QFT Exercises 3

Due on 13.11.14

Topics covered: Dirac equation, Lorentz and discrete transformations, field operators
We use natural units: $\hbar = c = 1$.

1. (20%) Determine under what conditions the operator γ_5 is a constant of motion for the free Dirac particle. Find the eigenvalues and projectors for this operator.

2. (20%) Let us define

$$\psi_L = \frac{1}{2}(1 - \gamma_5)\psi, \tag{1}$$

$$\psi_R = \frac{1}{2}(1 + \gamma_5)\psi, \tag{2}$$

where ψ is a Dirac spinor. Derive the equations of motion for these fields. Show that they are decoupled in the case of a massless spinor.

3. (25%) Consider the current

$$J_\mu = \bar{u}(\mathbf{p}_2) p^\rho q^\lambda \sigma_{\mu\rho} \gamma_\lambda u(\mathbf{p}_1) \tag{3}$$

where $p = p_1 + p_2$, $q = p_2 - p_1$ and $u(\mathbf{p}_1)$, $u(\mathbf{p}_2)$ are Dirac spinors. Show that J_μ can be written as

$$J_\mu = \bar{u}(\mathbf{p}_2) \left[F_1 \gamma_\mu + F_2 q_\mu + F_3 \sigma_{\mu\rho} q^\rho \right] u(\mathbf{p}_1) \tag{4}$$

and determine the functions $F_i = F_i(q^2, m)$.

4. (10%) Prove that the quantity $\bar{\psi}(x)\partial_\mu \gamma^\mu \psi(x)$ is a Lorentz scalar. How does it transform under parity?

5. (10%) Show that the helicity of the Dirac particle changes sign under space inversion.

6. (15%) Consider the Hamiltonian

$$H = \int d^3x \, \Psi^\dagger(\vec{x}) \left\{ -\frac{\hbar^2}{2m}\Delta + V(\vec{x}) \right\} \Psi(\vec{x}). \tag{5}$$

Show that in the Heisenberg picture the Heisenberg equation of motion for the operator $\Psi(\vec{x})$ has the same form as the Schrödinger equation for the wave function $\psi(\vec{x}, t)$.