

QFT Exercises 8

Due on 08.01.15

Notice: all solutions should be given to Dr. P. Giudice.

Topics: symmetries in QM, Noether theorem

1. (25%) Calculate the energy momentum tensor of the complex scalar field. Use it to obtain the Hamiltonian and the momentum operators (in terms of fields and conjugate momenta).
2. (25%) Using the canonical commutation relations (and the results of point 1.) show that $[P^\mu, \varphi(x)] = -i\partial^\mu \varphi(x)$, with $\varphi(x)$ a complex scalar field .
3. (25%) Prove that $e^{iQ}\phi(x)e^{-iQ} = e^{-iq}\phi(x)$, where $\phi(x)$ is a complex scalar field of charge q , and Q is its corresponding charge operator.
4. (25%) Consider the Lagrangian

$$\mathcal{L} = (\partial_\mu \phi^\dagger)(\partial^\mu \phi) - m^2 \phi^\dagger \phi$$

where $\phi = \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}$ is an $SU(2)$ doublet (already studied in exercise sheet 7, problem 3). Prove that the Noether charges, Q^a , satisfy the commutation relations of $SU(2)$. Evaluate $[Q^a, \phi_i]$ for $i = 1, 2$.