SM Exercises 3

To be handed in on 06.05.14

1. (15%) Consider the fermion bilinears

$$V^\mu = \bar{\psi}\gamma^\mu\psi, \tag{1}$$

$$A^\mu = \bar{\psi}\gamma^\mu\gamma^5\psi. \tag{2}$$

Find how these quantities transform under parity:

$$\psi_P(t,\vec{x}) = \hat{P}\psi(t,\vec{x})\hat{P}^{-1} = \gamma^0\psi(t,-\vec{x}). \tag{3}$$

Show that $V^\mu A_\mu$ is a pseudoscalar quantity.

2. (25%) Show that in the frame where the momentum k^μ of a massive vector boson is $k^\mu = (k^0, 0, 0, |\vec{k}|)$, the longitudinal polarization vector is given by

$$\epsilon^\mu(k,\lambda=0) = \frac{k^\mu}{M} + \frac{M}{k^0+|\vec{k}|}\left(-1, \vec{k}/|\vec{k}|\right) \tag{4}$$

where M is the mass of the vector boson. Find the transverse polarization vectors ($\lambda = \pm 1$) and notice that they do not depend on k^μ. Using the above equation, verify that

$$\sum_\lambda \epsilon^\mu(k,\lambda)\epsilon^{\nu*}(k,\lambda) = -g^{\mu\nu} + \frac{k^\mu k^\nu}{M^2}. \tag{5}$$

Thus, the large momentum behavior of the polarization sum is driven by the longitudinal modes.

3. (25%) In the Fermi theory, the decay of the muon ($\mu^- \to e^- + \bar{\nu}_e + \nu_\mu$) is determined by the following matrix element

$$M = \frac{G_F}{\sqrt{2}}\left[\bar{u}_{\nu_\mu}(k')\gamma^\mu(1-\gamma_5)u_\mu(p')\right]\left[\bar{u}_e(p)\gamma^\mu(1-\gamma_5)v_{\nu_e}(k)\right]. \tag{6}$$

Evaluate $|M|^2$ neglecting the masses of e, ν_e, ν_μ.

4. (35%) The decay rate of the muon is given by

$$\Gamma_\mu = \frac{1}{2m_\mu}\int \frac{d^3k}{(2\pi)^3\, 2E_k}\frac{d^3k'}{(2\pi)^3\, 2E_{k'}}\frac{d^3p}{(2\pi)^3\, 2E_p}|M|^2(2\pi)^4\delta^{(4)}(p'-p-k-k'), \tag{7}$$

where $|M|^2$ was obtained in the previous exercise. Compute the integrals in the rest frame of the muon and obtain Γ_μ.