathbb{N}: p_{k} \geq 1 / x\right\}$ belongs to the de Haan class $\Pi$ (the distribution $\left(p_{k}\right)_{k \in \mathbb{N}}$ is then light-tailed). Consider a deterministic weighted branching process generated by $\left(p_{k}\right)_{k \in \mathbb{N}}$. A nested Karlin's occupancy scheme is the sequence of Karlin's balls-in-boxes schemes in which boxes of the $j$ th level, $j=1,2, \ldots$ are identified with the $j$ th generation individuals and the hitting probabilities of boxes are identified with the corresponding weights. The collection of balls is the same for all generations, and each ball starts at the root and moves along the tree of the deterministic weighted branching process according to the following rule: transition from a mother box to a daughter box occurs with probability given by the ratio of the daughter and mother weights.

Assuming there are $n$ balls, denote by $\mathcal{K}_{n}^{(j)}$ the number of occupied (ever hit) boxes in the $j$ th level. I shall discuss a functional limit theorem for the vector-valued process $\left(\mathcal{K}_{\left\lfloor e^{T+u}\right\rfloor}^{(1)}, \ldots, \mathcal{K}_{\left\lfloor e^{T+u}\right\rfloor}^{(j)}\right)_{u \in \mathbb{R}}$, for each $j \in \mathbb{N}$, properly normalized and centered, as $T \rightarrow \infty$. The limit is a vector-valued process whose components are independent stationary Gaussian processes. I shall provide an integral representation of the limit process.

A more delicate functional limit theorem will also be given for

$$
\begin{aligned}
& \left(\mathcal{K}_{\left\lfloor e^{T+u}\right\rfloor}^{(1)}(1), \mathcal{K}_{\left\lfloor e^{T+u}\right\rfloor}^{(1)}(2), \ldots, \mathcal{K}_{\left\lfloor e^{T+u}\right\rfloor}^{(1)}\left(i_{1}\right) ; \ldots ;\right. \\
& \left.\quad \mathcal{K}_{\left\lfloor e^{T+u}\right\rfloor}^{(j)}(1), \mathcal{K}_{\left\lfloor e^{T+u}\right\rfloor}^{(j)}(2), \ldots, \mathcal{K}_{\left\lfloor e^{T+u}\right\rfloor}^{(j)}\left(i_{j}\right)\right)_{u \in \mathbb{R}}
\end{aligned}
$$

with arbitrary $j \in \mathbb{N}$ and arbitrary $i_{1}, \ldots, i_{j} \in \mathbb{N}$. Here, $\mathcal{K}_{n}^{(j)}(i)$ is the number of the $j$ th level boxes containing $i$ balls (out of $n$ ).

This talk is based on joint work with Z. Kabluchko (Münster) and V. Kotelnikova (Kyiv).

## References

[1] A. Iksanov, Z. Kabluchko and V. Kotelnikova, A functional limit theorem for nested Karlin's occupancy scheme generated by discrete Weibulllike distributions. J. Math. Anal. Appl. 507 (2022), 125798.
[2] A. Iksanov and V. Kotelnikova, Small counts in nested Karlin's occupancy scheme generated by discrete Weibull-like distributions. Stoch. Proc. Appl. 153 (2022), 283-320.

# Particle systems with coordination, and the seedbank coalescent with simultaneous switching 

Noemi Kurt (Frankfurt)

In this talk we consider a class of structured branching coalescing particle systems with coordination. By coordination we mean that the behaviour of particles or individuals is not (necessarily) independent, on the contrary, individuals tend to take simultaneous actions. Classical examples of such processes are the multiple merger coalescents, but also many other processes studied in recent years fall into this class. We show that such coordinated processes have moment duals, which happen to be multidimensional diffusions with jumps. In some cases, these may be interpreted as population models.
We will present some general results on coordinated processes, such as an invariance result for the expectation. Further, we discuss several examples in more detail, in particular the seedbank coalescent with simultaneous switches, which was introduced recently to describe a population of individuals which may switch between active and dormant states. Coordination in this context means that a substantial proportion of the population may switch from active to dormant and vice versa at the same time, for example due to an external trigger. We illustrate the impact of coordination by providing criteria for the process to come down from infinity.

This talk is based on joint work with Adrián González Casanova and András Tóbiás.

## References

[1] A. González Casanova, N. Kurt, A. Tóbiás, ALEA 18 (2021)
[2] J. Blath, A. González Casanova, N. Kurt, M. Wilke-Berenguer, Electron. J. Probab. 25 (2019)

## Detecting hyperbolic geometry in networks

## Nelly Litvak (Twente)

Geometric random graph models for complex networks formalize the natural idea that similar vertices are likely to connect. Because of that, these models are able to simultaneously capture many common structural properties of real-world networks, such as scale invariance and high clustering. For instance, many real-world networks can be accurately modeled by positioning vertices of a network graph in hyperbolic spaces. Nevertheless, if one observes only the network connections, the presence of geometry is not always evident. Currently, triangle counts and clustering coefficients are the standard statistics to signal the presence of geometry. In this work we show that triangle counts or clustering coefficients are insufficient because they fail to detect geometry induced by hyperbolic spaces. We therefore introduce a novel statistic, weighted triangles, which weighs triangles based on their evidence for geometry. We show analytically, as well as on synthetic and real-world data, that weighted triangles are a powerful statistic to detect hyperbolic geometry in networks.

This talk is based on joint work with Riccardo Michielan and Clara Stegehuis.

## The $\infty$-parent SLFV: Definition and growth properties

## Apolline Louvet (Bath)

The $\infty$-parent spatial $\Lambda$-Fleming Viot process, or $\infty$-parent SLFV, is a measure-valued population genetics process for expanding populations in a spatial continuum whose dynamics is reminiscent of the Eden growth model. As for other SLFV processes, its main feature is that the reproduction dynamics can be seen as "event-based" rather than "individual-based": the process is associated to a Poisson point process which indicates in which areas reproduction occurs at each instant. In this talk, I will first introduce three possible definitions of the process, each valid under more or less restrictive conditions on the underlying Poisson point process: as a set-valued process, as the unique solution to a martingale problem, or as the limit of coupled SLFVs. I will then present what is currently known of the growth properties of the process. I will focus in particular on the growth of the front, which can be investigated using a (self) duality relation satisfied by the process. These results are important as a first step towards studying genetic diversity at the front edge.

This talk is based on joint work with Amandine Véber.

## References

[1] A. Louvet, ESAIM - Probability and Statistics (2023)
[2] A. Louvet and A. Véber, Arxiv (2022)

## Discrete-time Hawkes process with inhibition

## Pascal Maillard (Toulouse)

We consider a discrete-time version of a Hawkes process defined as a Poisson auto-regressive process whose parameters depend on the past of the trajectory. We allow these parameters to take on negative values, modelling inhibition. More precisely, the model is the stochastic process $\left(X_{n}\right)_{n \geq 0}$ with parameters $a_{1}, \ldots, a_{p} \in \mathbb{R}, p \in \mathbb{N}$ and $\lambda \geq 0$, such that for all $n \geq p$, conditioned on $X_{0}, \ldots, X_{n-1}, X_{n}$ is Poisson distributed with parameter

$$
\left(a_{1} X_{n-1}+\cdots+a_{p} X_{n-p}+\lambda\right)_{+}
$$

We consider specifically the case $p=2$, for which we are able to classify the asymptotic behavior of the process for the whole range of parameters, except for boundary cases. In particular, we show that the process remains stochastically bounded whenever the linear recurrence equation $x_{n}=a_{1} x_{n-1}+a_{2} x_{n-1}+\lambda$ remains bounded, but the converse is not true. Relatedly, the criterion for stochastic boundedness is not symmetric in $a_{1}$ and $a_{2}$, in contrast to the case of non-negative parameters, illustrating the complex effects of inhibition. We conjecture these facts to hold as well in the general case.

This talk is based on joint work with Manon Costa and Anthony Muraro.

## Extremal process of multidimensional branching Brownian motion

## Bastien Mallein (Paris)

The branching Brownian motion is a particle system in which each particle evolves independently of one another. Each particle moves according to a Brownian motion in dimension $d$, and splits into two daughter particles after an independent exponential time of parameter 1. The daughter particles then start from their positions independent copies of the same process.

We take interest in the long time asymptotic behaviour of the particles reaching farthest away from the origin. We show that these particles can be found at a distance of order $\sqrt{2} t+\frac{d-4}{2 \sqrt{2}} \log t$ from the origin of the process, and that they can be grouped into a Poisson point process of families of close relatives, spreading in directions sampled according to the random measure $Z(\mathrm{~d} \theta)$ that plays the role of an analogue of the derivative martingale of the branching Brownian motion.

This talk is based on joint work with Julien Berestycki, Yujin H. Kim, Eyal Lubetzky and Ofer Zeitouni.

## References

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High-dimensional limits for random walks
Oleksandr Marynych (Kyiv)

We prove limit theorems for random walks with $n$ steps in the $d$-dimensional Euclidean space as both $n$ and $d$ tend to infinity. If a generic step of such a random walk has uncorrelated components and its squared norm has finite expectation, we show that a properly rescaled random walk trajectory, viewed as a random metric space, converges in probability in the GromovHausdorff sense to a deterministic metric space, called Wiener spiral, as $d, n \rightarrow \infty$. This result can be regarded as a counterpart of the classic functional weak law of large numbers for usual one-dimensional random walks in a sense that it establishes convergence of a rescaled random walk with finite first moment to a deterministic limiting object.

In case when the squared norm of the generic step has infinite expectation, its tail is regularly varying with index $-\alpha, \alpha \in(0,1)$, and the angular components of two different steps are asymptotically orthogonal, as $d \rightarrow \infty$, we prove that the random walk path converges in distribution in the Gromov-Hausdorff sense to a genuinely random metric space identified with the range of a certain $\ell_{2}$-valued random process that we call $\alpha$-stable crinkled subordinator. The latter can be thought of as a version of the usual $\alpha$-stable subordinator with each out of countably many jumps being in a direction orthogonal to the directions of all other jumps.

This talk is based on joint work with Zakhar Kabluchko and Kilian Raschel.

## References

[1] Z. Kabluchko, O. Marynych (2023+). Random Walks in the HighDimensional Limit I. Preprint.
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# Metastability of the contact process on evolving scale-free networks 

Peter Mörters (Cologne)

We study the contact process on scale-free inhomogeneous random graphs evolving according to a stationary dynamics, where the neighbourhood of each vertex is updated with a rate depending on its strength. We identify the full phase diagram of metastability exponents in dependence on the tail exponent of the degree distribution and the rate of updating.

This talk is based on joint work with Emmanuel Jacob and Amitai Linker.

## Local algorithms for Maximum Cut and Minimum Bisection on locally treelike graphs

Andrea Montanari (Stanford)

Given a graph $G$ of degree $k$ over $n$ vertices, I will consider the problem of computing a near maximum cut or a near minimum bisection in polynomial time. For graphs of girth $2 L$, I will describe a local message passing algorithm whose complexity is $O(n k L)$, and that achieves near optimal cut values among all $L$-local algorithms (under a conjecture about the solution of a certain variational problem).

Our algorithm is nearly optimal on random regular graphs and an immediate corollary of this result is that random regular graphs have nearly minimum max-cut, and nearly maximum min-bisection among all regular locally treelike graphs. This can be viewed as a combinatorial version of the nearRamanujan property of random regular graphs. The algorithm construction is based on ideas from spin glass theory. I will discuss a broader set of spin glass problems that can be solved by the same approach.

This talk is based on joint work with Ahmed El Alaoui and Mark Sellke.

## Cycles in Mallows random permutations

## Tobias Müller (Groningen)

We study random permutations of $1, \ldots, n$ drawn at random according to the Mallows distribution. For $n \in \mathbb{N}$ and $q>0$, the distribution Mallows $(n, q)$ samples a random permutation $\Pi_{n}$ of $1, \ldots, n$ in such a way that each has probability proportional to $q^{\operatorname{inv}(\pi)}$, where $\operatorname{inv}(\pi)$ is the number of inversions. That is, pairs $1 \leq i<j \leq n$ for which $\pi(i)>\pi(j)$. In a formula:

$$
\begin{equation*}
\mathbb{P}\left(\Pi_{n}=\pi\right)=\frac{q^{\operatorname{inv}(\pi)}}{\sum_{\sigma \in S_{n}} q^{\operatorname{inv}(\sigma)}}, \tag{A}
\end{equation*}
$$

for all $\pi \in S_{n}$ where $S_{n}$ denotes the set of permutations of $1, \ldots, n$.
This distribution was introduced in the late fifties by C.L. Mallows in the context of "statistical ranking models" and has since been studied in connection with a diverse range of topics.

In the present work we will consider the cycle counts. That is, for $\ell$ fixed we study the vector $\left(C_{1}\left(\Pi_{n}\right), \ldots, C_{\ell}\left(\Pi_{n}\right)\right)$ where $C_{i}(\pi)$ denotes the number of cycles of length $i$ in $\pi$ and $\Pi_{n}$ is sampled according to the Mallows distribution.

When $q=1$ then the Mallows distribution is simply the uniform distribution on $S_{n}$. A classical result going back to Kolchin and Goncharoff states that in this case, the vector of cycle counts tends in distribution to a vector of independent Poisson random variables, with means $1, \frac{1}{2}, \frac{1}{3}, \ldots, \frac{1}{\ell}$.

Surprisingly, the problem of finding analogues of this result for $q \neq 1$ has largely escaped attention until now. In the talk, I plan to discuss our proof of the fact that if $0<q<1$ is fixed and $n \rightarrow \infty$ then the cycle counts have linear means and the vector of cycle counts can be suitably rescaled to tend to a joint Gaussian distribution. Our results also show that when $q>1$ there is a striking difference between the behaviour of the even and the odd cycles. The even cycle counts still have linear means and when properly rescaled tend to a multivariate Gaussian distribution, while for the odd cycle counts on the other hand, the limiting behaviour depends on the parity of $n$ when $q>1$.

Time permitting, I may also discuss some results on the (probabilities of) properties of permutations that can be expressed in first order logic.

This talk is based on joint work with Jimmy He, Fiona Skerman and Teun Verstraaten.

## Components of dense random geometric graphs

## Mathew Penrose (Bath)

The random geometric graph (RGG) is obtained by placing $n$ vertices uniformly at random in a bounded region of $R^{d}$ and connecting any two vertices distant at most $r$ apart. We discuss large- $n$ asymptotics with $r=r_{n}$ a specified sequence.

Given a positive integer $k$, let $S_{n, k}$ be the number of components of order $k$ in this graph, and let $S_{n}:=\sum_{k} S_{n, k}$, the total number of components. Let $L_{n}=\max \left\{k: S_{n, k}>0\right\}$, the order of the largest component.

In the 'thermodynamic limit' where $n r_{n}^{d} \rightarrow c \in(0, \infty)$, a law of large numbers and central limit theorem were already known for $S_{n, k}$, and for $S_{n}$, and for $L_{n}$. We discuss newer results of this type in the 'dense limit' where $n r_{n}^{d} \rightarrow \infty$ slowly, and the 'sparse limit' where $n r_{n}^{d} \rightarrow 0$ slowly.

In a related result, we determine the large- $\lambda$ asymptotics for the probability that the origin lies in a cluster of order $k$ in a Poisson Boolean model with intensity $\lambda$.

This talk is based on joint work with Xiaochuan Yang.

## Invasion of cooperative parasites in structured host populations

## Cornelia Pokalyuk (Frankfurt)

Certain defense mechanisms of phages against the immune system of their bacterial host rely on cooperation of phages. Motivated by this example we analysed in [BP] the spread of cooperative parasites in host populations that were structured according to a configuration model. Building on these results we consider the case of a host population which is (genuinely spatially) structured according to a random geometric graph. We identify the spatial scale at which invasion of parasites turns from being an unlikely to a highly probable event and give in the critical regime upper and lower bounds on the invasion probability.

This talk is based on joint work in progress with Vianney Brouard and Marco Seiler.

## References

[BP] V. Brouard and C. Pokalyuk., Invasion of parasites in moderately structured host populations, Stoch. Proc. Appl., 153, pp 221-263 (2022)

## Some questions on random recursive models

Zhan Shi (Shanghai, Paris)

In probability theory, there are simple random recursive models originating from statistical physics and giving unsolved mathematical problems. I am going to present two examples of such models, and make elementary discussions on one of them, the random series-parallel graph.

This talk is based on joint work with Xinxing Chen, Bernard Derrida and Thomas Duquesne.

## A probabilistic approach to Lorentz balls

Mathias Sonnleitner (Passau)

In this talk, we present a probabilistic approach to understand volumetric and geometric properties of unit balls of finite-dimensional Lorentz spaces $\ell_{q, p}^{n}$ when the dimension $n$ tends to infinity. For the special case $p=1$, we present a probabilistic representation of a random vector sampled uniformly in such a Lorentz ball and use this to derive the asymptotic distribution of a coordinate, a weak Poincaré-Maxwell-Borel lemma, and limit theorems for $\ell_{r}$ norms. In this setting, we discover phenomena, some of which do not appear in the special case of $\ell_{p}$ balls.

This talk is based on joint work with Zakhar Kabluchko and Joscha Prochno.

## Continuous-time digital search tree and a border aggregation model

Debleena Thacker (Durham)

We consider the continuous-time version of the random digital search tree, and construct a coupling with a border aggregation model as studied in Thacker and Volkov (2018), showing a relation between the height of the tree and the time required for aggregation. This relation carries over to the corresponding discrete-time models. As a consequence we find a very precise asymptotic result for the time to aggregation, using recent results by Drmota et al. (2020) for the digital search tree.

This talk is based on joint work with Svante Janson.

## Random (Beta) Polytopes

## Christoph Thäle (Bochum)

This talks starts with an introduction to classical random polytope models in $\mathbb{R}^{d}$ : random convex hulls generated by uniform random points in convex bodies or on their boundary. We discuss various asymptotic results for the missed volume. Similar questions are then discussed for random polytopes on the $d$-dimensional unit sphere, which are generated by $n$ random points uniformly distributed in a spherically convex container set or on its boundary. In particular, we show how such a non-Euclidean model can be analysed by combining probabilistic tools with geometric estimates. Expanding the container set to a half-sphere gives rise to new phenomena and serves as our entrance card to the family of beta random polytopes. Their distinguished properties are discussed. We then present a selection of results demonstrating their central role in stochastic geometry, including random cones as well as random Voronoi and hyperplane tessellations.

## Scaling limit of an adaptive contact process

## Daniel Valesin (Warwick)

We introduce and study an interacting particle system evolving on the d-dimensional torus $Z_{N}^{d}$. Each vertex of the torus can be either empty or occupied by an individual of a given type; the space of all types is the positive real line. An individual of type $\lambda$ dies with rate one and gives birth at each neighbouring empty position with rate $\lambda$. Moreover, when the birth takes place, the new individual is likely to have the same type as the parent, but has a small chance to be a mutant; the mutation rate and law of the type of the mutant both depend on $\lambda$. We consider the asymptotic behaviour of this process when the size of the torus is taken to infinity and the overall rate of mutation tends to zero fast enough that mutations are sufficiently separated in time, so that the amount of time spent on configurations with more than one type becomes negligible. We show that, after a suitable projection (which extracts just the dominant type from the configuration of individuals in the torus) and time scaling, the process converges to a Markov jump process on the positive real lines, whose rates we determine.

This talk is based on joint work with Adrián González Casanova and András Tobias.

