Correction to “Equilibrium Contact Angle and Adsorption Layer Properties with Surfactants”

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The last line in eq 27 on page 7213 should read

\[
\int_0^R dx \left[ -P + \kappa Y \right] \delta h(x) + \left[ \frac{\partial}{\partial x} Y + \lambda_{\delta} \right] \delta h(R) = 0
\]

(27)

In the caption for Figure 2a on page 7213 it should read

(a) In the macroscopic approach, the equilibrium contact angle is determined by the solid–liquid interfacial tension \( \gamma \) and the liquid–gas and solid–gas interfacial tensions \( \gamma_s \) and \( \gamma_{sg} \) that depend on the respective surfactant concentrations \( \Gamma_d \) and \( \Gamma_a \) on the droplet and the adsorption layer.

The free energies \( g(\Gamma) \) and \( g_{sg} \) in section “Application for Simple Energy” on page 7215 written consistently in the surface number density \( \Gamma \) should read

\[
g(\Gamma) = \gamma^0 + k_B T \Gamma [\ln(\Gamma a^2) - 1]
\]

(45)

\[
g_{sg}(\Gamma) = \gamma_{sg}^0 + k_B T \Gamma [\ln(\Gamma_{sg} a^2) - 1]
\]

(46)

This results in the following modifications in the subsequent formulas (pages 7215 and 7216) and Figures 3 and 4 (page 7217):

\[
\gamma(\Gamma) = \gamma^0 + \Gamma \delta \gamma \Gamma = \gamma^0 - k_B T \Gamma
\]

(47)

\[
\gamma_{sg}(\Gamma) = \gamma_{sg}^0 + \Gamma \delta \gamma_{sg} = \gamma_{sg}^0 - k_B T \Gamma
\]

(48)

The mobility matrix in eq 60 on page 7216 should read

\[
\frac{\cos \theta_e}{\gamma} = \frac{\gamma^0 \cos \theta_0 - k_B T \Gamma_d}{\gamma^0 - k_B T \Gamma_d}
\]

(49)

\[
\Gamma_d = \frac{\Gamma_a}{\delta}
\]

(50)

\[
\cos \theta_e = \cos \theta_0 + \frac{k_B T}{\gamma^0} (\cos \theta_0 - 1) \Gamma
\]

(51)

\[
\cos \theta_e \approx \cos \theta_0 - \frac{k_B T}{\gamma^0} \Gamma_a \cos \theta_0
\]

(52)

\[
\cos \theta_e \approx \cos \theta_0 - \frac{k_B T}{\gamma^0} \Gamma_a
\]

(53)

\[
\chi(\Gamma) = 1 - \frac{k_B T}{f(h)} \Gamma \ln \delta
\]

(56)

\[
f(h, \Gamma) = \frac{1}{f(h)} \left[ 1 - \frac{k_B T}{f(h)} \Gamma \ln \delta \right]
\]

(57)

Figure 3. Profiles of film height \( h \) (top) and surfactant concentration \( \Gamma \) (bottom) evolving in the numerical simulations for large times. The simulations are performed for three different ratios \( \delta = \frac{a^2}{\Gamma_{\delta}} \) of the effective molecular length scales of the surfactant at \( a^2 \Gamma_{\delta} = 0.04 \) in (a) and three different mean surfactant concentrations \( \Gamma \) at \( \delta = 2 \) in (b) while keeping the remaining parameters fixed at \( \epsilon_1 = 0.2 \) and \( \epsilon_2 = 0.4 \). The insets show a zoom of the contact line region. Note that the surfactant concentration \( \Gamma_w \) which occurs on the wedge in the mesoscopic description corresponds to the concentration \( \Gamma_d \) on the droplet.

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\[
\mathbf{Q} = \begin{pmatrix} Q_{ib} & Q_{ib} \\ Q_{ib} & Q_{ib} \end{pmatrix} = \begin{pmatrix} h^3 & h^2 \Gamma \\ 3\eta & 2\eta \\ h^2 \Gamma & h^2 \Gamma + D \Gamma \end{pmatrix}
\] (60)

where \( \eta \) and \( D \) denote a viscosity and diffusivity, respectively.

**Formula 62** in the section “Generalization to Arbitrary Interface Energies” on page 7218 should read

\[
\chi = \frac{1}{\int_a^b} [g_{sb} (\Gamma) - g(\Gamma) - \gamma_{at}]
\] (62)

resulting in the modifications

\[
\mathcal{F}[h, \Gamma] = \int \left\{ \gamma_{at} + \frac{\lambda}{\int_a^b} \left[ g_{sg} (\Gamma) - g(\Gamma) - \gamma_{at} \right] \\
+ \xi [g(\Gamma) - \lambda \Gamma] - Ph \right\} dx
\] (63)

\[
\mathcal{F}_a = \int \{ g_{sg} - \lambda \Gamma \} \, dx
\] (65)

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**Figure 4.** Surfactant concentration on the droplet (top) and equilibrium contact angle (bottom) depending on the surfactant concentration in the adsorption layer. The analytically obtained equilibrium conditions (solid lines) are compared to values extracted from time simulations (diamonds) for three values of \( \delta \). The dashed (dotted lines) show the values obtained by parameter continuation for the domain size \( L_x/l = 400 \) (\( L_x/l = 6000 \)). The discrepancy in the equilibrium contact angle between the numerical and the analytical result can be attributed to finite size effects.

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