Introduction to Quantum Field Theory
Problem sheet 4

Deadline: Wednesday 2 December 2015 (12 am)
at Dr. Giudice's office (KP 301) and Ms Sonja Esch (KP 310)

Topics covered: Canonical quantisation of a scalar field.

1. Consider the theory of a quantised complex scalar field.
   a) (2 P) The annihilation operator $a(k)$ is given by

$$a(k) = \int d^3x \, \mathrm{e}^{\mathrm{i}kx} \left(\omega_k \phi(x) + \mathrm{i}\,\dot{\phi}(x) \right) |_{k^0 = \omega_k}.$$

   Verify that $a(k)$ is time-independent if ϕ satisfies the Klein-Gordon equation.

   b) (1 P) Show that $[a(k), b(k')] = 0$.

   c) (3 P) Show the following equations:

$$\begin{aligned} \left[H, a^\dagger(k)\right] &= \omega_k \; a^\dagger(k) \\ \left[H, a(k)\right] &= -\omega_k \; a(k) \\ \left[H, b^\dagger(k)\right] &= \omega_k \; b^\dagger(k) \\ \left[P^j, a^\dagger(k)\right] &= k^j \; a^\dagger(k) \\ \left[P^j, b^\dagger(k)\right] &= k^j \; b^\dagger(k) \\ \left[Q, a^\dagger(k)\right] &= a^\dagger(k) \\ \left[Q, b^\dagger(k)\right] &= -b^\dagger(k) \end{aligned}$$

   d) (2 P) Calculate

$$\begin{aligned} &H a^\dagger(k_1) a^\dagger(k_2) |0\rangle \\ &H a^\dagger(k_1) b^\dagger(k_2) |0\rangle \\ &Q a^\dagger(k_1) a^\dagger(k_2) b^\dagger(k_3) |0\rangle \end{aligned}$$

   e) (1 P) Calculate the inner product between the two states $a^\dagger(k_1)a^\dagger(k_2)|0\rangle$ and $a^\dagger(k_1')a^\dagger(k_2')|0\rangle$.

2. (3 P) The momentum operator in the theory of a quantised real scalar field ϕ is given by

$$\vec{P} = -\int d^3x \, (\partial_0 \phi) \nabla \phi.$$

   Calculate the normal-ordered $:\vec{P}:$ in terms of creation and annihilation operators.