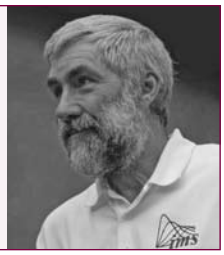
Terence's Stuff: *Moments have had their moment*

Is it just a passing fancy, or is it here to stay? Terry Speed ponders the fads and fashions of statistics, and consigns moments to history.



One of my fascinations is with fads and fashions in statistics, more precisely, with the waxing and waning of interest in what we might call *core* statistical theory and practice. If you have been around a while, as I have, you will have seen some long-established parts of our subject disappear from the mainstream. Of the ones that have arrived on the scene in my time, some are still limping along, struggling to stay alive, studied or practised by a faithful few, while others had their moment in the sun, and vanished without a trace. I mourn the passing of some; others I don't miss so much, because I now think their importance was overrated, and I was not sad to see a few die. These were the ones which were popular for a while, but which I thought were going nowhere, and that's where they went. Of course I conveniently forget the ones about which I made the same prediction, that became mainstream. *Sic transit gloria mundi* is a recurrent theme in religion, literature and popular culture, and apparently also in statistics.

One of the topics I lament is moments. Not too many years ago, everyone knew lots about moments, including their relationship with cumulants, and their generating functions. Karl Pearson vigorously promoted moments as a data analysis tool, emphasizing his family of frequency curves determined by their first four moments. Neyman was an early calculator of moments under simple random sampling without replacement from finite populations. Fisher re-discovered cumulants, and defined k-statistics based on random samples from infinite populations. His derivation of the joint cumulants of k-statistics was a combinatorial tour-de-force, but it was still rather complicated. That led Tukey to simplify it, and to show that it was easier to deal with finite than infinite populations, the reverse of the situation to that point. Pearson's *method of moments* was killed off in the 1920s by Fisher's *method of maximum likelihood*—though Pearson never conceded this, beginning a paper published in 1936, the year he died, with "Wasting your time fitting curves by moments, eh?" In a very personal attack on the dead Pearson, Fisher replied *"Yes!"* Similarly, the bootstrap seems to have killed off some of the other uses of moments. The reality is that we don't need moments any more, either in theory or in practice.

A topic I once loved, but don't mourn, is sufficiency. This was a unique gift of Fisher's, from statistics to measure theory and abstract analysis. As with moments, many of our heroes have contributed to its theory, including Neyman, Kolmogorov, Savage, and Le Cam. But did it ever have any practical value? Once you obtain the sufficient reduction of a sample under an assumed model, you can, they told us, throw away the rest of the data, as all inference can be done as well with what you have kept. That is, as long as you don't want to look at your data, or examine your model, or consider alternative models. To be sure, there are lots of lovely theorems and challenging exercises involving sufficiency, and it is still taught in some place, but still…

Robustness theory and applications expanded rapidly in the 1970s. I thought, and hoped, it would catch on and become mainstream. It hasn't, yet, but it hangs on, and I remain optimistic.

What about something I was not unhappy to see move on? Well, I am instinctively sceptical when I see a piece of fancy mathematics brought out to illuminate statistics, which happens frequently enough. Over thirty years ago, Efron showed that the notion of curvature in a parametric statistical problem could provide insights, and this led to a burst of research revisiting several aspects of parametric inference using the language of differential geometry. I think it has died down now, and have the impression it had little lasting impact on our subject, so wasn't sorry to see this. Perhaps I'm wrong. At the time I didn't think the gains justified the investment required, and I think this quite frequently in similar contexts. It is always hard to know how accommodating one should be. Perhaps one day a mathematical theory hitherto unfamiliar to statisticians (Riemannian connexions, group representations, Hodge theory, toric algebra, etc.) will turn out to have sufficiently important implications for mainstream statistics, that we'll all be compelled to master it. I can't rule this out, but I think the record to date justifies my scepticism.

It's always dangerous to pronounce something dead, and perhaps even more so to rejoice in the fact. Yet as new ideas arise and vie for our attention, something has to go. I think moments are dead, and some other topics are on the way. *Le Roi est mort, vive le Roi!*