University of Limerick

Operational Risk

Triona Comerford

BSc. Financial Mathematics

0432628

Advisor: Volkert Paulsen

January 21st 2008

Contents

0.1 abstract . . . 3
0.2 Introduction . . . 3

1 What is Operational Risk **5**
1.1 Examples of Operational Risk . . . 5
1.1.1 Barings Bank . . . 5
1.1.2 Société Générale . . . 7
1.2 The Basel Committee and The Basel II Accord . . . 9
1.3 A note on Market Risk and Credit Risk . . . 11
1.3.1 Market Risk . . . 11
1.3.2 Credit Risk . . . 11
1.4 Approaches to Operational Risk . . . 12
1.4.1 Basic Indicator Approach . . . 12
1.4.2 Standardised Approach . . . 13
1.4.3 Advanced Measurement Approach . . . 15

2 A Homogeneous Loss Distribution Approach **18**
2.1 The Actuarial View . . . 18
2.1.1 Extreme Value Theory . . . 19
2.1.2 Generalised Pareto Distribution . . . 19
2.1.3 The Pickands-Balkema-de Haan Theorem . . . 21
2.2 Fitting the Generalised Pareto Distribution . . . 22
2.2.1 Peaks Over Threshold . . . 22
2.2.2 Maximum Likelihood Estimate . . . 23

2.2.3 Mean Excess Function 24
2.2.4 Extremal Losses 25
2.2.5 Model Fitting 27
2.2.6 Severity Distribution 27

3 Simulation of Sample Data 30
3.1 Using statistical package R 30

4 Appendices 34
4.1 Figures 37
4.2 Tables 37

5 Extra Stuff 38

6 Risk Management 39

7 References 41
7.1 Books 41
7.2 Reports 42
7.3 Websites 42

0.1 abstract

0.2 Introduction

Operational Risk is, *"the risk of loss resulting from inadequate or failed internal processes, people, and systems or from external events"*, and, *"internationally active banks and banks with significant operational risk exposures are expected to use an approach appropriate for the risk profile and sophistication of the institution"* as proposed by the Basel II Accord. Operational risk is a relatively new concept in the financial sector. The risks can be applied to any organisation in a business sector, but it is of particular relevance to the banking regime where regulators are responsible for establishing safeguards to protect against systemic failure of the banking system and the economy.
Operational Risk brings about a need for precise modelling and measurement methodologies. This Basel II Accord was the brainchild of the Basel Committee, which was founded in 1974. Formally known as the Cooke Committee, in 1988 the Basel Committee published its first Accord. Referred to as, the Basel Capital Accord, this was a huge stepping stone for the financial industry on capital regulation. The framework of this publication centered on credit risk and market risk. Although Basel I succeeded in promoting financial stability, for larger and more complex banks, there were some shortcomings. The Basel I capital ratios neither reflect risk adequately nor measure bank strength accurately (http://en.wikipedia.org/wiki/Basic indicator approach). With the evolution of risk management and measurement, the Basel Committee published a new Accord, entitled Basel II, in 2003. This regulatory framework reflects the changes in practices throughout the financial sector. In this project, the Basel II and various books will be a primary source of research.
Today, banks are faced with many sources of risk. Only until recent times, have banks began to understand the concept of Operational Risk and realise the effect of Operational Risk on capital. Although Market Risk and Credit

Risk play a large role in the risks which affect banks, these risks are independent of the bank taking a risk, and so can be modelled easily using known qualitative and quantitative data. However it is very difficult to model Operational Risk as the portfolio of risks are not known.

The classification of risks over the years has played a major part in banks neglecting Operational Risk. When financial losses which involved derivatives occurred, the effect which was then noted was classified as a market risk. The same approach was adapted for credit risk. If the cause of a loss is studied and it is apparent that the customer is in default, then it is regarded as a credit risk. Therefore, they showed an arrogant approach in accepting the reality of Operational Risk and they have also displayed a level of weakness in proposed approaches to measuring Operational Risk. This is one of the main reasons why Operational Risk has been edged further into the spotlight in recent years.

Operational Risk can occur due to a variety of different sources, for example, fraud, natural disasters i.e. hurricane, flawed ethics practices, and also failure of IT equipment. As such events are difficult to predict, it is an even more complicated task to model these occurrences. To model such a risk, one could use past data to develop such a model. It is very important to note that one would create an insufficient model by collecting data over a period of time say 20 years. As the banking systems and its employees regularly change and banks themselves merge, so the data used would be incorrect. More recently, due to the negative impact on a banks reputation and its capital, banks are very reluctant to release figures which may show any notable losses to the public. So, to overcome this, one could use a package to generate their own set of data which would be comparable to those corresponding to Operational Risk. In this project I will use the R package to simulate a data set for analysis.

Chapter 1

What is Operational Risk

In this chapter are some examples on Op. Risk and the background on the Basel Committee which led to the development of regulations in the banking sector. Atrocities such as the September 11th terrorist attacks, rogue trading in banks like AIB, Barings Bank and also the Y2K meltdown scare. By discussing these topics and also credit risk and market risk which are the building blocks that contributed to the cultivation of Op. Risk, this chapter aims to provide the reader with an understanding of how Op. Risk came about and the 3 main approaches as proposed by the Basel Committee in quantifying Op. Risk.

1.1 Examples of Operational Risk

1.1.1 Barings Bank

The Barings Bank scandal which sent shock waves reverberating throughout the financial industry worldwide is one of the most infamous examples of Op. Risk today. Barings Bank was Britain's oldest banking institution and was brought down within a matter of days. One of the bank's employees, Nick Leeson, made a loss of $1.4 billion and so caused one of the oldest banks in the world to collapse and was declare itself insolvent in February 1995 ($http://en.wikipedia.org/wiki/Barings_Bank$).

Shortly after graduating from university, Leeson began working with Morgan Stanley but soon changed and began employment with Barings Bank in London. He was then placed in a back office in Jakarta, Indonesia where he successfully sorted a mess regarding $100million share of certificates. This clearly had a positive impact on his reputation within the organisation. After this, in 1987, Leeson was transferred to the banks office in Singapore, Barings Securities Singapore Limited (BSS). With his recent success behind him, Leeson applied for and was granted a position as a general manager with the authority to hire traders and back office staff in 1992. Having passed the necessary exam, Leeson began trading on the Singapore Mercantile Exchange (SIMEX). Leeson had very minimal experience in trading but soon became the head trader and head of the back office staff. Looking back now, this was a very peculiar promotion but at the time no eyebrows from senior managers were raised. Leeson performed 2 types of trading;

- Transacting futures and options orders for clients or for other firms within the Barings organisation, and
- Arbitraging price differences between Nikkei futures traded on the SIMEX and Japan's Osaka exchange

Arbitrage is a low risk strategy in which Baring's had hoped Leeson and his team would gain a significant number of small profits rather than a small number of significant profits. However, this was not the case. Barings' senior managers did not fully underestimate the potential risks and losses which were to follow. He began unauthorized speculation in futures on Nikkei 225 stock index and Japanese government bonds. This high leverage trading strategy could result in either a huge gain or a huge loss.
Leeson opened a secret trading account, 88888, which he claims it was initially set up to hide the mistakes of his traders. However Leeson used this account to shield his lucrative losses from colleagues, senior managers and even auditors. From the very beginning, Leeson was making huge mistakes and so created major losses for Barings. At the end of 1992, the account

88888 had a closing balance of minus $2million. But by the end of 1993, this deficit had jumped to $23million, and another year on, account 88888 was a staggering $208million in the red, to the blissful ignorance of senior management in Baring's. The Japanese stock market was still in a slow decline, and Leeson remained on course with his consistent bad bets. In January 1995, an earthquake shook Japan and the stock market plummeted. An unpredictable natural event caused account 88888 to suffer a monumental blow. Such was the economic impact of the disaster that it steered the account into a deficit of £827million (*http : //www.riskglossary.com/link/barings − debacle.htm*) or $1.4billion.

The responsibility for this catastrophic loss lay in the hands of the management committee of Baring's Bank. Over the course of approximately 3 years, one of Baring's employees single-handedly brought down the internationally renowned bank. Leeson's senior managers merely scratched the surface whenever questions were asked regarding his performance. During Leeson's spell as a trader in Barings, it is unmistakable that he was clearly a rogue trader who proved to be a massive risk to the company.

Nick Leeson and the Barings Bank shame is a prime example of Op. Risk due. The combination of an earthquake (external event), flawed recording procedures, unauthorized trading and bad governance ground the bank to a halt in a moderately short period of time.

1.1.2 Société Générale

More recently, one of France's biggest banking institutions, Société Générale announced on the 27th of January 2008 something that shook the financial industry to it's core. France's second biggest bank, released an official press release which stated that Société Générale had amassed a debt of €4.9billion due to a trading fraud by one of it's employees. This was the single biggest fraud in financial history.

The bank were initially apprehensive in releasing the name of the employee who was solely responsible for the fraud and so the press release from the

27th January goes on to refer to the person as "the trader". The trader had been employed by the Group in the year 2000 and began working in several different middle-offices where he acquired a good understanding of the bank's processing and control procedures. Then in 2005, he began trading in the arbitrage department. It was here where he developed an initial portfolio A comprising of genuine operations using financial instruments (futures) which reproduced changes in the main European stock market indices (Eurostoxx, the Dax, the FTSE, etc.). The futures in the portfolio were genuine and consistent and the margin calls were checked and settled by or paid to the bank (*http://www.sp.socgen.com/sdp/sdp.nsf/V3ID/D22EA4F2E1FB3487C12573DD005BC223/file/08005gb.pdf*).

The futures in portfolio A were in appearance offset by the fictitious instruments in portfolio B which meant that the only visible risk was very low residual risk. So the trader was able to hide a very sizeable speculation position, which was neither consistent nor related to his normal business activity for the bank.

As a result of Société Générale's internal investigations, it was found that the trader combined several fraudulent methods to avoid the controls which the bank had in place to protect from such risk,

- He ensured that the characteristics of the fictitious operations limited the chances of control: for example he chose very specific operations with no cash movements or margin call and which also did not require immediate confirmation
- the trader misused IT access codes in order to cancel certain operations
- he falsified documents allowing him to justify the entry of fictitious operations and ensured that these fictitious operations involved a different financial instrument to the one he had just cancelled. This was carried out so that he could minimise the chances of being controlled.

Over the course of several days, and by January 20th, the internal investigations found that the fraudulent position which the trader found himself

to be in was worth approximately €50billion. The team behind the internal investigations in Société Générale, began proceedings to unwind the traders position so volume levels could be kept under 10%. However the conditions in the market were unfavourable, and on the night of the 20th January, there was a significant drop in the Asian markets. The Hang Seng fell by 5.4% before the European markets opened. The position was unwound over the next three days in a controlled fashion to ensure Société Générale would not exceed approximately 8% of volumes traded on the relevant futures indices (Eurostoxx, the Dax, and the FTSE). The position was finally closed on the evening of January 23rd with a resulting total loss of €4.9billion (*http://www.sp.socgen.com/sdp/sdp.nsf/V3ID/D22EA4F2E1FB3487C12573DD005BC223/file/08005gb.pdf*).
According to the same article, "*specific control procedures have been implemented so that techniques devised by the trader to avoid controls can no longer be applied*" and "*additional controls will be launched*". It is clear that the management within Société Générale were lacking in implementing stringent control measures when it came to trading. The bad governance by the French bank's management team proved to be one of the biggest financial losses in history. Further investigations by Société Générale's internal team and the ongoing police inquiry will later show exact details and how much money was lost.

1.2 The Basel Committee and The Basel II Accord

As previously mentioned, the Basel Committee was set up in 1974. It was formally introduced by the central-bank governors of a group of 10 countries. The committee's members are represented by their own central bank and their regulatory bodies for banking supervision which is independent of the central bank. It's members hail from Belgium, Canada, France, Germany, Italy, Japan, Luxembourg, the Netherlands, Spain, Swe-

den, Switzerland, United Kingdom and the United States of America (*http : //www.bis.org/bcbs/history.htm*). The committee was initially set up to encourage financial institutions in implementing common standards and common practices within their supervisory organisation approaches. Rather than creating legal obligations for these organisations.
In 1988, the Committee decided to introduce a capital measurement system commonly referred to as the Basel Capital Accord (http://www. bis.org/bcbs/history.htm). This Accord sought to implement a credit risk measurement framework with a minimum capital standard of 8% by the end of 1992. However, the Basel Committee revised their original Accord and proposed the Basel II Accord which refined the guidelines that were put forward in the Capital Accord. They sought a more comprehensive approach to cover more risks that a bank may experience. One of the Basel II Accord's main goals is to help stabilise the financial industry through assisting in the governing of Op. Risk regulatory policies in financial institutions. The Basel II Accord set forth an improved capital framework which is also referred to as the three pillar approach;

1. Minimum Capital Requirement: This involves calculating the minimum capital to be set aside for the risks a bank may take on. These risks can be credit, market and operational risks. This is the main building block in the regulatory process as it quantifies risk which will then lead to reducing these risks.

2. Supervisory Review Process: The Supervisory Review Process is a more intense a rigorous adaptation of the Minimum Capital Requirement. Risks must be measured in a reliable way so capital set aside will cover all risks a bank may endure.

3. Market Discipline: This approach seeks to improve the solvency of a bank. By better regulating the disclosure of risk exposures and capital levels of the banks market.

1.3 A note on Market Risk and Credit Risk

1.3.1 Market Risk

Market Risk is the risk due to market price changes. Factors which influence the market are; Equity Risk(the risk that the stock price will change), Interest Rate Risk(the risk that the interest rates will change), Currency Risk(the risk that the foreign exchange rates will change), Commodity Risk(the risk that commodity prices will change) (http://en.wikipedia.org/wiki/Market risk). An example of market risk is where a trader may hold a portfolio of forwards. The trader knows today's market value however, the future value of the market is unknown.

To protect against market risk, banks implement asset allocation and diversification to protect against market risk as different portions of the market tend to under perform at different times, also called systematic risk (http://www.investorwords.com/ 2987/market risk.html). For traders, the Greek letters are used to assess their exposure to risk. These are derived from the Black-Scholes pricing formula. Many large financial institutions have designed specific departments, i.e. Global Treasury, to deal with this risk. Unlike Op. Risk, market risk begins with a known portfolio of risks, so it can be easily identified along with constant supervision.

1.3.2 Credit Risk

Credit Risk is the risk of the counter party defaulting and the pledged collateral does not cover the company's claim. In other words, the company is exposed to a loss. Credit risk can take many forms, as counter parties can range from individuals to sovereign governments and the obligations which they undertake can range from personal loans to derivative transactions.

Like market risk, credit risk modelling starts with a known portfolio of risks and it is independent of the bank taking risks. Credit risk exposures can be measured as money lent, mark to market exposure, or potential exposure on

a derivative. For banking institutions, i.e. Bank of Ireland, credit risk would arise due to lending activities, derivatives and securities.
To manage such a risk, many financial institutions have established a framework of credit policy which is governed by a team of highly skilled and experienced employees. They assess the probability of default, their credit exposure and also the rate at which they would recover if bankruptcy or the counter party defaulted. Credit Risk is managed with a long term focus on achieving a good return on investment over a period of time. Since this credit risk that is associated with large financial institutions is more complicated, it can be assessed by a credit analyst who will then assign a credit rating. Standard & Poor's, Moody and Fitch's credit rating system is the most widely used and is shown in Table 1 of Appendices (http://www2.standardandpoors.com/portal/site/sp/en/eu/page.article/2,1,4,0,1148449204344.html).

1.4 Approaches to Operational Risk

Given the continuing evolution of analytical approaches for Op. Risk, the Basel Committee does not specify the approach or distributional assumptions used to generate the Op. Risk measure for regulatory capital purposes. However, a bank must be able to demonstrate that its approach captures potentially severe 'tail' loss events (http://www.bis.org/publ/bcbs107.pdf). This chapter focuses on the 3 main approaches which a bank can implement in the calculation of a suitable capital charge to account for Op. Risk.

1.4.1 Basic Indicator Approach

Op. Risk is said to be a percentage (15%) of the gross income. This is the most simplistic approach and so Basel II has recommended that banks with significant international operations not to adopt this approach. It is advised that smaller or more local banks adopt the Basic Indicator Approach(BIA). As this model does not require complex calculations, financial institutions can calculate their capital without any difficulties. Based on the original

Basel Accord, banks using the basic indicator approach must hold capital for operational risk equal to the average over the previous three years of a fixed percentage of positive annual gross income (Report; A Review of Operational Risk Quantitative Methodologies Within the Basel II Frameworkl). If the annual gross income is negative, then it should not be included when calculating the average. The Basel Committee has defined the following formula for the calculation of regulatory capital for Op. Risk (Operational risk : the new challenge for banks by Gerrit Jan van den Brink Basingstoke : Palgrave, 2002),

$$\textit{Regulatory Capital Charge For Op. Risk} = \alpha \times \textit{Gross Income}$$

where the gross income the is net interest income plus net non-interest income.
However, with simplicity there can be disadvantages. It is difficult to identify a relationship between Op. Risk and the risk indicator chosen, which could be costs incurred by staff or technology. Financial institutions can encounter difficulties when distinguishing between different business lines. One business line can be properly organized and fully aware of the necessity of a control environment, whilst another business line may not be interested in these themes an is therefore poorly organised (Operational risk : the new challenge for banks by Gerrit Jan van den Brink Basingstoke : Palgrave, 2002). From this, it can be concluded that the single indicator approach is not an effective tool for financial institutions in managing their capital.

1.4.2 Standardised Approach

The Standardised Approach is a slightly more advanced version of the BIA. As this method can be carried out without the use of sophisticated management systems, banks that are not internationally active can introduce the SA method. This method calculates the Op. Risk capital on the basis of gross income split per business (Report; National Bank of Belgium; Working Papers-Research Series; Basel II and Operational Risk: Implications for risk measurement and management in the financial sector). As each business line may be riskier than the next, they each have their own percentage. This can be broken down as follows; 12% for least risky businesses i.e. asset management, 15% for moderate business lines,i.e. corporate banking and then up to 18% for the most risky business i.e. trading and settlement. There are 8 different business lines;

- Corporate Finance
- Trading and Sales
- Retail Banking
- Commercial Banking
- Payment and Settlement
- Retail Brokerage
- Agency Service and Custody

As this approach is more sophisticated than the Basic Indicator Approach, banks must adhere to the strict guidelines as set by the Basel II Accord. Banks need to have an independent audit function and also an independent Op. Risk function in place for this method to be established.
Mathematically, Op. Risk for the Standardized Approach is

$$Op.RiskCharge = \sum_i \beta_i GI_i$$

where,
GI = average gross income of the previous 3 years' income
β = business line specific risk weighting factor which varies between 12% and 18%.

An alternative way of describing β, is that it represents a rough estimate of the relationship between the industry's loss experience and the indicator

that represents the banks activities in a particular business line.
It is clear that the SA is an improvement on the BIA, but we are still unable to clearly identify the relationship which exists between the OP. Risk for each business line and the capital that is charged for this OP. Risk. Moreover, a sound OP. Risk management, which may condense into a proper set of risk-mitigating measures, is not remunerated by this approach (Operational risk : the new challenge for banks by Gerrit Jan van den Brink Basingstoke : Palgrave, 2002).

1.4.3 Advanced Measurement Approach

The Advanced Measurement Approach (AMA) is a more complex approach that is built on the Standardised Approach (SA). As proposed by the Basel II Accord, an AMA encompasses all measurement techniques that lead to a precise measurement of the exposure of each business line of a financial institution to each category of operational loss events (Report; National Bank of Belgium; Working Papers-Research Series; Basel II and Operational Risk: Implications for risk measurement and management in the financial sector,pg 6).
By using this technique, the regulatory capital will equal the risk measure generated by the bank's internal OP. Risk measurement system using the quantitative and qualitative criteria (http://www.bis.org/ publ/bcbs107.pdf). Since this method is a more complex and detailed approach, a bank must clearly show that they have adequate resources which give a detailed account of the risk measure it has adapted and it refers to a 1 year period with a 99.9% Confidence Interval. Also, senior managers must be actively involved in the supervision of the OP. Risk framework. Due to the importance of this approach, an initial monitoring period is mandatory whereby a supervisor will deem the approach credible and appropriate.
According to section 664 of the Basel II Accord, for a bank to qualify for adapting an AMA model, it must clearly satisfy the minimum requirements (Website; http://www.bis.org/publ/bcbs128.pdf);

- It's board of directors and senior management, as appropriate, are actively involved in the oversight of the operational risk management framework.
- It has an operational risk management system that is conceptually sound and is implemented with integrity, and
- It has sufficient resources in the use of the approach in the major business lines as well as the control and audit areas.

For this measurement, the models must be based on a minimum observation period of 5 years. With the exception being when the financial institutions initially moved to AMA, there is a transition period of 3 years (Report; A Review of Operational Risk Quantitative Methodologies Within the Basel II Framework).
As there is no specific formula to calculate Op. Risk, this approach is a fundamental building block for doing so, and thus, for this project a homogeneous AMA model will be applied for the modeling of a sample data set for OP. Risk.
The AMA is the technique towards Op. Risk which is adapted by internationally active banks, but is subject to supervisory approval. The Basel Committee refers to AMA, as a set of "Op. Risk Measurement techniques", which quantify Op. Risk. For banks which have approval for an AMA approach, they have permission to develop their own empirical model to quantify the required capital for Op. Risk (http://en.wikipedia.org/wiki/AdvancedMeasurementApproach).
A summary of these approaches to Op. Risk can be seen in Table 2 of the Appendices.

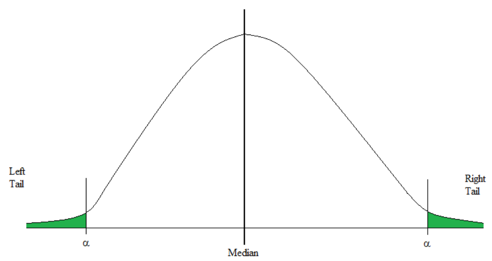

Figure 1.1: Normal Curve

Chapter 2

A Homogeneous Loss Distribution Approach

The most widely utilised AMA method is the Loss Distribution Approach (LDA). This method is an application of actuarial methods that combines a frequency distribution with a severity distribution. The frequency distribution is one which describes the occurrence of operational losses in an organisation, and the severity distribution is one that specifies the economic impact of the individual losses. This approach is best suited in modelling Op. Risk due to its ability to incorporate a frequency distribution and a severity distribution. For this reason, a homogeneous LDA will be examined in this chapter.

2.1 The Actuarial View

In an actuarial world, where the impact of risk and the uncertainty of undesirable events is analyzed for the purpose of insurance, the professionals in this industry face the same battle as the financial institutions in quantifying extreme risks. The insurance industry has a considerable amount of experience in the management of these extreme risks that can occur to people or property which can then lead to a claim against an insurance policy.

For this reason, many methods which belong to extreme value theory can be identified in the insurance industry today.

2.1.1 Extreme Value Theory

Extreme Value Theory, or EVT for short, is a set of mathematical methods which are used to estimate the tails of the loss severity distributions. As its name suggests, EVT is concerned with the modeling of extreme events. EVT is most naturally developed as a theory of large losses, rather than a theory of small profits, (Report;Extreme Value Theory for Risk Managers,pg 2).
EVT is used to model extreme or huge losses which lie beyond a predetermined threshold. As Op. Risk losses are exceptionally large losses, this concept is an important factor in assessing Op. Risk as it deals with extreme deviations from the median of probability distributions. A key theorem in EVT, is that for a wide class of distributions, losses which exceed high enough thresholds follow the Generalized Pareto Distribution, (Report;Estimating the Tails of Loss Severity Distributions using Extreme Value Theory,pg 2).
An example of an EVT application is the Peaks Over Threshold(POT) method. This method follows a Generalized Pareto distribution and one which I will apply in section 'Analysis of Simulated Data'.

2.1.2 Generalised Pareto Distribution

The GPD originated as a distribution which can model tails of a wide variety of distributions. Named after Italian economist Vilfredo Pareto, who initially used this distribution to describe the allocation of wealth among individuals. The results showed that a large percentage of the wealth of any society is owned by a much smaller percentage of that society. The GPD is not just limited to wealth or income but it can also be applied in the field of social, scientific, geographical and actuarial studies ($http://en.wikipedia.org/wiki/Pareto_distribution$).

The GPD is a right skewed distribution with shape parameter ξ and scale parameter β. For $\xi < 0$, the GPD has a zero probability above an upper limit of $\frac{-1}{k}$. For $\xi \geq 0$, the GPD has no upper limit. The GPD is a generalisation of the Pareto distribution and the exponential distributions. The mathematical function of the GPD is defined as follows;
The GPD is a two parameter distribution with distribution function

$$G_{\xi,\beta}(x) = \begin{cases} 1 - (1 + \frac{\xi x}{\beta}) & \xi \neq 0, \\ 1 - \exp(\frac{-x}{\beta}) & \xi = 0, \end{cases} \tag{2.1}$$

where $\beta > 0$,
and where $x \geq 0$ when $\xi \geq 0$,
and where $0 \leq x \leq -\beta/\xi$ when $\xi < 0$.
ξ is the important shape parameter of the distribution and β is an additional scaling parameter. When $\xi > 0$, the GPD is heavy-tailed and so this is closely related to Op. Risk measurement. (Report; Extreme Value Theory for Risk Managers, pg 4).

The following are some properties of the GPD;

1. For every $\xi \in \mathbb{R}$, $F \in MDA(H_\xi)$ if and only if,
   $\lim_{u \uparrow x_F} \sup_{0<x<x_F-u} \mid F_u(x) - G_{\xi,\beta(u)}(x) \mid = 0$
   for some positive function β.

2. Assume that N is Poisson(λ), independent of the iid sequence (X_n) with a GPD with parameters ξ and β. Write $M_N = max(X_1, ..., XN)$.
   Then,
   $P(M_N \leq x) = \exp\{-\lambda(1 + \xi\frac{x}{\beta})^{-1/\xi}\} = H_{\xi:\mu,\psi}$,
   where $\mu = \beta\xi^{-1}(\lambda^\xi - 1)$ and $\psi = \beta\lambda^\xi$.

3. Suppose X has GPD with parameters $\xi < 1$ and β. Then for $u < x_F$,
   $e(u) = E(X - u \mid X > u) = \frac{\beta+\xi u}{1-\xi}, \qquad \beta + u\xi > 0.$

(Modelling Extremal Events for insurance and finance, pg165).
$MDA(H)$ stands for the Maximum Domain of Attraction. One says that the *rv* X (the distributive function *(*F) of X, the distribution of X) belongs

to the maximum domain of attraction of the extreme value distribution H if there exist constants $c_n > 0$, $d_n \in \mathbb{R}$ such that $c_n^{-1}(M_n - dn) \xrightarrow{d} H$ holds. The shorthand method of writing this is $X \in MDA(H)$ (Modelling Extremal Events for insurance and finance, pg128).

2.1.3 The Pickands-Balkema-de Haan Theorem

The Pickands-Balkema-de Haan theorem, gives one an understanding as to why the GPD is the most appropriate distribution when using EVT.
Firstly consider a high threshold u, so that in any event u will be greater than any possible displacement δ which is associated with the data. One is interested in the amount by which observations exceed this threshold.
Let x_0 be the finite or the infinite right endpoint of the distribution F so that $x_0 = \sup x \in \mathbb{R} : F(x) < 1 \leq \infty$. The distribution function of the excesses over the threshold u is defined by,

$$F_u(x) = PX - u \leq x \mid X > u = \frac{F(x+u) - F(u)}{1 - F(u)} \tag{2.2}$$

for $0 \leq x < x_0 - u$.
For a large class of underlying distributions we can find a function $\beta(u)$ such that,

$$\lim_{u \to x_0} \sup_{0 \leq x < x_0 - u} \mid F_u(x) - G_{\xi,\beta(u)}(x) \mid = 0$$

where,
u denotes a high threshold such that β is a function of this high threshold u. The distribution function of the excesses may be approximated by $G_{\xi,\beta(u)}(y)$, i.e. a GPD where the shape and the scale parameters are given by ξ and β respectively. Thus one would use the GPD as the limiting distribution for the distribution of the excesses, as the threshold tends to the right endpoint (Report; Estimating the Tails of Loss Severity Distributions using Extreme Value Theory, pg7). In other words, if one choose a high enough threshold, one would expect the data to display GPD behaviour, see Figure 2.1.

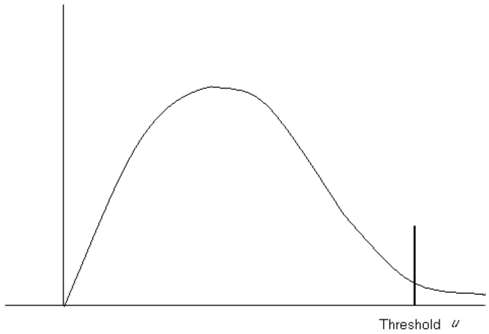

Figure 2.1: Threshold for Pickands-Balkema-de Haan Theorem

2.2 Fitting the Generalised Pareto Distribution

2.2.1 Peaks Over Threshold

The Peaks Over Threshold(POT) method is one which estimates high quantiles of loss severity distributions. In the POT model, the excess losses arising from Op. Risk are modelled with the Generalized Pareto distribution(GPD). This distribution arises naturally in a key limit theorem in EVT, (The Pickands-Balkema-de Haan theorem)and provides a simple tool for estimating measures of tail risk (Report; The Peaks Over Threshold Method for Estimating High Quantiles of Loss Distributions,pg 2).

By using POT, one obtains simple parametric formulae for measures of extreme risk for which it is relatively easy to give estimates of statistical error using the techniques of maximum likelihood estimates. Firstly, we must let Op. Risk losses be denoted by the random variables, $X_1, X_2, ...$, which are independent and identically distributed. And now, denote their common distribution function by $F_X(x) = P(X \leq x)$ where $x > 0$ (Report;Extreme Value Theory for Risk Managers,pg 3).
Recall, that the GPD is a two parameter distribution with distribution func-

tion

$$G_{\xi,\beta}(x) = \begin{cases} 1-(1+\frac{\xi x}{\beta}) & \xi \neq 0, \\ 1-\exp(\frac{-x}{\beta}) & \xi = 0, \end{cases} \tag{2.3}$$

where $\beta > 0$,
and where $x \geq 0$ when $\xi \geq 0$,
and where $0 \leq x \leq -\beta/\xi$ when $\xi < 0$.
ξ is the important shape parameter of the distribution and β is an additional scaling parameter. When $\xi > 0$, the GPD is heavy-tailed and so this is closely related to Op. Risk measurement. (Report; Extreme Value Theory for Risk Managers, pg 4). To apply this method, the GPD is fitted to the observed excesses over the threshold i.e. $x \geq u$ such that,

$$F(x) = P(X \leq x) = (1 - P(X \leq u))F_u(x-u) + P(X \leq u).$$

We now know that we can estimate $F_u(x-u)$ by $G_{\xi,\beta}(x-u)$ for large u. We can also estimate $P(X \leq u)$, which is the empirical distribution function evaluated at u. And so, this means that we can use the tail estimate

$$\widehat{F(x)} = (1 - F_n(u))G_{\xi,u,\sigma}(x) + F_n(u)$$

to approximate the distribution function F(x) and thus show that $\widehat{F(x)}$ is a GPD with shape parameter ξ, scale parameter $\tilde{\beta} = \beta(1-F_n(u))^{\xi}$ and location parameter $\tilde{\mu} = u - \tilde{\beta}((1-F_n(u))^{-\xi} - 1)/\xi$. (Report; Estimating the Tails of Loss Severity Distributions using Extreme Value Theory, pg8).

2.2.2 Maximum Likelihood Estimate

Suppose one now has their losses i.e. $X_1, X_2, ...,$ and so one needs a very good estimate for ξ and β. To do this, and thus fit the sample data, the method of probability weighted moments (PWM) can be adapted or a maximum likelihood estimation (MLE) can be used. The MLE method is favoured for heavy-tailed loss data, i.e. $\xi > -0.5$, and as the sample set is heavy-tailed, this method will be carried forward. A Maximum Likelihood Estimate (MLE) of these parameters is taken as this method will maximize the joint probability density of the losses and allow for any standard errors. The MLE can also show that the maximum likelihood estimates $(\hat{\xi}_n, \hat{\beta}_n)$ are asymptotically normally distributed. And so the following result,

$$n^{\frac{1}{2}} \begin{pmatrix} \hat{\xi}_n \\ \hat{\beta}_n \end{pmatrix} \xrightarrow{d} N \left[\begin{pmatrix} \xi \\ \beta \end{pmatrix}, \begin{pmatrix} (1-\xi)^2 & \beta(1+\xi) \\ \beta(1+\xi) & 2\beta^2(1+\xi) \end{pmatrix} \right] \tag{2.4}$$

will enable me to calculate approximate standard errors for the maximum likelihood estimates, (Report; Estimating the Tails of Loss Severity Distributions using Extreme Value Theory, pg 8,9).

An alternative approach to calculating the MLE, is firstly let H be the threshold, so the probability of falling under the threshold H is equal to the distribution function computed at H is *F(H)*. Then the probability of falling over the threshold H is *1-F(H)*. Under the MLE procedure, the unknown parameter can be directly estimated by maximizing the constrained log-likelihood function as follows;

$$\hat{\theta}^c_{MLE} = \arg_\theta \max \log \prod_{j=1}^{n} \frac{f_\theta(x_j)}{1 - F_\theta(H)} \tag{2.5}$$

(Report; Practical Operational Risk,pg 8).

2.2.3 Mean Excess Function

For the purposes of this paper, u denotes the threshold which separates the normal losses from the extremal losses. There is a slight problem posed when choosing a value for u. A value too high will result in a small numbers of excess losses and for a small value for u, the estimators become biased. To overcome this obstacle, one can use the mean excess function.
Let X be a *rv* with a right endpoint x_F, then

$$e(u) = E(X - u \mid X > u), \qquad 0 \le u < x_F \tag{2.6}$$

is known as the mean excess function of X, and so, *e(u)* is the mean of the excesses over the threshold u. (Modelling Extremal Events for insurance and finance, pg 294).
The mean excess function for the GPD is,

$$\tag{2.7}$$

To obtain an estimate for the mean excess function, one would use the result;

$$e_n(u) = \frac{\sum_{i=1}^{n}(X_i - u)^+}{\sum_{i=1}^{n} 1_{X_i > u}} \tag{2.8}$$

which is the sum of the excesses over u divided by the number of the data points that exceed u. This describes the expected overshoot of a threshold given that exceedance occurs. If the empirical plot follows a reasonably straight line with a positive gradient with a certain value of u, then this is an indication that the excesses over the threshold follow a GPD with a positive shape parameter, (The Peaks Over Threshold Method for Estimating High Quantiles of Loss Distributions, pg 6).

2.2.4 Extremal Losses

As Op. Risk is an extreme loss, one would need to look at how an extreme loss can be modelled. In applications used throughout the insurance and financial industry extremal loss severities can occur. These extremal losses can be modelled by a subexponential severity distribution. The subexponential distribution is used to model such losses as it is in the right tail where the extremal losses of interest occur. The tail of the distribution F needs to be a subexponential distribution rather than the whole distribution itself.
Subexponential is defined in terms of convolutions of probability distributions. For two independent, identically distributed random variables X_1,X_2 with common distribution function F the convolution of F with itself, F^{*2} is defined by:

$$Pr(X_1 + X_2 \leq x) = F^{*2}(x) = \int_{-\infty}^{\infty} F(x-y)F(dy)$$

If,

$$\overline{F^{*2}}(x) \sim 2\overline{F}(x) \qquad as \qquad x \to \infty.$$

then the n-fold convolution is given by;

$$\overline{F^{*n}}(x) \sim n\overline{F}(x) \qquad as \qquad x \to \infty.$$

Definition: A severity distribution F is said to be a subexponential if;

$$\frac{F^{*2}(t,\infty)}{F(t,\infty)} = 2 \qquad for \qquad t \to \infty$$

Some basic properties of the subexponential distribution;

(a) If $F \in S$, then uniformly on compact y-sets of $(0, \infty)$,

$$\lim_{x\to\infty} \frac{\overline{F}(x-y)}{\overline{F}(x)} = 1. \tag{2.9}$$

(b) If (3.15) holds, then for all $\varepsilon > 0$,

$$e^{\varepsilon x}\overline{F}(x) \to \infty, \qquad x \to \infty.$$

(c) If $F \in S$, then given $\varepsilon > 0$, there exists a finite constant K so that for all $n \geq 2$,

$$\frac{\overline{F^{n*}}(x)}{\overline{F}(x)} \leq K(1+\varepsilon)^n, \qquad x \geq 0$$

For example;

Take F, a Weibull distribution with parameters $0 < \tau < 1$ and $c > 0$, i.e

$$\overline{F}(x) = e^{-cx^\tau}, \qquad x \geq 0.$$

Then,

$$f(x) = c\tau x^{\tau-1}e^{-cx^\tau}, \qquad Q(x) = cx^\tau \qquad and \qquad q(x) = c\tau x^{\tau-1},$$

which decreases to 0 if $\tau < 1$. This leads to the following result;

$$x \mapsto e^{xq(x)}f(x) = e^{c(\tau-1)x^\tau}c\tau x^{\tau-1}$$

as it is integrable on $(0, \infty)$ for $0 < \tau < 1$. Therefor $F \in S$. In particular the following tail approximation for a compound distribution with a subexponential severity distribution is useful,

$$Let \qquad S = \sum_{k=1}^{N} X_k \qquad X_k \sim F, \qquad and \qquad F \qquad is \qquad subexponential,$$

Then,

$$P(S > t) \approx E(N) \qquad F(t, \infty)$$

2.2.5 Model Fitting

When one tries to fit the data to a model, some problems arise, such as *Which distribution should one use to fit the claim number?* and *Which distribution should one use to fit the claim severity?*

2.2.6 Severity Distribution

Truncated Lognormal Distribution

Let l,r denote a lower and upper threshold;
Let Y denote a lognormal(LN) distributed random variable.
i.e $\log Y \sim N(\mu, \sigma^2)$ where μ and σ^2 are the parameters of the LN distribution. This can also be written as $logY \sim LN(\mu, \sigma^2)$ for short. Let H denote it's distribution function. Then a random variable Z follows a truncated LN distribution if;

$$P(Z \in A) = P(Y \in A \mid Y \in [l, r]) \qquad for \qquad all \qquad events \qquad A$$

This means that the distribution function of Z can be expressed in the following way;

$$\begin{aligned} G(t) &= P(Z \le t) \\ &= P(Y \le t \mid Y \in [l, r]) \\ &= \frac{P(Y \le t \mid Y \in [l, r])}{P(Y \in [l, r])} \end{aligned}$$

$$= \begin{cases} 0 & \text{for } t \leq l \\ \frac{H(t)-H(l)}{H(l\ -H(r)} & \text{for } l \leq t \leq r \\ l & \text{for } t \geq r \end{cases} \tag{2.10}$$

Let X denote random variable that expresses the severity of an operational loss. Then let $X \sim F$.

Then for a threshold r, the excess distribution over r is defined by,

$$P(X - r \in A \mid X > r) \qquad for \qquad all \qquad events \qquad A$$

This excess distribution is a conditional distribution that has the following distribution function;

$$\begin{aligned} F_r(y) &= P(X - r \leq y \mid X > r) \\ &= \frac{P(r < X \leq r + y)}{P(X > r)} \\ &= \frac{F(r + y) - F(r)}{1 - F(r)} \\ &= \frac{F(r + y) - F(r)}{\overline{F}(r)} \end{aligned} \tag{2.11}$$

The distribution function of X can be expressed in the following way for $t > r$,

$$\begin{aligned} P(X \leq t) &= P(X - r \leq t - r) \\ &= P(X - r \leq t - r \mid X > r).P(X > r) \\ &= F_r(t - r).\overline{F}(r) \end{aligned} \tag{2.12}$$

Let the centre distribution of X be defined by

$$P(X \in A \mid X \leq r) \qquad for \qquad all \qquad events \qquad A$$

This means that the centre distribution has the following distribution function for $t \leq r$.

$$
\begin{aligned}
F^{(r)}(t) &= P(X \leq t \mid X \leq r) \\
&= \frac{P(X \leq t)}{P(X \leq r)} \\
&= \begin{cases} \frac{F(t)}{F(r)} & \text{for } t \leq r \\ 1 & \text{for } t > r \end{cases}
\end{aligned} \tag{2.13}
$$

The distribution of X can be seen as a mixture of the centre distribution and the excess distribution in the following way. Let Z denote a random variable(r.v) that is distributed by the center distribution and Y a random variable with excess distribution i.e $Z \sim F^{(r)}, \qquad Y \sim F_r,$
Then X and $Z.1_{Xleqr} + (r+Y).1_{X>r}$ have the same distribution. Proof:
First, one must show that the r.v

$$
M = Z.1_{X \leq r} + (Y + r).1_{X>r}
$$

has the same distribution function as X.

$$
\begin{aligned}
Pr(M \leq t) &= Pr(M \leq t \mid X \leq r)Pr(X \leq r) + Pr(M \leq t \mid X > r)Pr(X > r) \\
&= \frac{Pr(M \leq t, X \leq r)}{Pr(X \leq r)} Pr(X \leq r) + \frac{Pr(M \leq t, X > r)}{Pr(X > r)} Pr(X > r) \\
&= \frac{Pr(Z \leq t, X \leq r)}{Pr(X \leq r)} Pr(X \leq r) + \frac{Pr(Y + r \leq t, X > r)}{Pr(X > r)} Pr(X > r) \\
&= \frac{Pr(Z \leq t)Pr(X \leq r)}{Pr(X \leq r)} Pr(X \leq r) + \frac{Pr(Y + r \leq t)Pr(X > r)}{Pr(X > r)} Pr(X > r) \\
&= Pr(Z \leq t)F(r) + Pr(Y \leq t - r)Pr(X > r) \\
&= F^{(r)}(t)F(r) + F_r(t - r)(1 - F(r)) \\
&= F^{(r)}(t)F(r) = F(t) \qquad for \qquad t \leq r \\
& F^{(r)}(t)F(r) + F_r(t - r)(1 - F(r)) \qquad for \qquad t > r \\
&= F(r) + \frac{F(r + t - r) - F(r)}{1 - F(r)}(1 - F(r)) \\
&= F(r) + F(t) - F(r) \\
&= F(t)
\end{aligned} \tag{2.14}
$$

Chapter 3

Simulation of Sample Data

3.1 Using statistical package R

```
rm(list=ls()) library(evd) #generates a sample of a homogenuous
model based on EVT. #claim severity is a mixture of a truncated
lognormal and a generalizes Pareto distribution # the claim number
follows a Poisson distribution # parameter: # left : left
threshold # right : right threshold # meanl : meanlog parameter of
the lognormal distribution # sdl :  sdlog parameter of the
lognormal distribution # samplesize  : no of simulated
observations # shape : shape parameter of the GPD distribution #
scale : scale parameter of the GPD distribution # lambda  :
intensity of the Poisson distribution # alpha : mixing parameter

#output : Sample of the portfolio loss distribution

rheight<-function(samplesize, left=0, right=10, meanl=0, sdl=1,
scale=1, shape=1, alpha=0.9)
         {
           centersample<-rep(NaN,samplesize)
                for (i in (1:samplesize))
                  {
                    x<- -1
                    while (x<left | x>right)
                       x<-rlnorm(1,meanlog=meanl,sdlog=sdl)
                    centersample[i]<-x
                   }
            extremsample<-rgpd(samplesize,loc=right,scale,shape)
                randunif<-runif(samplesize)
                sample1<-centersample[randunif<alpha]
                sample2<-extremsample[randunif>alpha]
                sample<-c(sample1,sample2)
            return(sample)
```

```
          }
rtrunclnorm<-function(samplesize,left=0.1,meanlog=0,sdlog=1)
         {
           sample<-rep(NaN,samplesize)
           for (i in (1:samplesize))
             {
               x<--1
               while (x<left)
                  x<-rlnorm(1,meanlog,sdlog)
               sample[i]<-x
             }
           return(sample)
         }

SimHomogenEVT<-function(samplesize=10000,left=0.1, right=10 ,
meanl=0, sdl=1, scale=1, shape=1, lambda=10,alpha=0.9)
          {
             portloss<-rep(NaN,samplesize)
             for (i in 1:samplesize)
                {
                  claimnumber<-rpois(1,lambda)
                  claimseverities<-rheight(claimnumber,left,right,meanl,sdl,scal
                  portloss[i]<-sum(claimseverities)
                 }
              return(portloss)
            }

SimHomogenLognormal<-function(samplesize=10000,left=0.1,meanl=0,sdl=1,lambda=10)
         {
           portloss<-rep(NaN,samplesize)
           for (i in 1:samplesize)
             {
                claimnumber<-rpois(1,lambda)
                claimseverities<-rtrunclnorm(claimnumber,left,meanl,sdl)
                portloss[i]<-sum(claimseverities)
             }
           return(portloss)
         }

samplesize<-1000 l<-2000 r<-119037.1

alpha<-1-0.07801418 ml<-8.540138 sl<-1.25066 sc<-83902.06
sh<-0.2926496 intensity<-74



PortfoliolossEVT<-SimHomogenEVT(samplesize,left=l,right=r,meanl=ml,sdl=sl,scale=
summary(PortfoliolossEVT) VaREVT<-quantile(PortfoliolossEVT,0.99)
VaREVT mla<-7.81836 sla<-1.947136
```

```
PortlossLognormal<-SimHomogenLognormal(samplesize,left=l,meanl=mla,sdl=sla,lambd
summary(PortlossLognormal)
VaRLognormal<-quantile(PortlossLognormal,0.99) VaRLognormal
```

Above is the R program that was constructed to simulate Op Risk sample data. This program generates a sample of portfolio losses where the claim number is a poisson random variable and the claim severity is a mixture of a truncated lognormal and a GPD distributions.

The *rheight* command is a function which calculates a sample of loss severities form the normal distribution and the GPD distribution, as seen in FIG 3.

The *centersample* command is calculating a lognormal distributed sample set for the centre of distribution using the function *rlnorm*.

NAN is a command which initialises the vector space.

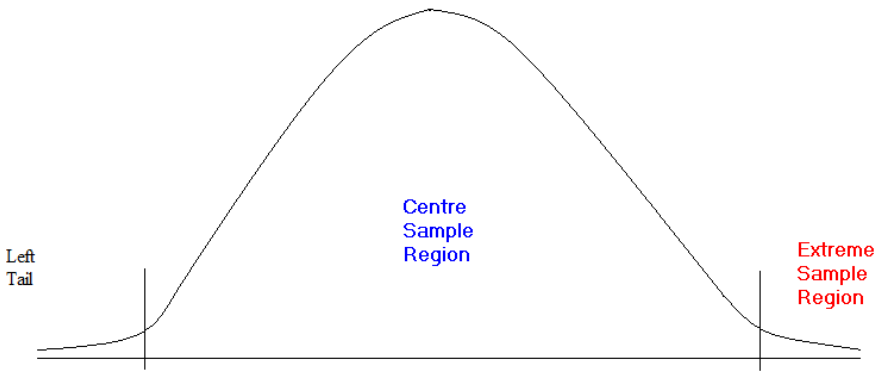

Figure 3.1: Centre Sample and Extreme Sample Regions

For the extreme sample to the right of the distribution, the program is generating a sample set which is a mixture of a GPD and a truncated Lognormal distribution. The *rgpd* function with its associated parameters in the round brackets, is the GPD distribution function. The *rtrunclnorm* is a command that is creating a sample set of a truncated lognormal distribution. The *rlnorm* is a density, quantile and random generation function for the lognormal distribution whose logarithm has mean *meanlog*, and standard deviation *sdlog*.

When this program is run using R, the following output is generated,

```
\begin{equation}
    summary(PortlossLognormal)
        Min.  1st Qu.   Median     Mean  3rd Qu.     Max.
      687600  1497000  1983000  2188000  2530000 11770000
\end{equation}
```

Figure 3.2: Simulated Data

FIG 4 is a simple representation of the results shown above.
687600 is the minimum loss which was generated, and 11770000
is the maximum loss which was generated.

Chapter 4

Appendices

AAA	An obligation rated 'AAA' has the highest rating assigned by Standard & Poor's. The obligor's capacity to meet its financial commitment on the obligation is extremely strong.
AA	An obligation rated 'AA' differs from the highest-rated obligations only to a small degree. The obligor's capacity to meet its financial commitment on the obligation is very strong.
A	An obligation rated 'A' is somewhat more susceptible to the adverse effects of changes in circumstances and economic conditions than obligations in higher-rated categories. However, the obligor's capacity to meet its financial commitment on the obligation is still strong.
BBB	An obligation rated 'BBB' exhibits adequate protection parameters. However, adverse economic conditions or changing circumstances are more likely to lead to a weakened capacity of the obligor to meet its financial commitment on the obligation.
BB	An obligation rated 'BB' is less vulnerable to nonpayment than other speculative issues. However, it faces major ongoing uncertainties or exposure to adverse business, financial, or economic conditions which could lead to the obligor's inadequate capacity to meet its financial commitment on the obligation.
B	An obligation rated 'B' is more vulnerable to nonpayment than obligations rated 'BB', but the obligor currently has the capacity to meet its financial commitment on the obligation. Adverse business, financial, or economic conditions will likely impair the obligor's capacity or willingness to meet its

CCC	An obligation rated 'CCC' is currently vulnerable to nonpayment, and is dependent upon favorable business, financial, and economic conditions for the obligor to meet its financial commitment on the obligation. In the event of adverse business, financial, or economic conditions, the obligor is not likely to have the capacity to meet its financial commitment on the obligation.
CC	An obligation rated 'CC' is currently highly vulnerable to nonpayment.
C	A subordinated debt or preferred stock obligation rated 'C' is currently highly vulnerable to nonpayment. The 'C' rating may be used to cover a situation where a bankruptcy petition has been filed or similar action taken, but payments on this obligation are being continued. A 'C' also will be assigned to a preferred stock issue in arrears on dividends or sinking fund payments, but that is currently paying.
D	An obligation rated 'D' is in payment default. The 'D' rating category is used when payments on an obligation are not made on the date due even if the applicable grace period has not expired, unless Standard & Poor's believes that such payments will be made during such grace period. The 'D' rating also will be used upon the filing of a bankruptcy petition or the taking of a similar action if payments on an obligation are jeopardized.
(+) or (-)	The ratings from 'AA' to 'CCC' may be modified by the addition of a plus (+) or minus (-) sign to show relative standing within the major rating categories.
NR	This indicates that no rating has been requested, that there is insufficient information on which to base a rating, or that Standard & Poor's does not rate a particular obligation as a matter of policy.

BASIC INDICATOR APPROACH	STANDARDISED A
Not allowed for international banks and institutions with high risk	Intermediate stage, no
Not risk sensitive	Calculate gross income

4.1 Figures

4.2 Tables

Chapter 5

Extra Stuff

Chapter 6

Risk Management

Quantitative risk management cannot be conducted on an ad hoc basis or by addressing selective sources of risk, (Book; Risk Modelling, Assessment, And Management, pg18).

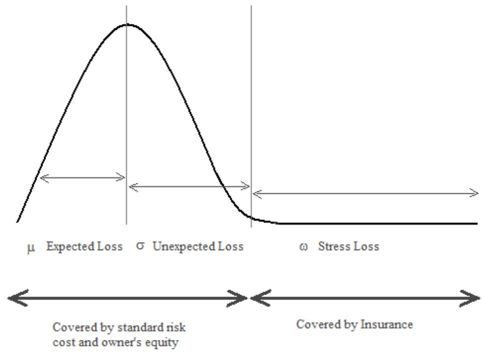

Figure 6.1: Representation of Operational Risk

(Book; Operational risk : the new challenge for banks, pg 110).

Chapter 7

References

7.1 Books

1. Operational risk : the new challenge for banks / Gerrit Jan van den Brink, Basingstoke : Palgrave, 2002

2. Mathematical methods in risk theory / Hans Bühlmann, Berlin : Springer Verlag, 1970

3. An introduction to mathematical risk theory / Hans U. Gerber, Philadelphia : Homewood, Ill. : S. S. Huebner Foundation for Insurance Education, Wharton School, University of Pennsylvania; distributed by R. D. Irwin, 1979

4. Loss Models From Data to Decisions / Stuart A. Klugman, Harry H. Panjer, Gordon E. Willmot / John Wiley & Sons, Inc., 1998

5. Risk Modelling, Assessment, And Management / 2^{nd} Ed. / Yacov Y. Haimes / John Wiley & Sons, Inc., 2004.

6. Modelling Extremal Events for insurance and finance / Paul Embrechts, Claudia Klüppelberg, Thomas Mikosch / Springer, 1997.

7.2 Reports

1. Dr. Carolyn V. Currie, Basel II And Operational Risk-Overview of Key Concerns, 2004
2. Ariane Chapelle / Yves Crama / George Hübner / Jean-Philippe Peters, National Bank of Belgium; Working Papers-Research Series; Basel II and Operational Risk: Implications for risk measurement and management in the financial sector, 2004
3. Rosella Giacometti / Svetlozar Rachev / Anna Chernobai / Marida Bertocchi / Giorgio Consigli, Practical Operational Risk, 2006
4. Alexander J. McNeil, Extreme Value Theory for Risk Managers, 1999
5. Svetlozar T. Rachev / Anna Chernobai / Christian Menn, Empirical Examination of Operational Loss Distribution.
6. Alexander J. McNeil / Thomas Saladin, The Peaks Over Threshold Method for Estimating High Quantiles of Loss Distributions, 1997
7. Alexander J. McNeil / Estimating the Tails of Loss Severity Distributions using Extreme Value Theory, 1996.
8. José Aparicio / Eser Keskiner, A Review of Operational Risk Quantitative Methodologies Within the Basel II Framework, 2004.

7.3 Websites

1. www.riskglossary.com
2. www.wikipedia.org
3. www.bis.org
4. www.standardandpoors.com

5. www.investorwords.com

6. www.sp.socgen.com