Dark energy*

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1 Introduction

After Einstein published his theory of General Relativity in 1916 he noticed that solutions of the original equations led to a non-static universe. To avoid that and driven by theoretical considerations (for a historical review see, e.g., Ref. [1]) he introduced the cosmological constant Λ. In 1929 Hubble found that the universe is indeed expanding, which led Einstein to again drop out the cosmological constant, which he thought should be reject "from the point of view of logical economy", as he stated in the second edition of his book *The meaning of Relativity* (1945). Already in the 1970's the cosmological constant was recovered to explain inflation.[1] A couple of decades later, in 1998, Perlmutter *et al.* [2] and Riess *et al.* [3, 4] found that the expansion of the universe was accelerating. As we shall see, this is not obtained in a universe filled with only matter and radiation, so a novel form of energy is needed, which has been called *dark energy*. Newer experiments like the WMAP [5] have independently confirmed these results. Remarkably this new form of energy should have the form of Einstein's cosmological constant (or a very similar one). Later, new theories for this dark energy has been developed trying to overcome some of the problems of the cosmological constant. To date, experiments have not been able to distinguish between them.

This summary is organized as follows: In Sec. 2 we present a brief review of the basics of the necessary cosmology to study a expanding universe and we introduce the cosmological constant. In Sec. 3 we present some of the current evidence for dark energy. In Sec. 4 we revisit the cosmological constant to try to find its physical interpretation and present some theoretical attempts to determine its value compared to the experimental value obtained. In Sec. 5 we introduce a couple of new dynamical theories as alternative to the cosmological constant. Finally the conclusions and open questions are presented in Sec. 6.

2 Cosmology of an expanding universe

In this section we will study how measurements of the expansion of the universe can be related to its energy content, and in particular how current observational results lead us to believe in the existence of a new type of energy different from matter and radiation.

[1]The reader is referred to the talk on inflation given at this very seminar.

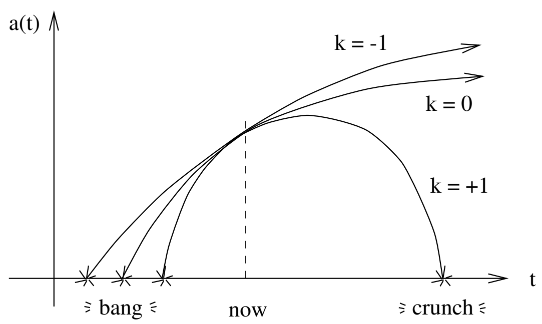


Figure 1: Possible geometries of the universe depending of the three possible values of k: open universe ($k = -1$), flat universe ($k = 0$) and closed universe ($k = 1$). From Ref. [7].

2.1 Friedmann-Lemaître cosmology

2.1.1 Homogeneity and isotropy

The geometry of the universe and its energy content can be related by the Einstein equation of general relativity [6],

$$G_{\mu\nu} := R_{\mu\nu} - \frac{1}{2}\delta_{\mu\nu}R = 8\pi G T_{\mu\nu}, \tag{1}$$

where $G_{\mu\nu}$ is the Einstein tensor, $R_{\mu\nu}$ is the Ricci tensor, R the Ricci scalar and $T_{\mu\nu}$ the energy-momentum tensor.

The Einstein equation (1) is in general difficult to solve, but particular solutions can be obtained in the presence of symmetries, namely *homogeneity* and *isotropy.*

For an isotropic and homogeneous universe we can use the Friedmann-Robertson-Walker (FRW) metric [6, 1],

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2(t)\left(\frac{\mathrm{d}r^2}{a - kr^2} + r^2(\mathrm{d}\theta^2 + \sin\theta\mathrm{d}\phi^2)\right), \tag{2}$$

where r, θ, ϕ are the comoving coordinates, $a(t)$ is the scale factor and k describes the geometry of the space, as shown in Fig. 1. There seems to be enough evidence for a (at least close to) spatially flat universe [5] and we will assume $k = 0$ from now on, except otherwise noted.

2.1.2 Dynamics of a universe filled with a perfect fluid

Now we will consider the universe to be filled with a *perfect fluid* with pressure p, energy density ρ and energy-momentum tensor

$$T^{\mu}_{\nu} = \mathrm{diag}(\rho, p, p, p). \tag{3}$$

Inserting Eqs. (2) and (3) into Eq. (1) and simplifying we obtain

$$H^2 := \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G\rho}{3} - \frac{k}{a^2}, \tag{4a}$$

$$\frac{\ddot{a}}{a} = -\frac{4}{3}\pi G(\rho + 3p). \tag{4b}$$

where H is the Hubble parameter.

It is known that perfect fluids can be well approximated by the equation of state [6]

$$\omega = p/\rho, \tag{5}$$

where ω is constant. Introducing this into the so-called Friedmann equations (4), we obtain the solution

$$a(t) \propto (t - t_0)^{\frac{2}{3(1+\omega)}}, \tag{6a}$$

$$\rho \propto a^{-3(1+\omega)}, \tag{6b}$$

for any $\omega \neq -1$. The case $\omega = -1$ is of special interest and will be studied later.

In particular, it can be shown [8] that $\omega = 1/3$ for radiation and $\omega = 0$ for matter (dust), from where we have the solutions:

$$\begin{aligned} \text{Radiation:} \quad & a(t) \propto (t - t_0)^{1/2}, \quad \rho \propto a^{-4}, \\ \text{Matter:} \quad & a(t) \propto (t - t_0)^{2/3}, \quad \rho \propto a^{-3}. \end{aligned} \tag{7}$$

We can see that in both cases (and in general any combination of them) this leads to a decelerated expansion of the universe $\ddot{a} < 0$. In fact, we can see that for an accelerated expansion of the universe, from Eq. (4b) we obtain the condition

$$\ddot{a}/a = -\frac{4\pi G}{3}(\rho + 3p) = -\frac{4\pi G}{3}(1 + 3\omega)\rho > 0, \tag{8}$$

from where

$$\omega < -1/3. \tag{9}$$

2.1.3 Evidence for an accelerated expansion

Many cosmologists were convinced that the expansion of the universe should be in fact decelerating as it is clear from the fact that a new *deceleration parameter* $q := -\frac{\ddot{a}a}{a^2}$ was defined [9]. It came as a surprise the discovery that in fact the expansion of the universe is accelerating [3, 4, 2] as shown in Fig. 2. In Sec. 3 we will present some of the main sources of observational evidence.

At this point it became clear that some of the hypothesis used to obtain the deceleration of the expansion $\ddot{a} < 0$ should be revisited, i.e., one of the following should be false:

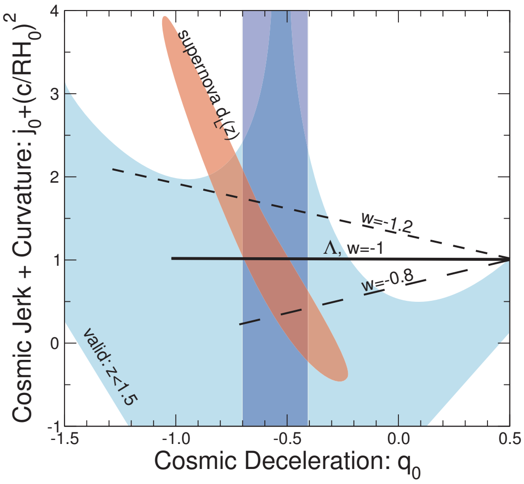


Figure 2: Measurements of the *deceleration parameter* showed that in fact the expansion of the universe is accelerating. Here we show the constraints on the deceleration parameter from supernovae Type Ia measurements. From Ref. [9].

1. General relativity is a valid theory unmodified.
2. The universe is isotropic and homogeneous.
3. The universe is only composed of matter and radiation.

All of these options have been extensively studied and discussed in the literature. Revision of 1, i.e., that general relativity must me modified is studied, e.g., in Refs. [7, 10, 11] while hypothesis 2, i.e., that the universe is not isotropic and homogeneous enough is studied, e.g., in Refs. [12, 13]. In these notes we will focus on 3, i.e., that there exists a new form of energy in the universe: *dark energy*.

2.2 The cosmological constant

After the publication of general relativity in 1915-1916, in 1917 Einstein applied his theory to the whole universe and noticed that there is not a static solution of the original equations as he expected [14], and so introduced a modification to the original equations in the form a new parameter, the *cosmological constant* Λ.

While Einstein introduced the new term to the LHS of the original equation (1), i.e., as a modification of the geometry of the universe, today, it has become a common practice to include the new term to the RHS, i.e., as a modification of its energy content,

$$G_{\mu\nu} = 8\pi G + \Lambda g_{\mu\nu} \tag{10}$$

Now, introducing Eqs. (2) and (3) into Eq. (10), we obtain

$$H^2 := \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G\rho}{3} - \frac{k}{a^2} + \frac{\Lambda}{3}, \tag{11a}$$

$$\frac{\ddot{a}}{a} = -\frac{4}{3}\pi G(\rho + 3p) + \frac{\Lambda}{3}. \tag{11b}$$

as opposed to (4). Notice in particular the presence of the last term in (11b) and how the effect of $\Lambda > 0$ is to increase $\ddot{a}$, making $\ddot{a} > 0$ if enough to compensate the first terms. Furthermore, it is possible to identify the new term with a new perfect fluid. To that end, let us rewrite Eq. (10) as

$$G_{\mu\nu} = 8\pi G T_{\mu\nu} + \Lambda = 8\pi G(T_{\mu\nu} + T^{\Lambda}_{\mu\nu}), \tag{12}$$

where we have defined

$$T^{\Lambda}_{\mu\nu} := \frac{\Lambda}{8\pi G} g_{\mu\nu} \equiv \rho_\Lambda g_{\mu\nu}. \tag{13}$$

Now we can identify Eqs. (13) and (3), obtaining,

$$T^{\text{p.f.}}_{\mu\nu} = \operatorname{diag}(\rho, p, p, p) = \rho_\Lambda \operatorname{diag}(1, -1, -1, -1) \tag{14}$$

$$\implies \rho = \rho_\Lambda = -p, \tag{15}$$

i.e., the new term with the cosmological constant behaves as a perfect fluid with

$$\rho_\Lambda = \frac{\Lambda}{8\pi G}, \quad \omega := \frac{p}{\rho} = -1 < -\frac{1}{3}. \tag{16}$$

If we compare Eqs. (16) and (9), we see that the new perfect fluid leads to an accelerated expansion of the universe (if its relative energy density is enough to compensate the effect of matter and radiation.)

We will come back to the cosmological constant and its physical interpretation in Sec. 4.

3 Observational evidence

3.1 Supernovae SN Ia

In 1998 Riess *et al.* [3, 4] and Perlmutter *et al.* [2] noticed that the expansion of the universe was accelerating by using data from Type Ia Supernovae measurements.

The evolution of the expansion of universe is often described using the concept of *redshift*, whose origin is the increase in λ of a photon travelling in the universe as it expands, and is defined as

$$1 + z := \frac{\lambda_0}{\lambda} = \frac{a_0}{a}, \tag{17}$$

where λ_0, a_0 are the quantities given at the present epoch.

The expansion of the universe also influences the concept of distance. In fact, in an expanding universe several definitions of distance are possible, e.g., the physical (or proper) distance, which scales proportional to $a(t)$; or the comoving distance, which is independent of $a(t)$. In particular the *luminosity distance* turns out to be very useful from a phenomenological point of view. The motication is the following: in a Minkowski space, the energy flux $\mathcal{F}$ at a distance d of an object with absolute luminosity L_s is given by $\mathcal{F} = \frac{L_s}{4\pi d^2}$, what leads to the following definition of luminosity distance [6]

$$d_{\mathrm{L}} := \frac{L_s}{4\pi\mathcal{F}}. \tag{18}$$

At this point we will relate information about the expansion of the universe with measurable quantities, namely the redshift and the luminosity distance. In particular, we will show how how to obtain $H(t)$ or equivalently[2] $H(z)$ from measurements of d_{L} and z. Since we can also obtain $H(z)$ from Eq. (1) this will lead us to a test of the theory and a constraint of the parameters like the energy content of the universe.

To this end, we first need astronomical objects whose standard luminosity is know, the so-called *standard candles*. Several types of standard candles have been proposed (see Ref. [6] and references therein), like FRIIb radio galaxies or Gamma ray Bursts. But the first and arguably the most important standard candle to date (and the one who led to the 2011 Nobel Prize for Physics) is the Supernovae Type Ia.[3]

We consider an object at $\chi = \chi_s$ that emits light with absolute luminosity $L_s = \frac{\Delta E_1}{\Delta t_1}$ and an observer at $\chi = \chi_0$ who sees an apparent luminosity $L_0 = \frac{\Delta E_0}{\Delta t_0}$. Hence,

$$1 + z \equiv \frac{\lambda_0}{\lambda_1} = \frac{\nu_1}{\nu_0} = \frac{\Delta t_0}{\Delta t_1} = \frac{\Delta E_1}{\Delta t_0}, \tag{19}$$

where we have used the relation for the velocity of wave $c = \nu\lambda$, the energy-frequency relation $E \propto \nu$ and the conservation of photons $\nu_0 \Delta t_0 = \nu_1 \Delta t_1$. From here,

$$L_s = L_0(1+z)^2. \tag{20}$$

We know that light travelling along the χ direction satisfies the geodesic equation

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2(t)\mathrm{d}\chi^2, \tag{21}$$

[2]In this case the knowledge of $H(t)$ and $H(z)$ are equivalent due the relation (17).

[3]For Supernovae Type Ia the luminosity of distant supernovae is extrapolated from measurements of nearby ones. It has been pointed out that this may lead to inaccuracies since more distant supernovae were formed in a different epoch of the evolution of the universe and hence when the composition of the universe was quite different. For a more comprehensive discussion see Ref. [15].

from where

$$\chi_s = \int_0^{\chi_s} \mathrm{d}\chi = \int_{t_1}^{t_0} \frac{\mathrm{d}t}{a(t)} = \frac{1}{a_0}\int_0^z \frac{\mathrm{d}z'}{H(z')}, \tag{22}$$

and using Eq. (2) for a flat universe with $k = 0$ the observed flux can be written as

$$\mathcal{F} = \frac{L_0}{4\pi(a_0\chi_s)^2} = \frac{L_0}{4\pi}\left(\int_0^z \frac{\mathrm{d}z'}{H(z')}\right)^{-2}. \tag{23}$$

Finally, substituting Eqs. (20) and (22) into Eq.(20) we obtain

$$d_\mathrm{L} = (1+z)\int_0^z \frac{\mathrm{d}z'}{H(z')}, \tag{24}$$

or solving for $H(z)$,

$$H(z) = \left\{\frac{\mathrm{d}}{\mathrm{d}z}\left(\frac{d_\mathrm{L}(z)}{1+z}\right)\right\}^{-1}. \tag{25}$$

Eq. (25) shows that it is possible to obtain $H(z)$ from observational measurements of the luminosity distance d_L and the redshift z of astronomical objects. We will now show that is also possible to obtain it from theoretical grounds (i.e., the Einstein equation).

From Eq. (4a) with $\rho = \sum_i \rho_i(t)$ and using Eq. (6b) we obtain

$$H^2 = \frac{8\pi G}{3}\rho = \frac{8\pi G}{3}\sum_i \rho_i^{(0)}(a/a_0)^{-3(1+\omega_i)} = \frac{8\pi G}{3}\sum_i \rho_i^{(0)}(1+z)^{3(q+w_i)}, \tag{26}$$

or using $\Omega_i := \frac{\rho_i}{(3H_0^2/8\pi G)} \equiv \frac{\rho}{\rho_\mathrm{crit}}$,

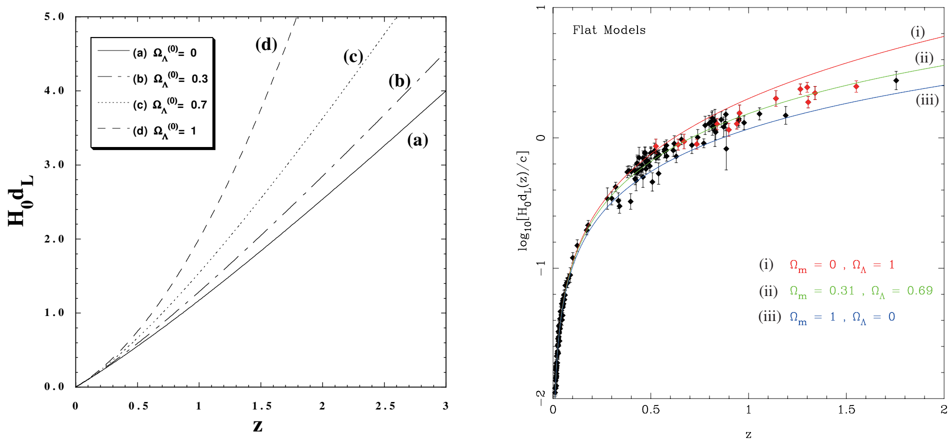
$$H = H_0\left\{\sum_i \Omega_i^{(0)}(1+z)^{3(1+w_i)}\right\}^{1/2}. \tag{27}$$

Eq. (27) could be directly compared with the observations using Eq. (25), but this is not mathematically convenient since it involves derivatives of observational data, and therefore we solve for d_L using Eq. (24) so we can better compare with raw data,

$$d_\mathrm{L} = \frac{1+z}{H_0}\int_0^z \frac{\mathrm{d}z'}{\sqrt{\sum_i \Omega_i^{(0)}(1+z)^{3(1+\omega_i)}}}, \tag{28}$$

which, as we see, depends on the composition of the universe.

The theoretical result of Eq. (28) is plotted for different parameters (i.e., for different universe energy content) in Fig. 3(a) and the comparison with the observational results and the best fit, shown in Fig. 3(b), gives a first estimate of $\Omega_\mathrm{m} \sim 0.3$ and $\Omega_\Lambda \sim 0.7$. Newer data sets and later analysis have improved the precision of this estimates and we will show the combined results with other evidences in Sec. 3.4.



(a) (b)

Figure 3: Plot of the theoretical luminosity distance as a function of the redshift with different parameters (from Ref. [6]) and plot of the observational data with the best fit (from Ref. [16].)

3.2 Cosmic Microwave Background

The Cosmic Microwave Background (CMB) has turned out to be one of the most powerful observational tools in cosmology, giving some of the most precise measurements and arguably some of the most important results of the field, especially with the WMAP experiment, including the evidence of the (at least approximate) flatness of the universe [5].

The experiments measure the temperature as a function of the direction in the sky $T(\hat{\mathbf{n}})$, and this is expanded in terms of the spherical harmonics (see e.g., Ref. [17])

$$a_{lm} = \int \mathrm{d}\hat{\mathbf{n}}\; T(\hat{\mathbf{n}}) Y_{lm}(\hat{\mathbf{n}}), \tag{29}$$

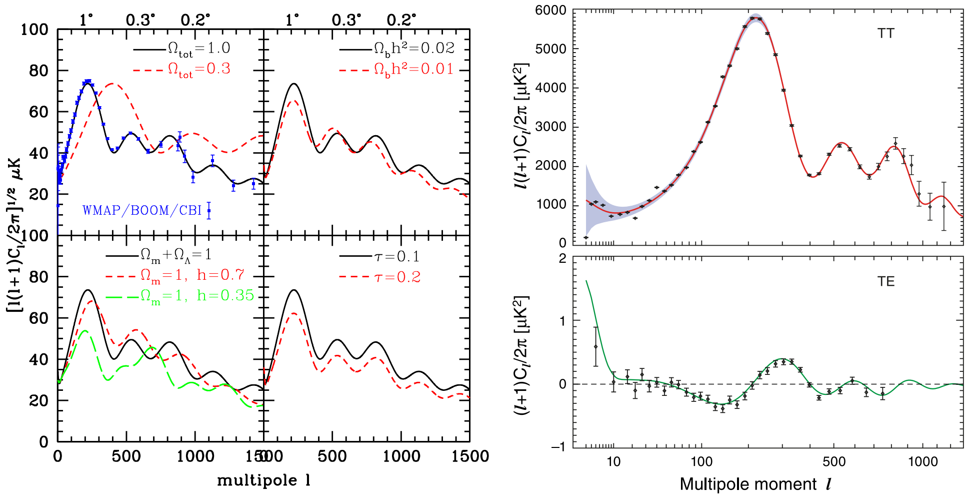
and the power spectrum is constructed

$$C_l = \langle |a_{lm}|^2 \rangle. \tag{30}$$

This power spectrum can in turn be obtained through theoretical considerations given a model of the universe, and so it can be compared, giving a constraint on the parameters in a similar way to the previous section. This can be seen, e.g., in Fig. 4(a). The best-fit for WMAP 7-year data is shown in Fig. 4(b) and the best-fit parameters are given in Table 1.

3.3 The age of the universe

The age of stellar objects can be determined using observational methods. A number of groups have constrained the age of certain cosmological objects



(a) (b)

Figure 4: (a) Best fit of the power spectrum as a function of the multipole (solid line) and theoretical prediction with the variation of one of the parameters (dashed line), namely Ω_{tot}, $\Omega_b h^2$, $\Omega_{\text{m}} + \Omega_\Lambda$ and τ. Some experimental points are given in the graph in the top left. Notice how the variation of a parameter causes the deviation from the best fit. From Ref. [17]. (b) Observational points and best fit from WMAP7 data. From Ref. [5].

Parameter	Estimated value
Age of the universe	$t_0 = (13.75 \pm 0.13)$ Gyr
Hubble's constant	$H_0 = (71.0 \pm 2.5)\ \text{km} \cdot \text{Mpc}^{-1} \cdot \text{s}^{-1}$
Dark energy density	$\Omega_\Lambda = 0.734 \pm 0.029$
Dark matter density	$\Omega_c = 0.222 \pm 0.026$
Baryon density	$\Omega_b = 0.0449 \pm 0.0028$

Table 1: Some of the best-fit parameter estimates from WMAP 7-year data. Notice the non-zero dark energy density.

using a variety of techniques (see Ref. [6] and references therein). For example, the age of the Globular clusters in the Milky has been constrained to be $t_1 = (13.5 \pm 2)$ Gyr, and the age of the Globular cluster M4 to $t_2 = (12.7 \pm 0.7)$ Gyr. WMAP observations estimate the age of the universe in $t_0 = 13.75 \pm 0.13$ Gyr. Thus, for consistence, we expect the age of universe to be bound by $t_0 > 12$ Gyr. We will see that this bound is not met by a universe without a cosmological constant, and that its introduction remarkably gives the right value for t_0.

The age of the universe can be computed using the definitions of the Hubble parameter H and the redshift z as

$$t_0 = \int_0^{t_0} \mathrm{d}t = \int_0^{\infty} \frac{\mathrm{d}z}{(1+z)H(z)}. \tag{31}$$

Considering that the radiation period ($z \gtrsim 1000$) is negligible, we can approximate $\Omega_r \sim 0$, and in a universe without cosmological constant, $\Lambda = 0$ from where $\Omega_m^{(0)} = 1$, and hence

$$t_0 = \frac{1}{H_0} \int_0^{\infty} \frac{\mathrm{d}z}{(1+z)^2\sqrt{1+\Omega_m^{(0)} z}} = \frac{2}{3H_0} \sim 9\,\mathrm{Gyr}, \tag{32}$$

but if we include the cosmological constant,

$$t_0 = \frac{1}{H_0} \int_0^{\infty} \frac{\mathrm{d}z}{(1+z)\sqrt{\Omega_m^{(0)}(1+z)^3 + \Omega_\Lambda^{(0)}}} = \frac{2}{3H_0\sqrt{\Omega_\Lambda^{(0)}}} \ln\left(\frac{1+\sqrt{\Omega_\Lambda^{(0)}}}{\sqrt{\Omega_m^{(0)}}}\right), \tag{33}$$

and for the measured values $\Omega_m^{(0)} = 0.3$ and $\Omega_\Lambda^{(0)} = 0.7$,

$$t_0 \sim 13.1\,\mathrm{Gyr}, \tag{34}$$

and thus the problem of the age of the universe is solved by the introduction of the appropriate amount of dark energy in the universe.

3.4 Combined results

The result of the combination of these and other observational constraints is plotted in Fig. 5, from where we can see that measurements are compatible with a cosmological constant with $\omega = -1$ and predict a matter density $\rho_m \lesssim 0.3\rho_{\mathrm{crit}}$ and therefore $\rho_\Lambda \gtrsim 0.7\rho_{\mathrm{crit}}$. For a flat universe this means a composition of $\sim 30\%$ of matter and $\sim 70\%$ of dark energy.

4 The cosmological constant revisited

4.1 The cosmological constant as vacuum energy density

We have previously seen that the cosmological constant is mathematically equivalent to a perfect fluid with $\rho_\Lambda = \frac{\Lambda}{8\pi G}$ and $\dot{\rho}_\Lambda = 0$, i.e., a fluid with a

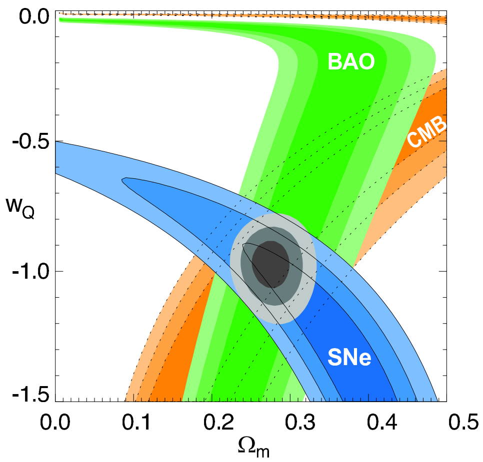


Figure 5: Plot of the 68.3%, 95.4% and 99.7% confidence-level curves for ω and Ω_m using constraints from Supernoae Type Ia (SNe), Cosmic Microwave Background from WMAP (CMB) and Baryonic Acoustic Oscillations (BAO). From Ref. [18].

constant energy density per unit volume. It can also be shown that if every inertial observer is to see the same vacuum then the energy-momentum tensor associated with the vacuum should have the form $T_{\mu\nu} \propto g_{\mu\nu}$, which equivalent to the cosmological constant as seen in Eq. (13). This lead us to interpret the effect of the cosmological constant as that of the energy density of the vacuum, i.e., the energy the space itself has.

The idea of a non-zero energy of the vacuum is not strange to physics. In quantum mechanics it can be associated with the ground state energy of a system, and it has been shown to have a measurable effect, famously in the form of a Casimir force [19].[4]

We could try to naïvely compute the vacuum energy density from the zero-point contribution of a field with mass m in the following way (see, e.g., Ref. [6]):

$$\begin{aligned}\rho_{\mathrm{vac}} &= \frac{1}{2}\int_0^\infty \frac{\mathrm{d}^3\vec{k}}{(2\pi)^3}\sqrt{k^2+m^2} \\ &= \frac{1}{4\pi^2}\int_0^\infty \mathrm{d}k k^2\sqrt{k^2+m^2} \\ &\sim \frac{k_{\max}^4}{16\pi^2}. \end{aligned} \tag{35}$$

This integral is not surprisingly divergent. In particle physics this integral

[4]Note that the difference between the case in quantum mechanics and the cosmological constant is that in the former case it is an effect of a difference between the energy densities of two vacuums while in the latter it is an effect of the *absolute* vacuum energy density.

needs to be made finite by *renormalization*. The most elementary "renormalization" procedure is arguably that of a cutoff. If we set a usual cutoff at the Planck scale, we obtain $\rho_\Lambda^{\rm th} \sim 10^{74}\,{\rm GeV}^{-4}$ while the experimental value is known to be $\rho_\Lambda^{\rm th} \sim 10^{-47}\,{\rm GeV}^{-4}$, i.e., a difference of 120 orders of magnitude. Even if we set the cutoff at the QCD the scale the result would be $\rho_\Lambda^{\rm th} \sim 10^{-3}\,{\rm GeV}^{-4}$, which is still unacceptable.

4.2 Supersymmetry and the cosmological constant

Supersymmetry emerges as an extension of the Standard Model introducing a new symmetry Q between fermions and bosons,

$$Q|\text{boson}\rangle \propto Q|\text{fermion}\rangle, \quad Q|\text{fermion}\rangle \propto Q|\text{boson}\rangle. \tag{36}$$

Among the theoretical benefits of Supersymmetry (see, e.g., Ref. [20]), it (1) solves the hierarchy problem of the Higgs boson of the Standard Model, (2) leads to gauge coupling unification, (3) fits better some measurable quantities, e.g. the Weinberg angle,

$$\sin^2\theta_W = \begin{cases} 0.2100 \pm 0.0026 & \text{SM}, \\ 0.2335 \pm 0.0017 & \text{MSSM}, \\ 0.2316 \pm 0.0002 & \text{experimental}, \end{cases}$$

and (4) includes dark matter candidates.[5]

Supersymmetry has been proposed to solve the great discrepancy between the theoretical and experimental value of the dark energy density. Fermions and bosons have opposite contributions to ρ_λ (see, e.g., Ref. [14]), so the fact that in supersymmetry the number of both of them is the same can lead to a sufficient compensation. In fact, exact supersymmetry leads to $\rho_\Lambda = 0$ [21, 14].

This result is only valid for *exact* supersymmetry, where we know that supersymmetry must indeed be broken (otherwise we should have already seen the superpartners). But this breaking mechanism is not known yet to date, and moreover no physically realistic theory has been able yet to explain the value of ρ_Λ from supersymmetry breaking.

But in any case the argument for a supersymmetric explanation of the cosmological constant value suffers a yet unsolved problem: since supersymmetry was exact at early times of the universe (i.e., at large energies), right after the Big Bang $\rho_\Lambda = 0$ and increases with the supersymmetry breaking at lower energies $T_{\rm SUSY}$, which the opposite as expected, since a large ρ_Λ is necessary in the early stages of the expansion of the universe to originate inflation, and a low value is compatible with today's measurements [22].

[5] The reader may refer to the talk on dark matter given at this very seminar.

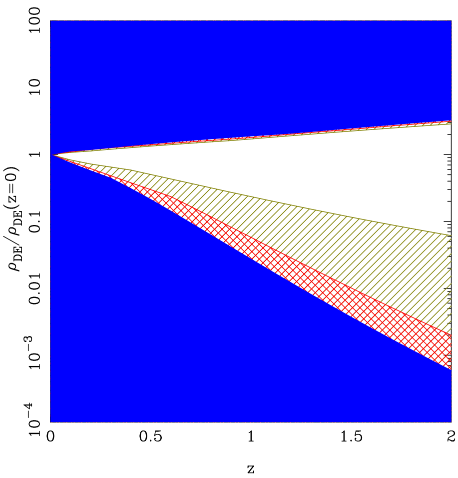


Figure 6: The evolution of ρ_Λ as constrained by WMAP and supernovae data plotted with respect to the redshift z. Note that the data is compatible with dark energy of both the form of a cosmological constant and from a dynamical model with varying $\rho_\Lambda(z)$. From Ref. [6].

4.3 Fine-tuning and the coincidence problem

The idea of the cosmological constant as seen in the general theory of relativity immediately rises several problematic questions. Here we will only briefly sketch the most basic ones:

Firstly, the density for matter evolves as the universe expands as $\rho_m \propto a^{-3}$ while for dark energy ρ_Λ is constant, so after $\sim$ 13.7 billion years, ρ_m has varied hundreds of orders of magnitude. If that is the case, why precisely *today* is $\rho_m \sim \rho_\Lambda$? [8, 1]. This is sometimes called the "coincidence problem". Secondly, the exact value of the cosmological constant seems to be extremely fine-tuned to allow the formation of structures [23] and the universe as we know it today [6]. If this value comes from particle physics, it seems that a extreme "fine-tuning" is needed.

There has been a long scientific discussion on these and related problems and different possible solutions has been proposed (including especially antrophic arguments). For a historical review of these problems and a deeper discussion the reader is referred to Refs. [1, 6] and references therein.

5 Dynamical models

The problems seen in Sec. 4.3 have lead to the development of dynamical models, which set the cosmological constant strictly to $\rho_\lambda = 0$ by some yet unknown mechanism and include new elements to explain the presence and quantity of dark energy in the universe. Moreover a dynamical model seems

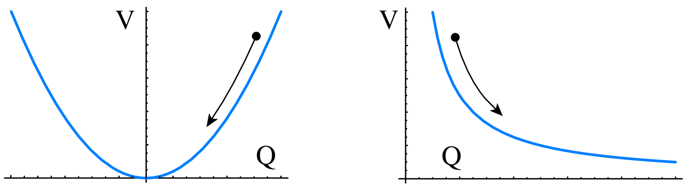


Figure 7: Two examples of possible quintessence potentials. In the first one the field is relaxing towards the local minimum and in the second, representative of tracker potencials, the field is evolving towards the global minimum. From Ref. [9].

compatible with all current data, as can be seen in Fig. 6. We will now review the basics of two of the most important dynamical models and cite some others.

5.1 Quintessence

The quintessence dark energy (see, e.g., Ref. [22]) is inspired in particle physics and proposes the existence of a new scalar field

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + V(\phi), \tag{37}$$

with energy-momentum tensor

$$T_{\mu\nu} = \partial_\mu\phi\partial_\nu\phi - g_{\mu\nu}\mathcal{L}, \tag{38}$$

which is spatially homogeneous and varies in time according to

$$\ddot{\phi} + 3H\dot{\phi} + \frac{\mathrm{d}V}{\mathrm{d}\phi} = 0. \tag{39}$$

From Eq. (38) it can be seen quintessence behaves like a perfect fluid with

$$\rho = \frac{1}{2}\dot{\phi}^2 + V(\phi), \quad p = \frac{1}{2}\dot{\phi}^2 - V(\phi), \tag{40}$$

what means using Eq. (5),

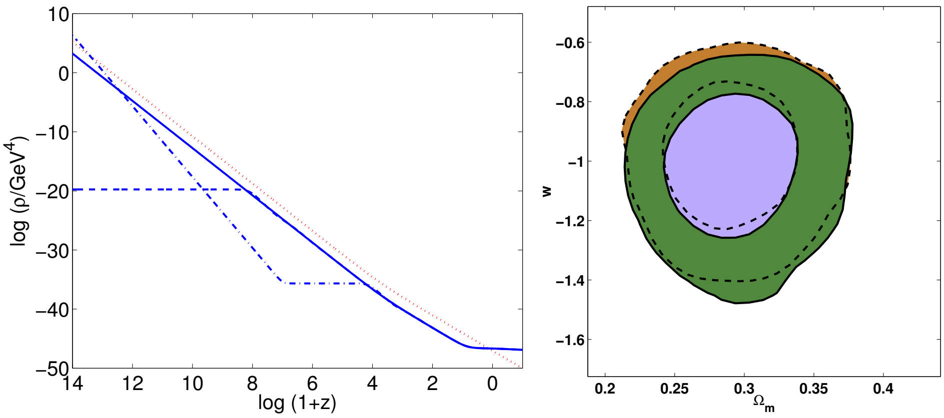
$$\omega = \frac{\frac{1}{2}\dot{\phi}^2 - V(\phi)}{\frac{1}{2}\dot{\phi}^2 + V(\phi)} \sim -1, \quad \text{if } \dot{\phi}^2 \ll V(\phi), \tag{41}$$

so we see that quintessence is a dark energy candidate as long as it varies slowly enough in time.

From Eq. (39) we see that quintessence evolves as a damped oscillator, so to achieve a slowly enough variation today potentials like the ones in Fig. 7 are possible. In fact the concrete realization of these potentials is very diverse, and some examples are presented in Tab. 2.

Potential	Authors
$V_0 \exp(-\lambda\phi)$	Ratra and Peebles, Wetterich and others
$m^2\phi^2, \lambda\phi^4$	Frieman *et al.*
V_0/ϕ^α	Ratra and Peebles
$V_0 \exp(\lambda\phi^2)/\phi^\alpha$	Brax and Martin
$V_0 \sinh^{-\alpha}(\lambda\phi)$	Sahni and Starobinsky and others
$V_0(\exp^{\alpha\phi} + \exp^{\beta\phi})$	Barreiro, Copeland and Nunes
$V_0(\exp(M_p/\phi) - 1)$	Zlatev, Wang and Steinhardt
$V_0[(\phi - B)^\alpha + A]e^{-\lambda\pi}$	Albrecht and Skordis

Table 2: Some proposed quintessence potentials. From Ref. [22].



(a) (b)

Figure 8: (a) Quintessence tracking of the matter density. From Ref. [6]. (b) Observational constrains on ω. From Ref. [24].

One the interesting features of quintessence is that in certain cases the dark energy density can evolve following the matter density even for a wide range of initial conditions as shown in Fig. 8(a), thus solving the “coincide problem”. This is called a tracker field. In these cases of course the fine-tuning problem persists, since the theoretical parameters of the potencial have to be carefully chosen.

5.2 Phantom dark energy

Up-to-date data does not rule out the possibility of a dark energy with $\omega < -1$, as can be seen from Fig. 8(b) This possibility has recently attracted the attention of the scientific community and it is called “phantom dark energy”.

The solution of Eq. (4) for $a(t)$ in this case is

$$a(t) \simeq a(t_{\rm eq}) \left[(1+\omega)\frac{t}{t_{\rm eq}} - \omega \right]^{2/3(1+\omega)} , \quad \omega < -1, \tag{42}$$

where the densities for matter and dark energy are equal at $t_{\rm eq}$, which leads to

$$a \to \infty,\ H \to \infty,\ \rho \to \infty \quad \text{for } t \to \left(\frac{\omega}{1+\omega} \right) t_{eq}, \tag{43}$$

i.e., the dark energy density increases instead of decaying, which leads to an infinite energy density in a finite amount of time and essentially a singularity in space-time, known as Big Rip [22]. It is also possible to construct models in which $\omega < -1$ today but do not lead to a Big Rip.

5.3 Other models

There are many different models of dark energy. The main difference between these models is the relation between p and ρ, obtained from particle physics inspiration (e.g. quintessence, spintessence, ...), string theory (e.g. braneworld models, ...) or just put in by hand (e.g. Chaplygin gases, ...), and the different phenomenology obtained from Eq. (4). Other models include: mass-varying neutrinos, axions, pseudo-goldstone bosons, K-essence, tachyons, ghost condensates, dilatonic dark energy, chameleon fields or de-Sitter vacua. We will not study any of these in these noted. Instead, for a review of these and other models, the reader is referred to Ref. [6].

6 Conclusions and future directions

We have seen how cosmological observations of Supernovae Type Ia, Cosmic Microwave Background and others lead to believe in the existence of a novel form of energy which we call dark energy. The effect of this energy is that of a repulsive gravity that accelerates the expansion of the universe. This dark energy behaves like a perfect fluid with equation of state parameters $\omega \sim -1$ and it accounts for ~ 0.7 of the total energy density of the universe.

Many questions still remain open about dark energy. It is not known whether it is constant (i.e., its effect is that of a cosmological constant) or it evolves in time. It is not yet well understood what, if any, is the relationship between dark energy and the vacuum energy density. Or if it is the result of a new field, or what type of field it could be. Also, recently it has come to the fore the question of whether it could have the form of a phantom energy with $\omega < -1$ and what would be the consequences of that. And especially, it is not known why it has precisely the right value to allow Large-Scale-Structure formation and the universe as we know it.

References

[1] P. J. E. Peebles and B. Ratra, Rev. Mod. Phys. **75**, 559 (2003).

[2] S. Perlmutter et al., Astrophys. J. **517**, 565 (1999).

[3] A. G. Riess et al., Astron. J. **116**, 1009 (1998).

[4] A. G. Riess et al., Astron. J. **707**, 1009 (1999).

[5] N. Jarosik et al., ApJS **192**, 14 (2011).

[6] E. J. Copeland, M. Sami, and S. Tsujikawa, Int. J. Mod. Phys. **D15**, 1753 (2006).

[7] S. M. Carroll, arXiv:gr-qc/9712019.

[8] M. Ross, *Introduction to Cosmology* (John Wiley and Sons, 2003), 3rd ed.

[9] R. R. Caldwell and M. Kamionkowski, JCAP **0409**, 009 (2004).

[10] G. Dvali and S. Turner, arXiv:astro-ph/0301510.

[11] S. Nojiri and S. D. Odintsov, arXiv:hep-th/0601213.

[12] E. Barausse, S. Matarrese, and A. Riotto, Phys. Rev. D **71**, 063537 (2005).

[13] C. M. Hirata and U. Seljak, Phys. Rev. D **72**, 0803501 (2005).

[14] S. Weinberg, Rev. Mod. Phys. **61**, 1 (1989).

[15] D. H. Perkins, *Particle astrophysics* (Oxford University Press, 2009).

[16] T. R. Choudhury and T. Padmanabhan, Astron. Astrophys. **429**, 807 (2005).

[17] M. Kamionkowski, arXiv:9796.2986 [astro-ph].

[18] M. Kowalski, arXiv:0804.4142 [astro-ph].

[19] H. B. G. Casimir and D. Polder, Phys. Rev. **73**, 360 (1948).

[20] Aitchison, *Supersymmetry in Particle Physics* (Cambridge University Press, 2007).

[21] B. Zumino, Nucl. Phys. B p. 535 (1975).

[22] V. Sahni, arXiv:astro-ph/0403324.

[23] V. Springel, arXiv:astro-ph/0504097.

[24] J. Weller and A. M. Lewis, Mon. Not. Roy. Astron. Soc. **346**, 987 (2003).