SM Exercises 2

To be handed in on 29.04.14

1. (10%) Let $U(\theta)$ be a unitary transformation within the group $SU(2)$, $U(\theta) = e^{-\frac{i}{2}\theta\cdot\sigma}$. Show that $\det U = 1$.

2. (20%) Within the $SU(2)$ group the transformation for particles are $\chi' = U\chi$ whereas for anti-particles $\chi^{*'} = U^*\chi^*$. Show that if χ belongs to the fundamental representation of $SU(2)$

$$U^* = (i\sigma_2) \cdot U \cdot (-i\sigma_2). \tag{1}$$

Show that the doublet $-i\sigma_2\chi^*$ transforms in the same way as χ. In particular, if $\psi = \begin{pmatrix} u \\ d \end{pmatrix}$ transforms as the doublet representation of $SU(2)$, $\psi = \begin{pmatrix} -d \\ u \end{pmatrix}$ also does.

3. (35%) Show that under a gauge transformation the covariant derivative

$$D^\mu q^j(x) = \left[\delta_{jk}\partial^\mu - ig\sum_a T^a_{jk}A^\mu_a(x)\right] q^k(x)$$

has the same transformation law as the field on which it acts.

4. (35%) Show that under a gauge transformation the QCD field strenght tensor, $F^{\mu\nu}_a = \partial^\mu A^\nu_a - \partial^\nu A^\mu_a + gf_{abc}A^\mu_b A^\nu_c$, transforms in a homogeneous way:

$$F^{\mu\nu}_a \to F^{\mu\nu}_a + gf_{abc}\theta_b F^{\mu\nu}_c.$$