

Exercise Sheet 3

1 Bifurcations

For the following equations find the critical value of r where a bifurcation takes place, determine the type of bifurcation and draw the bifurcation diagram x^* vs. r .

$$\dot{x} = r + x - \ln(1 + x) \quad \dot{x} = x - rx(1 - x) \quad \dot{x} = x + \frac{rx}{1 + x^2}$$

2 Imperfect transcritical bifurcation

Consider the system $\dot{x} = h + rx - x^2$ showing a transcritical bifurcation for $r = 0$ and $h = 0$.

1. Draw the bifurcation diagram for $h < 0$, $h = 0$ and $h > 0$.
2. Indicate which regions in the (r, h) -area belong to qualitative different phase portraits and identify the different bifurcations at the boundaries between the different regions
3. Draw for every region the potential $V(x)$.