Introduction to the Standard Model
Problem sheet 4

Deadline: Monday 11 May 2015 (12 am)
at Dr. Giudice's office (KP 301) and Dr. Piemonte's office (KP 412)

Topics covered: Generators, global and local symmetries

1. The quark triplet $q = \begin{pmatrix} u \\ d \\ s \end{pmatrix}$ transforms under flavour SU(3) as $q \to q' = Uq, \quad U \in \mathrm{SU}(3)$.

   Let $\bar{q}$ be another triplet transforming as $\bar{q} \to \bar{q}\,' = U^* \bar{q}$.

   a) (2 P) Define the generators $\overline{T}_a$ by $U^* = \exp(-\mathrm{i}\,\alpha_a \overline{T}_a)$, where $U = \exp(-\mathrm{i}\,\alpha_a T_a)$. Show that $\overline{T}_a = -T_a^*$ and check that the $\overline{T}_a$ fulfil the SU(3) Lie algebra.

   b) (1 P) Show that $\bar{q}$ represents antiquarks, i.e. that their quantum numbers are opposite to those of the quarks.

   c) (2 P) In the case of SU(2), the representation with generators T_a, $a = 1, 2, 3$, and the one with generators $\overline{T}_a = -T_a^*$ are equivalent, i.e. there exists a unitary matrix S, such that $\overline{T}_a = S T_a S^{-1}$. Find S.

2. The SU(3) d-coefficients are defined by $[T_a, T_b]_+ = \frac{1}{3}\delta_{ab}\mathbb{1} + d_{abc} T_c$.

   a) (2 P) Calculate d_{123}, d_{146}, d_{366} and d_{888}.

   b) (1 P) Show that $d_{abc} = 2\,\mathrm{Tr}\,([T_a, T_b]_+ T_c)$.

   c) (2 P) Show that the d_{abc} are completely symmetric.

3. Scalar QED:

   a) (1 P) For a free complex scalar field $\phi(x)$ there is a global U(1) symmetry given by

   $$\phi(x) \longrightarrow \phi' = \mathrm{e}^{-\mathrm{i}q\alpha}\phi(x).$$

   Calculate the corresponding Noether current.

   b) (2 P) Write down the Lagrangian $\mathscr{L}$ for $\phi(x)$ interacting with the Maxwell field $A_\mu(x)$. Derive the field equation for $\phi(x)$. Write the field equation by using the covariant derivative D_μ.

   c) (2 P) Calculate the Noether current corresponding to the global U(1) symmetry of $\mathscr{L}$, and calculate its divergence $\partial_\mu j^\mu$ using the field equations.

   d) (1 P) Check whether the interaction term in (b) is proportional to $j^\mu(x) A_\mu(x)$, where j^μ is the Noether current from (c).

   e) (2 P) Draw the interaction vertices belonging to $\mathscr{L}$. Use lines with arrows to distinguish ϕ from ϕ^*.