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Acronyms

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cdf	(cumulative) distribution function
chf	characteristic function
CLT	central limit theorem
dRi	directly Riemann integrable
FT	Fourier transform
GWM	Galton-Watson measure
GWP	Galton-Watson process
GWPI	Galton-Watson process with immigration
GWT	Galton-Watson tree
gf	generating function
iff	if, and only if
iid	independent and identically distributed
i.o.	infinitely often
LT	Laplace transform
mgf	moment generating function
MRCA	most recent common ancestor
MRW	Markov random walk
RP	renewal process
RW	random walk
SLLN	strong law of large numbers
SRP	standard renewal process = zero-delayed renewal process
SRW	standard random walk = zero-delayed random walk
ui	uniformly integrable
WLLN	weak law of large numbers