Supporting Information for:
Instabilities of layers of deposited molecules on chemically stripe patterned substrates: Ridges vs. drops

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1 Implementation in AUTO-07p

For the treatment with the continuation toolbox AUTO-07p, we transform the set of equations that determines the steady state solutions \( \text{and} \) the accompanying linear stability problem into a set of first order ordinary differential equations. The application of the continuation approach to a similar thin film equation is outlined in Ref. 1 while explanations of technical details and example codes can be found in the tutorials drop, hetdrop, and lindrop of Ref. 2.

In particular, we transform the nonautonomous Eq. (5) of the main text

\[
\partial_x^2 h_0(x) + \Pi(h_0, x) + C = 0
\]

that is of second order and the nonautonomous Eqs. (7)

\[
\beta h_1 = -Q(h_0)(\partial_x^2 - q^2) \\
\times \left[ (\partial_x^2 - q^2)h_1 + (\partial_x \Pi(h_0, x))h_1 \right] \\
- (\partial_x Q(h_0))\partial_x \left[ (\partial_x^2 - q^2)h_1 + (\partial_x \Pi(h_0, x))h_1 \right]
\]

that is of fourth order into an autonomous system of seven first order ODEs on the interval \([0, 1]\). To this end, we first define the independent variable \( \xi := x/L \) with \( L \) denoting the
physical domain size. Next, we introduce the variables

\[ u_1(\xi) = h_0(L\xi) - \bar{h}, \]
\[ u_2(\xi) = \left. \frac{dh_0}{dx} \right|_{x=L\xi}, \]
\[ u_3(\xi) = h_1(L\xi), \]
\[ u_4(\xi) = \left. \frac{dh_1}{dx} \right|_{x=L\xi}, \]
\[ u_5(\xi) = \left. \frac{d^2h_1}{dx^2} \right|_{x=L\xi}, \]
\[ u_6(\xi) = \left. \frac{d^3h_1}{dx^3} \right|_{x=L\xi}, \]
\[ u_7(\xi) = L\xi. \]

Here, \( \bar{h} \) denotes the mean film thickness. With the notation \( \dot{u}_i(\xi) = du_i(\xi)/d\xi \), the system of first order ODEs reads

\[ \dot{u}_1 = Lu_2 \]
\[ \dot{u}_2 = -L[\Pi(\bar{h} + u_1, u_7) + C] \]
\[ \dot{u}_3 = Lu_4 \]
\[ \dot{u}_4 = Lu_5 \]
\[ \dot{u}_5 = Lu_5 \]
\[ \dot{u}_6 = L \left\{ -\frac{\beta u_3}{Q_0} + q^2 u_5 - \partial_x^2(h_1 \partial_h \Pi_0) \right. \\
- \frac{\partial_x Q_0}{Q_0} \left[ (u_6 - q^2 u_4 + \partial_x(h_1 \partial_h \Pi_0)) \right] \\
\left. + q^2 \left[ u_5 - q^2 u_3 + \Pi'(\bar{h} + u_1)u_3 \right] \right \} \]
\[ \dot{u}_7 = L \]
with $Q_0 = Q(u_1 + \bar{h})$ and

\begin{align}
\partial_x (h_1 \partial_h \Pi_0) &= \Pi''(\bar{h} + u_1, u_7) u_2 u_3 + \Pi'(\bar{h} + u_1, u_7) u_4 \\
&\quad + \Pi'_x(\bar{h} + u_1, u_7) u_3 , \tag{15}
\end{align}

\begin{align}
\partial^2_x (h_1 \partial_h \Pi_0) &= \Pi'''(\bar{h} + u_1, u_7) u_2^2 u_3 \\
&\quad + \Pi''(\bar{h} + u_1, u_7)(\partial^2_x h_0) u_3 \\
&\quad + 2 \Pi''(\bar{h} + u_1, u_7) u_2 u_4 \\
&\quad + \Pi'(\bar{h} + u_1, u_7) u_5 \\
&\quad + 2 \Pi'_x(\bar{h} + u_1, u_7) u_4 \\
&\quad + 2 \Pi''_x(\bar{h} + u_1, u_7) u_2 u_3 \\
&\quad + \Pi''_{xx}(\bar{h} + u_1, u_7) u_3 . \tag{16}
\end{align}

In the last two equations, primes denote derivatives w.r.t. $h$ while the index $x$ denotes a derivative w.r.t. $x$ at constant $h$. The equation for $x = u_7$ allows to write the heterogeneity function in an autonomous way. This is necessary because AUTO-07p only solves autonomous ODEs.

**References**
