

Introduction to the Standard Model

Problem sheet 11

Deadline: Monday 6 July 2015 (12 am)
at Dr. Giudice's office (KP 301) and Dr. Piemonte's office (KP 412)

Topics covered: Glashow-Weinberg-Salam model

1. a) (1 P) Explain the meaning of the upper labels of the components of the Higgs doublet $\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$.
- b) (2 P) Show that an arbitrary value of the Higgs field ϕ of length $|\phi| = v/\sqrt{2}$ can be transformed to

$$\phi \rightarrow \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$$

by an SU(2) transformation.

2. a) (2 P) Derive the mass terms for the Higgs scalar $\rho(x)$ and the gauge fields $W_\mu^a(x)$ and $B_\mu(x)$ from the Higgs Lagrangian, assuming the Higgs mechanism.
- b) (2 P) The Z -boson field is defined by

$$Z_\mu = \cos \theta_W W_\mu^3 - \sin \theta_W B_\mu.$$

Why is the linear combination representing the photon field,

$$A_\mu = \sin \theta_W W_\mu^3 + \cos \theta_W B_\mu,$$

chosen to be orthogonal to it?

3. a) (2 P) Write down the covariant derivative acting on the electron-neutrino doublet $\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L$. Express the neutral gauge fields W_μ^3 and B_μ in terms of Z_μ and A_μ . From the prefactor of the term $A_\mu e_L^-$ read off the expression of the electric charge e_0 in terms of g and g' .
- b) (4 P) Write down the terms of the Lagrangian describing the interactions between electrons and electron-neutrinos with the gauge fields. Express the neutral gauge fields in terms of Z_μ and A_μ and show that the neutral current interactions are given by

$$\mathcal{L}_{\text{NC}} = -e_0 J_\mu^{(\text{em})} A^\mu - \frac{g}{\cos \theta_W} J_\mu^{(\text{n})} Z^\mu,$$

where $J_\mu^{(\text{em})}$ is the electric current, and

$$J_\mu^{(\text{n})} = J_\mu^3 - \sin^2 \theta_W J_\mu^{(\text{em})}$$

the weak neutral current.