Introduction to Nonlinear Physics, WS 2016/17

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http://pauli.uni-muenster.de/~okamp/complex/complex.html

Exercise Sheet 2

1 Growth of tumors

The growth of cancer tumors can be described by the Gompertz-law

$$\dot{N} = -aN \ln(bN) \,.$$

Here is $N(t)$ proportional to the number of cells in the tumor, while a and b are parameters ($a, b > 0$). As long as N is big enough, the predictions of this model agree very well with experimental data on tumor growth.

1. Sketch the phase portrait and the solution $N(t)$ for different initial conditions N_0.
2. Classify via linear stability analysis the fixed points of the model.
3. What is the biological meaning of the parameters a and b?

2 Linear Stability analysis

Use linear stability analysis to classify the fixed points of the following equations:

$$\dot{x} = 1 - e^{-x^2} \qquad \dot{x} = \ln x \qquad \dot{x} = -\tan x$$

If this is not possible use a graphical analysis!

3 Potentials

Derive the potentials $V(x)$ for the following equations and sketch them. Where are the fixed points and what is their stability?

$$\dot{x} = x(1-x) \qquad \dot{x} = 3 \qquad \dot{x} = -\sinh x$$