

## Exercise Sheet 6

### 1. Turing instability

Consider the reaction-diffusion system

$$\begin{aligned}\dot{u} &= f(u, v) + D_u u'' \\ \dot{v} &= g(u, v) + D_v v''\end{aligned}$$

with the diffusion coefficients  $D_u, D_v > 0$  with  $D_v > D_u$ . The functions  $f(u, v)$  and  $g(u, v)$  are nonlinear.

1. Start with  $D_u = D_v = 0$  and determine via the Jacobian  $J$  under which conditions the homogeneous state  $(u, v) = (0, 0)$  is stable.
2. Make the ansatz (see lecture)

$$u(x, t) = \sum_q u_q(t) \cos(qx), \quad v(x, t) = \sum_q v_q(t) \cos(qx)$$

and derive the  $q$ -dependend Jacobian  $J_q$ . Under which conditions are the Fourier modes stable?

3. Determine the wavenumber  $q_c$ , which gets unstable first.