SM Exercises 4

To be handed in on 13.05.14

1. (20%) The weak and electric charges are defined by

$$\begin{aligned} T_+(t) &= \frac{1}{2}\int d^3x\, J_0(x) = \frac{1}{2}\int d^3x\, \nu_e^\dagger(1-\gamma_5)e, \\ T_-(t) &= T_+^\dagger(t), \\ Q(t) &= \int d^3x\, J_0^{em}(x) = -\int d^3x\, e^\dagger e. \end{aligned}$$

   Using the canonical commutation relations for fermions

$$\left\{\psi_i^\dagger(\mathbf{x},t), \psi_j(\mathbf{x}',t)\right\} = \delta_{ij}\,\delta^3(\mathbf{x}-\mathbf{x}')$$

   show that

$$[T_+(t), T_-(t)] = 2\,T_3(t)$$

   with

$$T_3(t) = \frac{1}{4}\int d^3x\left[\nu_e^\dagger(1-\gamma_5)\nu_e - e^\dagger(1-\gamma_5)e\right].$$

   Since $T_3 \neq Q$, $T_\pm, Q$ do not form a closed algebra.

2. (30%) Consider a theory with three complex scalar fields ϕ_1, ϕ_2, ϕ_3 with charges $2, 4, -1$ respectively under a $U(1)$ global symmetry. Write down the most general Lagrangian (with no terms including more than four fields) that is compatible with such a symmetry. Assume that, via the Higgs mechanism, the fields acquire vacuum expectations values given by $\langle\phi_1\rangle = v_1$, $\langle\phi_2\rangle = v_2$, $\langle\phi_3\rangle = 0$. Show that, in agreement with the Goldstone theorem, the spectrum contains a massless particle.

3. (35%) Consider an $SU(2)$ gauge theory. Choose the Higgs field to be an $SU(2)$ triplet of real scalar fields. Write the Lagrangian for the Higgs sector. Assuming $\mu^2 < 0$ and $\lambda > 0$ (in the usual notation), show that the Lagrangian describes a massive scalar (find its mass) and two massless Goldstone bosons. Study the terms that give mass to the gauge bosons and show that only two of them become massive.

4. (15%) How many gauge bosons become massive when an $SU(m)$ gauge symmetry is spontaneously broken down to $SU(n)$ (with $n < m$)? Explain. Consider a scalar field transforming under the fundamental representation of $SU(4)$. Could that field spontaneously break $SU(4)$ down to $SU(2)$?