Abstracts

Bruno Anglès

Stark Units in Positive Characteristic II : several variable L-series

In 2012, F. Pellarin introduced certain several variable zeta functions, and, shortly afterwards, by developing a variant of L. Taelman's work, B. Anglès, F. Demeslay, F. Pellarin and F. Tavares Ribeiro showed that these latter zeta functions were intimately connected to the arithmetic of the "twist" of the Carlitz module by its shtuka function. In this talk, using properties of "several variable" Stark units and in the context of general A, we will present some new results concerning special values of "several variable" Goss zeta functions. This talk is based on a work in progress with T. Ngo Dac and F. Tavares Ribeiro.

Esmail Arasteh Rad

Local theory of global G-shtukas

Moduli stacks of global G-shtukas appear as function field analogs for Shimura varieties. In particular one may hope that the Langlands correspondence for function fields can be realized on their cohomology. In this talk we explain the theory of "local" shtukas and corresponding moduli spaces, we further explain their applications to study the moduli of global G-shtukas (for a parahoric Bruhat-Tits group scheme G over a smooth projective curve). In particular, we discuss the local model theory and uniformization theory of the moduli of global G-shtukas, in analogy with that of Rapoport and Zink for Shimura varieties.

Alex Samuel Bamunoba

On coefficients of Carlitz cyclotomic polynomials

Let n be a positive integer, and $\Phi_n(x)$ be the nth elementary cyclotomic polynomial. D. Lehmer showed in "Some properties of the cyclotomic polynomial" that the geometric mean of $\{\Phi_m(1) : m \in \mathbb{Z}_+, m \leq n\}$ tends to e, as $n \to \infty$. In this talk, I will replace the index set $\mathbb{Z}_+$ with $\mathbb{F}_q^+[t]$ and give the Carlitz $\mathbb{F}_q[t]$-module analogue of Lehmer's result.

Alp Bassa

Rational points on curves over finite fields and Drinfeld modular varieties

In this talk we will be concerned with rational points on algebraic curves over finite fields. The central result in this area is without doubt the Hasse-Weil Theorem, which implies strong bounds on the number of rational points. As noticed by Ihara, for curves of large genus this bound can be improved significantly. Over quadratic finite fields modular curves of various type (classical, Drinfeld, Shimura) have been used successfully to construct high genus curves with many rational points, hence giving good asymptotic results for large genus curves. In this talk I will explain how these results can be generalised to all non-prime finite fields using certain special curves on Drinfeld modular varieties. This result is a joint work with Beelen, Garcia, Stichtenoth and has independently been obtained by Gekeler.

Gebhard Böckle

A Hecke stable decomposition of spaces of cusp forms

The conjecture of Maeda predicts that the space $S(k, SL(2,\mathbb{Z}))$ of classical cusp forms of level 1 and weight k is simple as a module for the Hecke algebra over $\mathbb{Q}$ and that the splitting field over $\mathbb{Q}$ of the Hecke algebra has Galois group the symmetric group on m elements where $m = dim S(k, SL(2,\mathbb{Z}))$.
For $A = \mathbb{F}_q[t]$, calculations of the Hecke action on the space $S(k, SL(2, A))$ of Drinfeld cusp forms of level 1 and weight k show that an analog of the above does not hold. Moreover Hecke eigensystems in weight k reappear up to twist as eigensystems in certain weights $k' \neq k$. Some of this is explained by the theory of hyperderivatives of Bosser-Pellarin.
In this talk we explain how one can use representation theory of $SL(2, K)$ to decompose $S(k, \Gamma)$ into certain natural Hecke stable subquotients, for any congruence subgroup Γ of $GL_2(A)$. We also reinterpret the results of Bosser-Pellarin in terms of representation theory. At the end we present some data for the Hecke action on the subquotients. Parts of this is joint work with R. Perkins and A. Petrov.

Florian Breuer

Drinfeld modular forms in higher rank: analytic theory

Chieh-Yu Chang

Logarithmic interpretation of multiple zeta values in positive characteristic

In this talk, we will introduce positive characteristic multiple zeta values (MZV's) initiated by Thakur in 2004. Our main result is to relate each MZV as certain coordinate of the logarithm of certain t-module at an algebraic point (joint work with Y. Mishiba). It generalizes the work of Anderson-Thakur in 1990.

SoYoung Choi

On the zeros and coefficients of certain weakly holomorphic Drinfeld modular forms

Let $M^!_{k.m}$ be the space of weakly holomorphic Drinfeld modular forms of weight k and type m for $GL_2(\mathbb{F}_q[T])$ that are holomorphic on the Drinfeld upper half plane except possibly at the cusp ∞. We construct a certain natural basis for $M^!_{k.m}$ and investigate various properties of the basis elements. In precise, we find that the basis elements satisfy a generating function and find the duality between coefficients of the basis elements. We also obtain the congruence properties of coefficients of these forms under some conditions. Finally we show that almost all elements in the basis have the property that the zeros in the fundamental domain for $GL_2(\mathbb{F}_q[T])$ the forms lie on the unit circle $|z| = 1$.

Alina Cojocaru

Reductions modulo primes of a generic Drinfeld module

Given a generic Drinfeld module, we consider its reductions modulo primes and, in particular, the elementary divisors of the emerging finite Drinfeld modules. For arbitrary rank, we study the distribution of the first elementary divisor as the prime of reduction varies. For rank 2, we also study the growth of the second elementary divisor (the exponent) as the prime of reduction varies and unravel the key arithmetic data which determines the two elementary divisors of the module. This is based on joint work with D. Shulman and with M. Papikian.

Luca Demangos

Quantum j invariant and Real Multiplication program for global function fields

(Joint work with T. M. Gendron)
We introduce a multi-valued modular invariant defined on the moduli space of quantum tori, parametrized by the real quadratic elements over $\mathbb{F}_q(T)$. We show that the product of the values taken by such an invariant on each real quadratic f is the primitive generator of the relative Hilbert Class Field associated to f. We then propose a notion of quantum exponential and quantum Drinfeld module to possibly generate in an analogous way the relative ray class fields of $\mathbb{F}_q(T)(f)$.

Ahmad El-Guindi

Explicit formulas and vanishing conditions for certain coefficients of Drinfeld-Goss Hecke eigenforms

We obtain a closed form polynomial expression for certain coefficients of Drinfeld-Goss double-cuspidal modular forms which are eigenforms for the degree one Hecke operators with power eigenvalues, and we use those formulas to prove vanishing results for an infinite family of those coefficients.

Ernst-Ulrich Gekeler

On Drinfeld modular forms of higher ranks

I will give a survey on modular forms for the full modular group $\mathrm{GL}(r, \mathbb{F}_q[T])$, where $\mathbb{F}_q$ is the finite field with q elements. This includes:

- the fundamental domain $\underline{F}$ on the Drinfeld space and its image in the Bruhat-Tits building
- the building map and properties of its fibers
- the growth/decay of coefficient forms, Eisenstein series, para-Eisenstein series on $\underline{F}$
- vanishing sets of modular forms, their smoothness and intersection properties.

Nathan Green

Tensor Powers of rank 1 Drinfeld Modules and Zeta Values

We study tensor powers of rank 1 sign-normalized Drinfeld A-modules, where A is the coordinate ring of an elliptic curve over a finite field. Using the theory of A-motives, we find explicit formulas for the A-action of these modules. Then, by developing the theory of vector valued Anderson generating functions, we give formulas for the coefficients of the logarithm and exponential functions associated to these A-modules, as well as formulas for the fundamental period. We then express zeta values in terms of the bottom coordinate of the logarithm function evaluated at certain vectors.

Urs Hartl

The Langlands-Rapoport Conjecture for global G-Shtukas

The conjecture of Langlands and Rapoport gives a group theoretic description of the points on a Shimura variety with values in a finite field. We will formulate the function field analog of this conjecture for moduli stacks of global G-shtukas and prove it in certain cases.

Oliver Lorscheid

Hecke operators, buildings and Hall algebras

Serre's theory of trees can be applied successfully to calculations with automorphic forms whenever strong approximation was sufficiently well working for PGL(2). This is the case if the ground field is a rational function field. In general, the class group becomes an obstruction, and a global variant of Serre's theory is needed.

In this talk, we introduce such a variant: the graph of a Hecke operator. We explain a structure theorem for elliptic function fields and its applications to automorphic forms. We investigate its connection to Ronan's theory of adelic buildings and we line out how Burban and Schiffmann's result about the Hall algebra of an elliptic curve might be used for the calculation of graphs of Hecke operators for higher rank groups.

This is work in progress joint with Robert Kremser and Roberto Alvarenga.

Maxim Mornev

Shtuka cohomology and special values of Goss L-functions

The motivic Tamagawa number conjecture gives a formula for special values of L-functions in terms of motivic cohomology. Among other things it integrates the analytic class number formula for number fields and the BSD conjecture in a single framework. While this conjecture is exceptionally hard, it has a tractable analogue in positive characteristic as discovered by Lenny Taelman. Here the usual L-functions are replaced by Goss L-functions which take values in a positive characteristic field, and one is interested in Goss L-functions attached to Drinfeld modules. In the talk I will explain how special values of these L-functions can be expressed in terms of shtuka cohomology which plays the role of motivic cohomology in this story.

Tuân Ngô Dac

Stark Units in Positive Characteristic I : zeta functions

In this talk, we introduce the module of Stark units attached to a Drinfeld module. For Drinfeld-Hayes modules, we explain how the associated module can be completely determined from Anderson's equivariant harmonic series. We apply this to obtain a class formula à la Taelman and to prove a several variable log-algebraicity theorem, generalizing Anderson's log-algebraicity theorem. This talk is based on a joint work with B. Anglès and F. Tavares Ribeiro.

Matthew Papanikolas

Limits of Bernoulli-Carlitz numbers and Eisenstein series

Unlike the classical case, the Bernoulli-Carlitz numbers BC_m for the rational function field K over a finite field of order q do not satisfy Kummer type congruences in a natural way. On the other hand, when m has the form $q^{dj} - s$ for a fixed positive integer s, works of Anglès, Ngo Dac, Pellarin, Perkins, and Tavares Ribeiro show that BC_m can have a v-adic limit as j goes to infinity, where v is a finite place of K of degree d. In a different direction, if one considers instead m of the form $aq^{dj} + b$, where a and b are positive integers, the above results do not apply. In the first case the index approaches a negative integer p-adically, whereas in the second the index goes to a positive integer, and this discrepancy leads to fundamentally different behavior, philosophically much like one sees in the difference between positive and negative Carlitz zeta values. Nevertheless, we show in this second case that BC_m does approach a v-adic limit as j goes to infinity, and using similar methods we show that the Eisenstein series E_m also have a v-adic limit in the sense of Serre. Furthermore, we show that the limiting value of BC_m is always algebraic, and in fact in only a constant field extension of K. Joint work with G. Zeng.

Mihran Papikian

Drinfeld-Stuhler modules

We study $\mathcal{D}$-elliptic sheaves in terms of their associated modules, which we call Drinfeld-Stuhler modules. We prove some basic results about Drinfeld-Stuhler modules and their endomorphism rings, and then examine the existence and properties of Drinfeld-Stuhler modules with large endomorphism algebras, which are analogous to CM and supersingular Drinfeld modules. Finally, we examine the fields of moduli of Drinfeld-Stuhler modules.

Federico Pellarin

On certain multiple sums

In this talk, we will review the properties of certain nested multiple sums of powers. We will notably describe a useful algebra structure. These multiple sums are ubiquitous in the theory of Thakur's multiple zeta values, but also, they sometimes occur in the expansion at the cusp infinity of certain vectorial modular forms with values in Tate algebras. We will discuss some results (also in collaboration with Bajpai and Perkins) and address some questions. If time allows it, we shall also describe a connection between an algebra of what we call regular multiple zeta values in Tate algebras and an algebra of finite multiple zeta values inspired by recent works of Kaneko and Zagier.

Rudolph Perkins

Twisting Drinfeld Cuspforms by Characters

We discuss a recent preprint wherein we answer a question of D. Goss, showing that is is possible to twist the eigensystems of cuspidal Drinfeld Hecke eigenforms (cuspforms) by characters. As usual, the effect of these twisting operators on Goss's power series expansion at the cusp at infinity appears complicated, and even for cuspforms with A-expansions in the sense of Petrov, the effect is worse than a simple "twisting of the A-expansion coefficients by a character." The upshot of our results, as Goss originally pointed out, is that the local L-factors one obtains from a Drinfeld module of rank 1, defined over the rational function field over a finite field, may also be obtained from the local L-factors of a cuspidal Drinfeld Hecke eigenform.

Richard Pink

Drinfeld modular forms in higher rank: algebraic theory

Zev Rosengarten

Tamagawa Numbers of Linear Algebraic Groups over Function Fields

In 1981, Sansuc obtained a formula for Tamagawa numbers of reductive groups over global fields, conditional on some then as-yet unknown conjectures which have since been proven. This allows one to easily obtain the same formula for all connected linear algebraic groups over number fields. Passing to the general, not necessarily reductive, case for function fields, however, is much more difficult, due to difficulties arising from the imperfection of such fields. Indeed, Sansuc's formula is not correct for general linear algebraic groups over function fields. We propose a modification of Sansuc's formula in the function field setting and prove it in many interesting cases, particularly, for all groups that are either commutative or pseudo-reductive. The commutative case, in particular, involves a substantial generalization of the classical Poitou-Tate nine-term exact sequence from finite group schemes to group schemes of higher dimension.

Dinesh Thakur

Some recent results and conjectures in function field arithmetic

Maria Valentino

Diagonalizability of the U-operator for Drinfeld cusp forms of congruence subgroups

Let $S_{k,m}(\Gamma)$ be the space of Drinfeld cusp forms of weight k and type m for a congruence subgroup $\Gamma \in GL_2(\mathbb{F}_q[t])$. We will address the problem of the diagonalizability of the Atkin

U-operator acting on the spaces $S_{k,m}(\Gamma)$ for $\Gamma = \Gamma_1(t), \Gamma(t), \Gamma_0(t)$. Different approaches will be considered for different congruence subgroups. In particular, we will use Teitelbaum's interpretation of cusp forms as harmonic cocycles for $\Gamma = \Gamma_1(t), \Gamma(t)$ and a level raising argument for $\Gamma = \Gamma_0$. We will see that U is not always diagonalizable and we formulate a conjecture, supported by numerical search and proofs in some specific cases, about non diagonalizability in even characteristic.

Christelle Vincent

Towards computing the structure of algebras of Drinfeld modular forms

We first present the work of Voight and Zureick-Brown, who compute the structure (generators and relations) of the canonical ring of a stacky curve, using as input the signature of the curve (which is simply the data of the number and orders of each elliptic point along with the cuspidal divisor). One application of this work is computing the structure of the algebra of modular forms for a given modular group: in that case the canonical ring of the modular curve is isomorphic to the corresponding algebra of modular forms. We then finish by explaining the obstacles to applying their work to compute algebras of Drinfeld modular forms in general and discuss how these obstacles might be overcome.

Fu-Tsun Wei

Kronecker limit formula of mirabolic Eisenstein series over function fields

Mirabolic Eisenstein series plays an important role in the study of L-functions coming from automorphic representations. In this talk, I will establish an analogue of the Kronecker limit formula for the mirabolic Eisenstein series on GL(n), which connects the first derivative of these series with Drinfeld-Siegel units.
One application is to derive a Colmez-type formula of CM Drinfeld modules, which expresses the "Taguchi" height of CM Drinfeld modules in terms of the logarithmic derivative of the corresponding zeta functions. From the integral representation of the Rankin-Selberg L-functions and Godement-Jacquet L-functions associated to cuspidate representations over function fields, we obtain two product formulas of their special values in the end.