QFT Exercises 7

Attention: For this week only the exercises are due one day earlier than usual, on Wednesday 10.12.14

Topics: symmetries, Noether theorem

1. (25%) Find the energy-momentum tensor of the massive vector field ($\mathscr{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m^2 A_\mu A^\mu$). Check whether it is symmetric.

2. (25%) Consider a field $\phi(x)$ with Lagrangian density $\mathscr{L} = \mathscr{L}_0 + \mathscr{L}_1$ such that $\mathscr{L}_0$ has a certain symmetry but $\mathscr{L}_1$ does not. That is, in the absence of $\mathscr{L}_1$ the system would have a conserved Noether current. Show that, in the presence of $\mathscr{L}_1$, the divergence of the would-be Noether current is $\delta\mathscr{L}_1$.

3. (25%) Consider the Lagrangian

$$\mathscr{L} = (\partial_\mu \phi^\dagger)(\partial^\mu \phi) - m^2 \phi^\dagger \phi \tag{1}$$

   where $\phi = \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}$ is an $SU(2)$ doublet. Show that this Lagrangian has $SU(2)$ symmetry and find the corresponding Noether currents and charges.

4. (25%) The Lagrangian of a real scalar field is given by

$$\mathscr{L} = \frac{1}{2}\partial_\mu \phi^T \partial^\mu \phi - \frac{m^2}{2}\phi^T \phi, \tag{2}$$

   where $\phi = \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{pmatrix}$. Prove that this Lagrangian is $SO(3)$ invariant and find the Noether charges.