Numerical simulation of supersymmetric Yang-Mills theory

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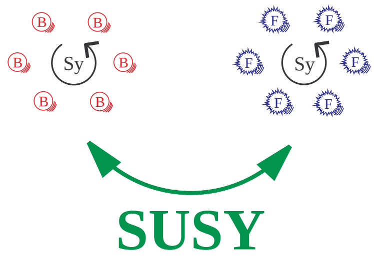
We report on large-scale numerical simulations of supersymmetric Yang-Mills (SYM) theory by the DESY-Münster Collaboration. The spectrum of light composite particles is investigated and confronted with theoretical expectations based on unbroken supersymmetry for large volumes and small gaugino masses.

1 Introduction

The recently found Higgs scalar particle at the Large Hadron Collider (LHC)[1,2] completes the *Standard Model (SM)* of elementary particle interactions. All known matter is composed of a small number of fundamental constituents, the quarks and leptons. Subatomic interactions of these particles, namely strong, weak and electromagnetic interactions, are described in the Standard Model within the framework of *Quantum Field Theory*. The scalar field corresponding to the Higgs particle is the source of masses of quarks and leptons and of the vector boson fields mediating the interactions. All known experimental data in the presently available energy range can be described by the Standard Model.

In spite of the completeness and beauty of the Standard Model the question what happens at still higher energies cannot be answered with certainty. The simplest possibility would be that the Standard Model in the present form is valid at all energies till infinity. Otherwise there are physical laws Beyond the Standard Model (BSM). One argument in favour of the existence of BSM physics is the large number of free parameters in the Standard Model. By just a minimal change of some free parameters the entire world would become completely different from the one we know. For instance, the existence and stability of atoms and ordinary matter relies heavily on the fact that neutrons are slightly more massive than protons. The reason for this small difference, besides the different electromagnetic self-interactions, is the small mass difference of up- and down-quarks as constituents. It is an intriguing question, why this small mass difference happens to be just the one realised in our world.

A theoretical hint towards the existence of BSM physics comes from the investigation of the change of effective ("running") couplings as a function of the energy. It is a long known fact that the three running couplings (electromagnetic, weak and strong) become almost equal at high energies above, say, 10^{16} GeV. At this *Grand Unification Theory (GUT)* scale the basic symmetry group underlying the Standard Model changes from SU(3)⊗SU(2)⊗U(1) to a larger embedding group like SU(4)⊗SU(4)[3], SU(5)[4], or SO(10)[5]. The small discrepancy of the three running couplings near the GUT scale disappears if the Standard Model is extended by *supersymmetry (SUSY)* to a *Supersymmetric Standard Model (SSM)*. Supersymmetry is an extension of the Poincaré symmetry of space-time corresponding to an extension of the Poincaré algebra by one or several supersymmetry charges to a *super-Poincaré algebra* that relates bosons to fermions (for a review see Ref. 6).



The supercharges change the spin by $\frac{1}{2}$, hence the supersymmetry multiplets contain particles with different spins, in particular, bosons and fermions at the same time. Since at present energies no such supermultiplets with degenerate masses are observed, supersymmetry – if it is realised in Nature – has to be a broken symmetry.

Another important reason why BSM physics has to exist, is the *triviality* of the infinite cut-off limit: some renormalised couplings such as the electromagnetic coupling, quartic scalar coupling and (generalised) Yukawa couplings have to vanish, i. e. such interactions cannot be realised in the framework of a relativistic Quantum Field Theory. In case of the electromagnetic coupling this problem is traditionally described by the emergence of a *Landau pole*[7] at high energies, where the running electromagnetic coupling diverges. As a consequence of triviality in the Standard Model, for a given cut-off there is a region in parameter space of renormalised couplings that is allowed. In the limit of an infinite cut-off this region shrinks to the origin at zero renormalised couplings. Since the renormalised couplings in the Standard Model are known from the experimentally known particle masses, there is a cut-off value at which the couplings are at the border of the allowed region. Higher cut-offs are impossible and therefore BSM physics has to appear not later than at this energy scale. The allowed region of renormalised quartic and Yukawa couplings can be investigated in numerical simulations. (See for instance Ref. 8.) If the Higgs boson mass is about 125–126 GeV, it is in particular the lower Higgs boson mass bound that can imply the values of the energy scale where new BSM physics has to appear[9].

Supersymmetry may also relieve the constraints arising from triviality. For instance, in $\mathcal{N} = 2$ *supersymmetric Yang-Mills (SYM)* theory every renormalisable quartic and Yukawa coupling is proportional to the non-abelian gauge coupling, which is asymptotically free and can, therefore, describe non-trivial interactions (see for instance Ref. 10,11). In case of the Standard Model, an advantage of GUTs is that the U(1) gauge symmetry corresponding to the electromagnetic interaction becomes part of a non-abelian gauge symmetry and therefore the Landau pole problem disappears. Also, from the point of view of the triviality problem of quartic and Yukawa couplings, a SUSY GUT is better (less restrictive) than a GUT without supersymmetry. More generally, in SUSY theories the radiative corrections are less important and the infinities in renormalisation are fewer[6].

Every supersymmetric extension of the Standard Model contains as a building block the $\mathcal{N}=1$ SYM theory, which is the object of investigation in our project. It is the supersymmetric extension of Yang-Mills theory describing the carriers of gauge interactions, the gauge particles, together with their supersymmetric partners, the gauginos. Gauginos are massless Majorana fermions, which are in the adjoint representation of the gauge group. In the continuum the (on-shell) Lagrangian of the theory is

$$\mathcal{L} = \mathrm{tr}\left[-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{\mathrm{i}}{2}\bar{\lambda}\gamma^{\mu}D_{\mu}\lambda - \frac{m_g}{2}\bar{\lambda}\lambda\right] , \tag{1}$$

where $F_{\mu\nu}$ is the non-Abelian field strength formed out of the gauge fields $A_{\mu}(x)$, $\lambda(x)$ is the gaugino field, and D_{μ} denotes the gauge covariant derivative in the adjoint representation. The supersymmetry of the theory is broken softly by the gaugino mass term. We are presently investigating the simplest non-abelian SYM theory with gauge group SU(2). Our nonperturbative studies are concentrating on the properties of the light particle spectrum. In particular, we determine the masses of the lightest composite particles by performing numerical simulations in the lattice-regularised theory. For our recent publications see Refs. 12–15.

2 Supersymmetry on the lattice

Poincaré (Lorentz) symmetry is broken by lattice regularisation. Since SUSY generators define an extension of the Poincaré algebra, it is not surprising that SUSY is, in general, also broken by the lattice. There are some recently exploited exceptions to this rule, especially in case of extended SUSY with several SUSY generators ($\mathcal{N} > 1$) and in most cases in lower dimensions (for a review see for instance Ref. 16). An interesting example in four dimensions is $\mathcal{N}=4$ SYM theory, which may be discretised in such a way as to preserve one scalar supersymmetry at nonzero lattice spacing[17]. (The other 15 supersymmetries are still broken by lattice artefacts of $O(a)$, where a denotes the lattice spacing.) In case of $\mathcal{N}=1$ SYM there is no such possibility, hence SUSY is broken in any known lattice formulation.

Our numerical calculations are based on the Curci-Veneziano lattice action[18], which is built in analogy to the Wilson action of QCD[19] for the gauge field ("gluon") and Wilson fermion action for the gaugino ("gluino"). Both supersymmetry and chiral symmetry are broken by lattice artefacts but are expected to be restored in the continuum limit if the gaugino hopping parameter (i.e. bare mass) is tuned to a critical value. The breaking of the chiral symmetry for nonzero lattice spacings could be avoided by using domain-wall[20, 21] or overlap[22] fermions and then there is no need of parameter tuning, but the SUSY breaking remains and the required numerical effort for simulations would substantially increase.

2.1 SYM theory on the lattice

The Curci-Veneziano lattice action for SYM theory is given by

$$S = S_g + S_f \; . \tag{2}$$

Here S_g is the gauge field action

$$S_g = \beta \sum_{pl} \left(1 - \frac{1}{N_c} \mathrm{Re}\,\mathrm{Tr}\, U_{pl} \right) , \tag{3}$$

with the gauge coupling $\beta \equiv 2N_c/g^2$ for an SU(N_c) gauge field. U_{pl} is the product of the gauge link fields along a plaquette. The fermionic part of the action (2) is

$$S_f \equiv \tfrac{1}{2}\overline{\lambda} Q \lambda$$

$$\equiv \tfrac{1}{2} \sum_x \left\{ \overline{\lambda}^a_x \lambda^a_x - K \sum_{\mu=1}^{4} \left[\overline{\lambda}^a_{x+\hat{\mu}} V_{ab,x\mu}(1+\gamma_\mu)\lambda^b_x + \overline{\lambda}^r_x V^T_{ab,x\mu}(1-\gamma_\mu)\lambda^b_{x+\hat{\mu}} \right] \right\} . \tag{4}$$

Here K is the hopping parameter which determines the gaugino mass, γ_μ denotes a Dirac matrix and $V_{x\mu}$ is the gauge field variable in the adjoint representation of the gauge group, which is obtained from the gauge field links in the fundamental representation $U_{x\mu}$ by

$$V^{ab}_{x\mu} \equiv 2\mathrm{Tr}\left(U^\dagger_{x\mu} T^a U_{x\mu} T^b \right) \tag{5}$$

(T^a are the generators of SU(N_c)). The gaugino field λ_x satisfies the Majorana condition

$$\overline{\lambda}_x = \lambda^T_x\, C , \tag{6}$$

with the charge conjugation Dirac matrix C.

Performing the path integral over the fermion field λ results in a *Pfaffian*:

$$\int [d\lambda] e^{-\frac{1}{2}\overline{\lambda} Q \lambda} = \int [d\lambda] e^{-\frac{1}{2}\lambda M \lambda} = \mathrm{Pf}(M) , \tag{7}$$

where M is the antisymmetric matrix defined as

$$M \equiv CQ = -M^T . \tag{8}$$

The square of the Pfaffian $\mathrm{Pf}(M)$ is equal to the determinant of the fermion matrix Q:

$$\det(Q) = \det(M) = [\mathrm{Pf}(M)]^2 . \tag{9}$$

The Monte Carlo simulations are performed by importance sampling with respect to a positive measure. Since for finite lattice spacing a the Pfaffian is not always positive, its sign has to be taken into account separately. Taking the non-negative square root of the determinant, the effective gauge field action is[18]:

$$S_{CV} = \beta \sum_{pl} \left(1 - \frac{1}{N_c} \mathrm{Re}\,\mathrm{Tr}\, U_{pl} \right) - \frac{1}{2} \log\det Q[U] . \tag{10}$$

The factor $\frac{1}{2}$ in front of $\log\det Q$ can be interpreted as corresponding to a flavour number $N_f = \frac{1}{2}$ of Dirac fermions. The gauge configuration for this fractional flavour number can be created, for instance, by the *two-step polynomial Hybrid Monte Carlo (TSPHMC)* algorithm[23], which is our choice for Monte Carlo updating.

The omitted sign of the Pfaffian can be taken into account by reweighting:

$$\langle A \rangle = \frac{\langle A\ \mathrm{signPf}(M) \rangle_{CV}}{\langle \mathrm{signPf}(M) \rangle_{CV}} , \tag{11}$$

where $\langle \ldots \rangle_{CV}$ denotes expectation values with respect to the effective gauge action S_{CV}. The reweighting with the Pfaffian sign in (11) may lead to a *sign problem* if a strong cancellation occurs among contributions with opposite sign. This could lead to very large statistical errors. However, as we have shown in previous papers by monitoring the sign of the Pfaffian[24, 25], for positive gaugino masses practically no sign problem occurs because the positive contributions dominate.

3 Light particle spectrum in SYM theory

The gauge coupling in SYM theory is asymptotically free at high energies and becomes very strong in the infrared limit. The low energy particle spectrum is expected to consist of hadron-like colourless states due to confinement, similar to QCD. The difference to QCD is that in SYM the quarks are replaced by Majorana fermions in the adjoint representation. Because of supersymmetry, the particles should belong to mass degenerate SUSY multiplets. The verification of this expectation is a central task for nonperturbative studies in the lattice regularisation.

3.1 Pseudoscalar and scalar mesons

The colourless states can be created from the vacuum state by gauge invariant operators which are built from the gluon and gluino field operators. (In this section we shall call, in analogy to QCD, the gauge field “gluon field” and the gaugino as the “gluino”.) A simple example of colourless composite states are the *adjoint mesons*. (The name “adjoint” refers to the fact that the composing fermions are in the adjoint representation.) The adjoint mesons are composite states of two gluinos with spin-parity 0^+ and 0^-. We denote the former by a-η' and the latter by a-f_0, where the prefix a refers to the adjoint representation. For projecting to these states we use the gluino bilinear operators $\mathcal{O} = \bar{\lambda}\Gamma\lambda$ where $\Gamma = \gamma_5, 1$ respectively. The resulting meson propagator consists of connected and disconnected contributions:

$$\begin{aligned} C_\Gamma(t) = \frac{1}{V_s} \sum_{\vec{x},\vec{y}} \Bigg\langle \underbrace{\mathrm{Tr}_{sc}[\Gamma Q^{-1}_{xx}]\mathrm{Tr}_{sc}[\Gamma Q^{-1}_{yy}]}_{\text{disconnected}} - 2\,\underbrace{\mathrm{Tr}_{sc}[\Gamma Q^{-1}_{xy}\Gamma Q^{-1}_{yx}]}_{\text{connected}} \Bigg\rangle \\ - \frac{1}{V_s} \left\langle \frac{1}{T} \sum_t \sum_{\vec{x}} \mathrm{Tr}_{sc}[\Gamma Q^{-1}_{xx}] \right\rangle^2 . \end{aligned} \tag{12}$$

where Tr_{sc} denotes a trace over spin and colour indices. The connected term can be used to extract the mass $m_{a\text{-}\pi}$ of the adjoint pion, which is an unphysical state in SYM (the last term in Eq. (12) is zero for $\Gamma = \gamma_5$). The vanishing pion mass is used to signal the chiral limit.

The numerical evaluation of the disconnected propagators is rather demanding. In order to reduce the large variance, the disconnected part has been calculated using the stochastic estimator method[26]. As it is the case in QCD, the disconnected diagrams are intrinsically noisier than the connected ones and dominate the level of noise in the total correlator.

An additional difficulty for calculating the mass of the scalar meson ($\Gamma = 1$ in Eq. (12)) is that in this case, in contrast to the pseudoscalar meson ($\Gamma = \gamma_5$), the vacuum expectation value appearing in the last term of Eq. (12) is non-zero. This has to be subtracted or has to be fitted as an additional contribution in the correlator. This can be done, for instance, by the method of fitting and error determination described in section 3.2 of Ref. 27, but the effect of the additional fit parameter is an increase of the statistical error.

Scalar particles with the same quantum numbers as a-η' and a-f_0 can also be created by purely gluonic operators built from the gauge links as in pure gauge theory or QCD. It is possible that the gluon operators create the same states as the above gluino operators. For this case the low energy effective theory was defined in Ref. 28. The other possibility is that there are two low-mass SUSY multiplets created by the two sorts of operators, which can eventually also be mixed with each other[29].

3.2 Gluino-glue

For completing a (chiral) SUSY multiplet, besides a scalar and pseudoscalar particle, a Majorana fermion particle is also required. Such particles are provided by the *gluino-glueballs*.

An operator for the gluino-glue particle is in the continuum

$$\tilde{O}_{g\tilde{g}} = \sum_{\mu\nu} \sigma_{\mu\nu} \mathrm{tr}\left[F^{\mu\nu}\lambda\right], \tag{13}$$

where $\sigma_{\mu\nu} = \frac{1}{2}\left[\gamma_\mu, \gamma_\nu\right]$ and $F^{\mu\nu}$ is the field strength tensor. A lattice version of this, which can be used in numerical simulations, is

$$O^{\alpha}_{g\tilde{g}} = \sum_{i<j,\beta} \sigma^{\alpha\beta}_{ij}\,\mathrm{tr}\left[P_{ij}\lambda^{\beta}\right], \tag{14}$$

where the indices i and j stand for the spatial directions. A choice for F_{ij} with the proper parity and time reversal transformation properties is the antihermitian part of the clover plaquette $U^{(c)}$

$$P_{ij} = \frac{1}{8\mathrm{i}g_0}(U^{(c)}_{\mu\nu} - (U^{(c)}_{\mu\nu})^{\dagger}). \tag{15}$$

For its definition and more details see Ref. 13.

The gluino-glue correlator has been obtained using different smearing techniques. The link fields are smeared using APE smearing, the fermionic fields using Jacobi smearing. In order to decrease lattice artefacts and statistical fluctuations in the Wilson-Dirac fermion matrix Q of the lattice action (4)-(5), the gauge link variables $U_{x\mu}$ have been replaced by *stout smeared* links[30].

4 Conclusions and outlook

The results for light particle masses are summarised in Fig. 1. The masses are shown as a function of the squared mass of the adjoint pion, which for small values is proportional to the gluino mass. Also shown are the extrapolations to the limit of vanishing gluino mass. All figures include the gluino-glue mass for comparison.

The detailed investigation of finite volume effects showed[13] the lattice volumes are sufficiently large, such that the finite volume effects are smaller than the statistical errors. The

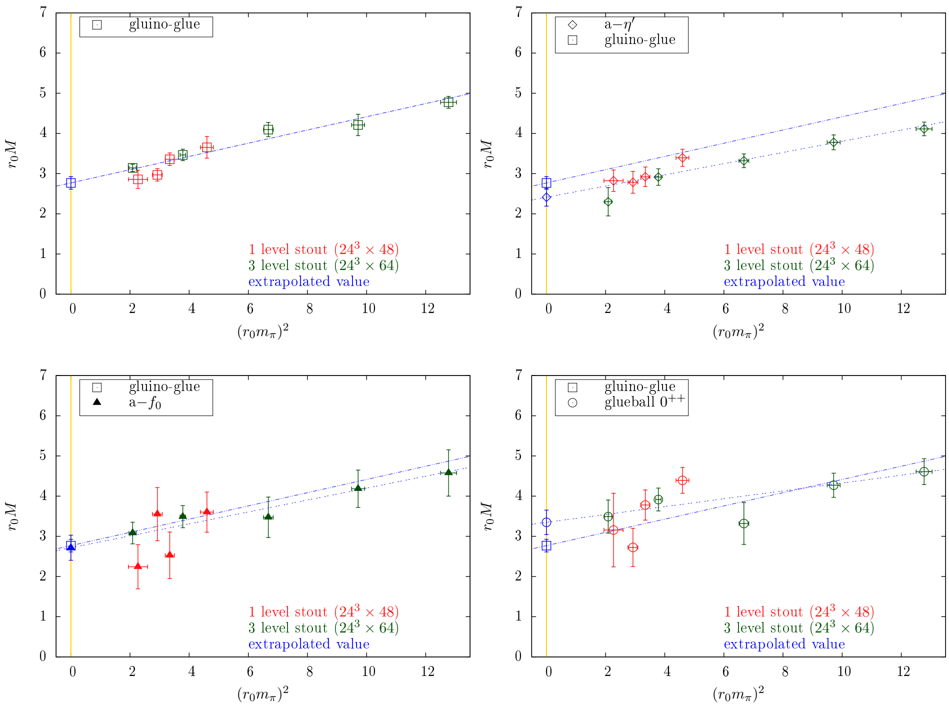


Figure 1. Light particle masses in SYM as a function of the squared mass of the adjoint pion. r_0 is the Sommer scale parameter.

simulations are performed at non-zero gluino masses where the supersymmetry is softly broken. The extrapolation to vanishing gluino mass, where supersymmetry is expected, is consistent with the emergence of a mass-degenerate chiral supermultiplet. Of course, besides the soft breaking by non-vanishing gluino masses, there are also additional SUSY breaking lattice artefacts. This should and could be diminished in future simulations closer to the continuum limit, that is at larger values of β.

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