Bound states in dark-matter phenomenology

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Long-range interactions
mediated by
massless or light particles

Bound states

Long-range interactions

- **Iconoclastic-ism:**

  Most DM research has focused on contact interactions.
  Prototypical WIMP scenario: $m_{DM} \sim m_{mediators} \sim 100$ GeV.

  What happens when $m_{DM} \gg m_{mediators}$?

- **Long-range interactions appear in a variety of DM theories:**
  - Self-interacting DM
  - Asymmetric DM
  - Dissipative DM
  - DM explanations of galactic positrons
  - DM explanations of IceCube PeV neutrinos
  - Little hierarchy problem, e.g. twin Higgs models
  - Sectors with stable particles in String Theory

  } Hidden sector DM

  - WIMP DM with m_{DM} > few TeV ! [Hisano et al. 2002]
    - Minimal DM [Cirelli et al.]
    - LHC implications for SUSY
    - Direct/Indirect detection bounds

Complications

- **Large logarithmic corrections:**

$$\delta\sigma/\sigma \quad \sim \quad \alpha \ln (m_{DM} / m_{mediator})$$

→ resummation techniques etc. [In WIMP pheno: Baumgart et al. (2014)]

- **Non-perturbative effects:**

Sommerfeld enhancement in the non-relativistic regime.

Usually invoked for DM annihilation into radiation, but in fact affects *all* processes with same initial state.

- **More processes:**

Radiative formation of bound states [Sommerfeld enhanced]

Bound states — Phenomenological implications

- **Asymmetric DM → stable bound states**
  - Kinetic decoupling of DM from radiation, in the early universe
  - DM self-scattering in haloes (screening)
  - Indirect detection signals (radiative level transitions)
  - Direct detection signals (screening, inelastic scattering)

- **Symmetric DM → unstable bound states**
  formation + decay = extra annihilation channel
  - Relic abundance [von Harling, KP (2014); Ellis et al. (2015)]
  - Indirect detection

A. Confining theories
Hadronic bound states ("non-perturbative non-perturbative bound states").

Cosmologically, they definitely form.
May leave a remnant weakly coupled long(-ish)-range interaction.

B. Weakly coupled theories
"Perturbative non-perturbative bound states", e.g. atoms.

Formation efficiency depends on the details:
(i) bound-state formation cross-section, and
(ii) thermodynamic environment (early universe, DM haloes)

C. Theories with contact interactions
Solitonic bound states, e.g. Q-balls. [Coleman (1985)]
Exist in SUSY models. [Kusenko (1997)]

Highly ordered systems. Typically, very special conditions of formation (low entropy), e.g. Q-ball formation after inflation but before reheating, via fragmentation of Affleck-Dine condensate. [Kusenko, Shaposhnikov (1998)]

Outline

- [Pheno] Effect of bound-state formation and decay on freeze-out of symmetric thermal-relic DM.
- [Formal] How (and why) to calculate bound-state formation cross-sections in weakly coupled theories, in QFT.
- [Discussion] Relevance to WIMP dark matter.

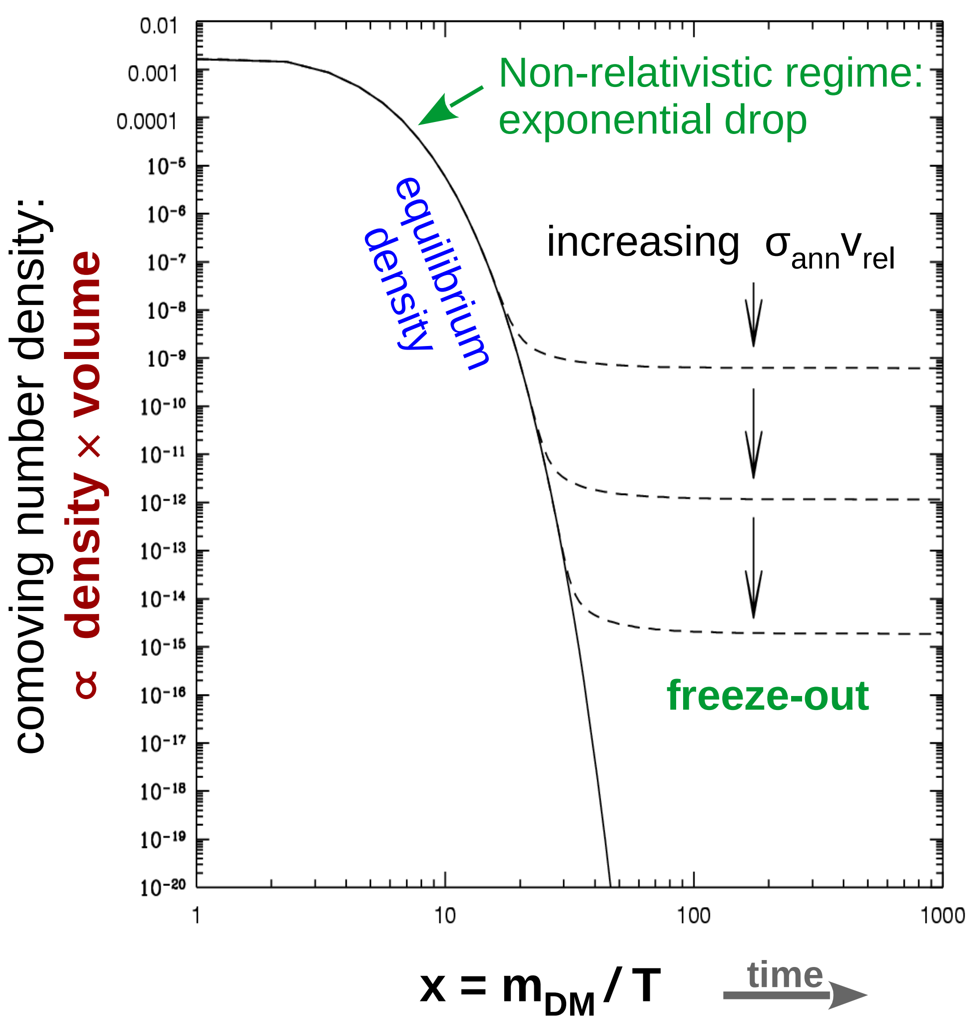
Relic density of symmetric DM with contact interactions

- Early universe: Hot thermal bath of elementary particles, $n_X = n_X(T)$, kept in chemical equilibrium via annihilations, $X + \bar{X} \leftrightarrow f + \bar{f}$

- As universe expands and cools
  → Density decreases
  → Annihilations become inefficient
  → Exponential decrease of $n_X(T)$ stalls: **freeze-out**
  → Relic density

$$\Omega_X \simeq 0.26 \times \left[\frac{3 \times 10^{-26}\ cm^3/s}{\langle \sigma_{ann} v_{rel} \rangle} \right]$$

Assumption:
$\sigma_{ann} v_{rel}$ is velocity-independent.
Valid for contact interactions.



Relic density of symmetric DM with long-range interactions

[von Harling, KP (2014)]

Toy model – Dark QED: **Dirac fermions (X, $\bar{X}$) of mass m, coupled to a massless dark photon γ, with dark fine-structure constant α.**

Very important parameter: $\zeta = \alpha / v_{rel}$

Annihilation

$$X + \bar{X} \rightarrow \gamma + \gamma$$

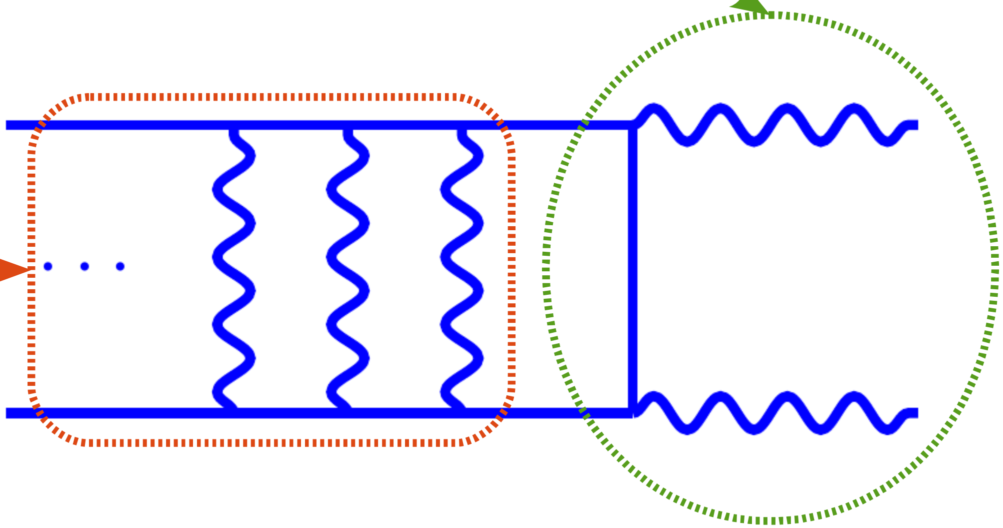
$$\sigma_{ann} v_{rel} = \sigma_0 \, S_{ann}(\zeta)$$

$$\sigma_0 = \pi \alpha^2 / m^2$$

$$S_{ann}(\zeta) = \frac{2\pi\zeta}{1 - e^{-2\pi\zeta}}$$

$$S_{ann}(\zeta \ll 1) \simeq 1$$

$$S_{ann}(\zeta \gtrsim 1) \simeq 2\pi\zeta$$



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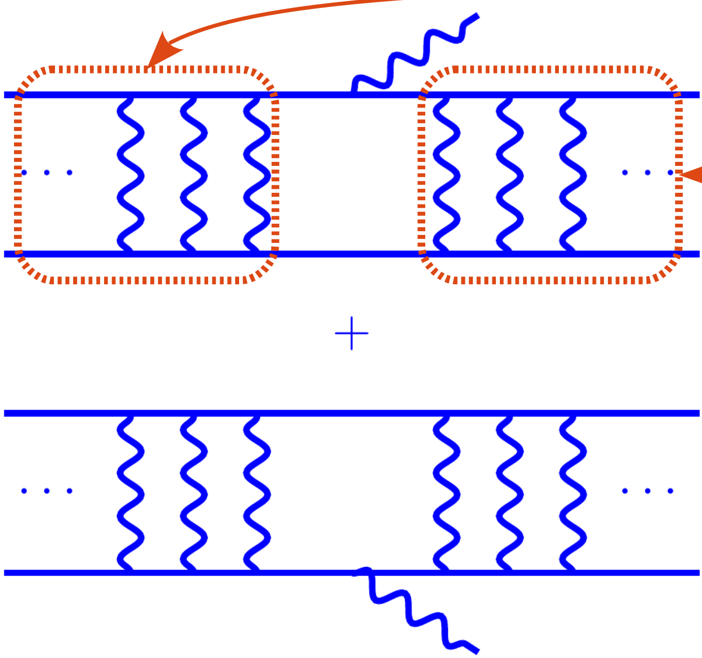


Bound state formation and decay

$$X + \bar{X} \rightarrow (X\bar{X})_{bound} + \gamma$$

$$(X\bar{X})_{bound} \rightarrow 2\gamma \;\; \text{or} \;\; 3\gamma$$

$$\sigma_{BSF} v_{rel} = \sigma_0 S_{BSF}(\zeta)$$

$$\sigma_0 = \pi \alpha^2 / m^2$$

$$S_{BSF}(\zeta) = \left[\frac{2^9}{3\, e^{4\zeta \operatorname{arccot}(\zeta)}} \frac{\zeta^4}{(1+\zeta^2)^2} \right] \frac{2\pi\zeta}{1 - e^{-2\pi\zeta}}$$

$$S_{BSF}(\zeta \ll 1) \simeq \frac{2^9 \zeta^4}{3} \ll 1$$

$$S_{BSF}(\zeta \gtrsim 1) \simeq \frac{2^9}{3 e^4} \times 2\pi\zeta \simeq 3.13 \times S_{ann}$$

Relic density of symmetric DM with long-range interactions

[von Harling, KP (2014)]

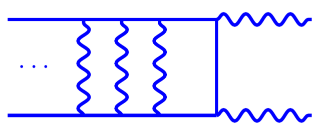
Processes

Toy model – Dark QED: **Dirac fermions (X, $\bar{X}$) of mass m, coupled to a massless dark photon γ, with dark fine-structure constant α.**

Very important parameter: $\zeta = \alpha / v_{rel}$

Annihilation

$$X + \bar{X} \rightarrow \gamma + \gamma$$



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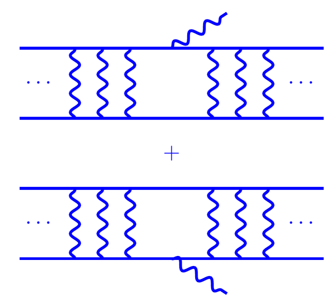

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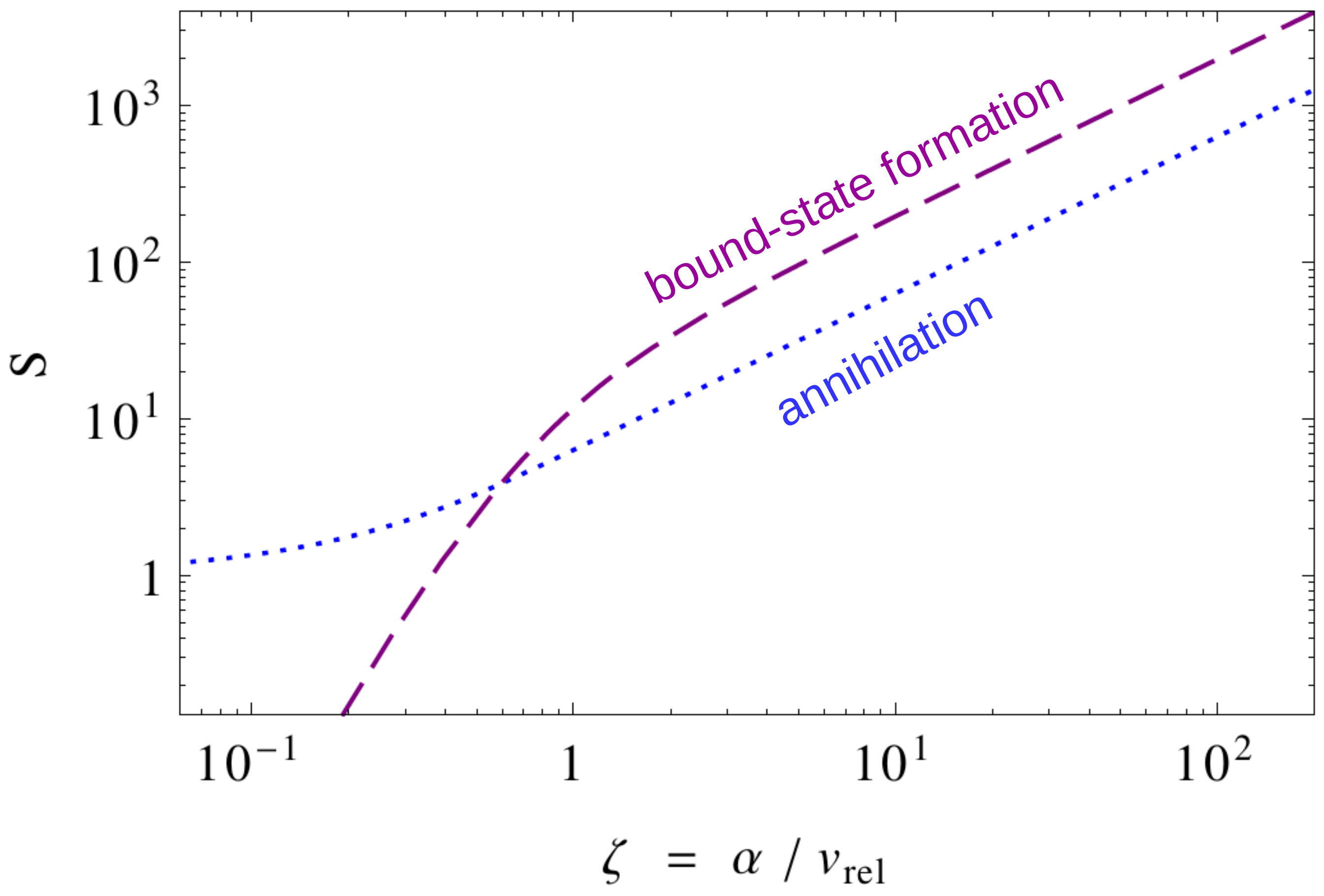
Relic density of symmetric DM with long-range interactions

[von Harling, KP (2014)]

Rates



BSF dominates over annihilation everywhere the Sommerfeld effect is important (ζ > 1) !

Relic density of symmetric DM with long-range interactions

[von Harling, KP (2014)]

Boltzmann equations

$$\frac{dn_X}{dt} + 3Hn_X = -\left(n_X^2 - n_X^{eq\ 2}\right)\langle\sigma_{ann} v_{rel}\rangle - n_X^2\langle\sigma_{BSF} v_{rel}\rangle + \left(n_{\uparrow\downarrow} + n_{\uparrow\uparrow}\right)\Gamma_{ion}$$

$$\frac{dn_{\uparrow\downarrow}}{dt} + 3Hn_{\uparrow\downarrow} = +\frac{1}{4} n_X^2\langle\sigma_{BSF} v_{rel}\rangle - n_{\uparrow\downarrow}\left(\Gamma_{ion} + \Gamma_{decay,\uparrow\downarrow}\right)$$

$$\frac{dn_{\uparrow\uparrow}}{dt} + 3Hn_{\uparrow\uparrow} = +\frac{3}{4} n_X^2\langle\sigma_{BSF} v_{rel}\rangle - n_{\uparrow\uparrow}\left(\Gamma_{ion} + \Gamma_{decay,\uparrow\uparrow}\right)$$

$(X\bar{X})_{\uparrow\downarrow} \rightarrow 2\gamma$: $\quad \Gamma_{decay,\uparrow\downarrow} = \alpha^5 (m/2)$

$(X\bar{X})_{\uparrow\uparrow} \rightarrow 3\gamma$: $\quad \Gamma_{decay,\uparrow\uparrow} = \frac{4(\pi^2-9)}{9\pi}\alpha^6 (m/2)$

$(X\bar{X})_{\uparrow\downarrow \text{ or } \uparrow\uparrow} + \gamma \rightarrow X + \bar{X}$: $\quad \Gamma_{ion}(T) = \frac{2}{(2\pi)^3} 4\pi \int_0^\infty d\omega \frac{\omega^2}{e^{\omega/T}-1}\sigma_{ion}(\omega)$

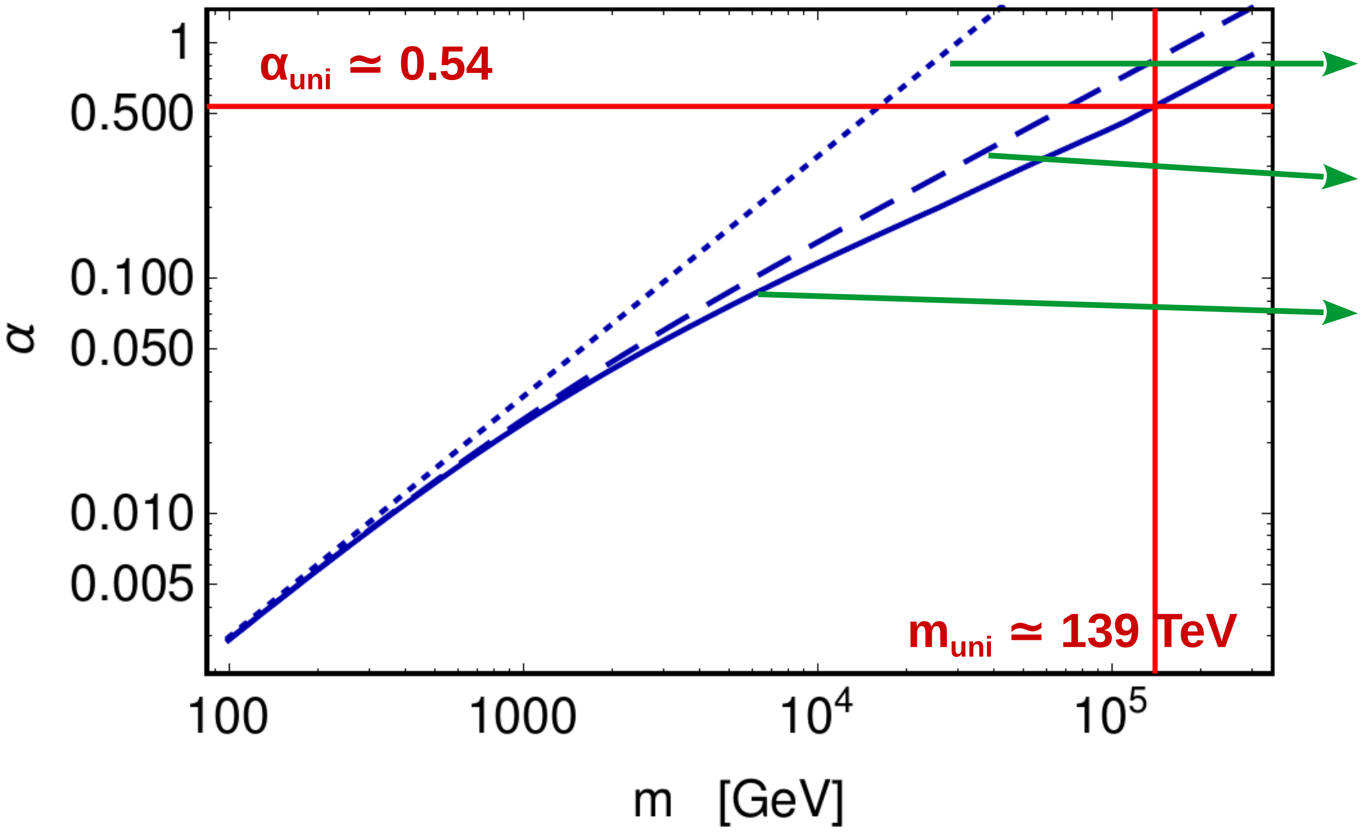
BSF important when $\Gamma_{decay} > \Gamma_{ion}(T)$

related to σ_{BSF}

Relic density of symmetric DM with long-range interactions

[von Harling, KP (2014)]

Determination of $\alpha(m)$ or $m(\alpha)$



Perturbative annihilation.

Sommerfeld-enhanced annihilation.

Sommerfeld-enhanced annihilation and BSF.

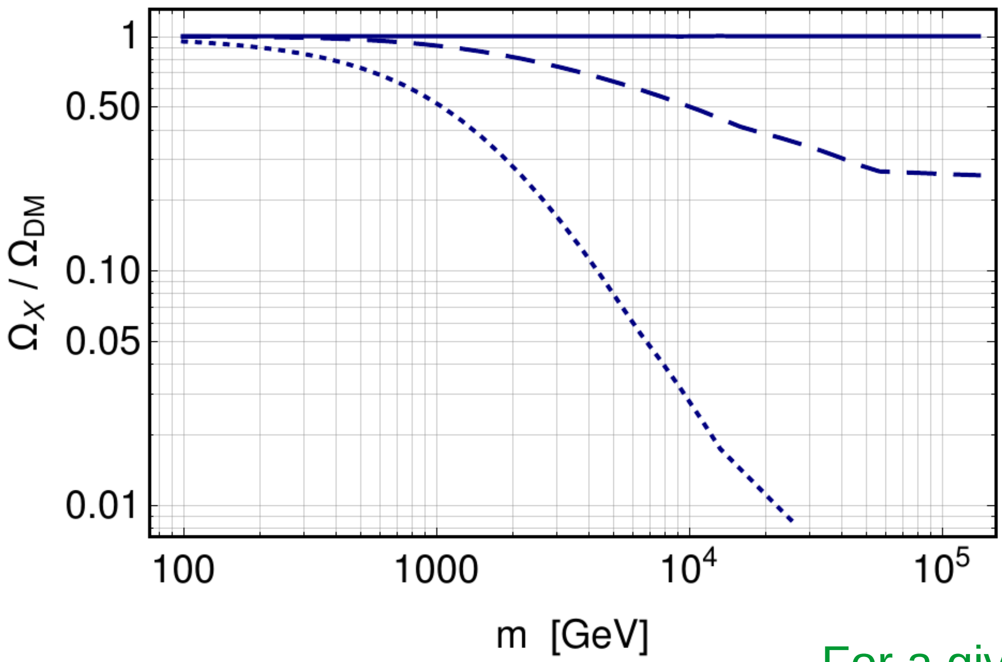
Relic density of symmetric DM with long-range interactions

[von Harling, KP (2014)]

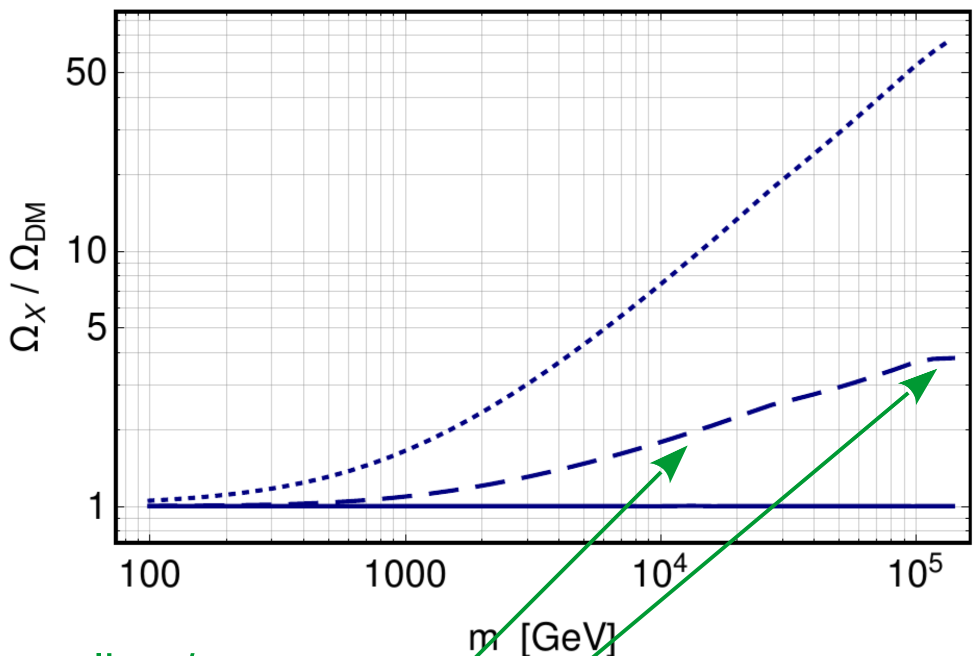
Effect on DM relic density

Much larger than experimental uncertainty of 1% !

Various determinations of α, plugged into full Boltzmann Eqs.



α determined from full Boltzmann Eqs, plugged into “partial” Boltzmann Eqs.



For a given coupling / mass,
SE annihilation alone results in

$\Omega_X / \Omega_{DM} \simeq 2$ @ 15 TeV

$\Omega_X / \Omega_{DM} \simeq 4$ @ 139 TeV

Relic density of symmetric DM with long-range interactions
[von Harling, KP (2014)]

Partial-wave unitarity

Saturation of interaction probability at large couplings.

$$\sigma_{inel,J} \, v_{rel} \leqslant \frac{(2J+1)\,4\pi}{m^2 \, v_{rel}}$$

feature of long-range inelastic processes

- **Implies upper limit on mass of thermal relic DM.** [Griest, Kamionkowski (1990)]
- **Can be realised only if DM possesses long-range interactions.**
  S-wave processes: **$m < m_{UNI}$ = 83 TeV → 139 TeV** (non-self-conjugate DM)
  [von Harling, KP (2014)]
- **All partial waves must have the same velocity dependence close to the unitarity limit.** Confirmed by explicit calculations for long-range interactions.
  - For annihilation, higher $J \Rightarrow$ higher powers of α. [Cassel (2009)]
  - For BSF, higher partial waves give significant contribution,
    e.g. BSF with vector emission: $\mathcal{M} \propto \sin\theta \Rightarrow J=0$: 62%, $J=2$: 24% ...
    ⇒ Unitarity limit on m_{DM} even higher? [KP, Postma, Wiechers (2015)]

Relic density of symmetric DM with long-range interactions

[von Harling, KP (2014)]

Partial-wave unitarity

Saturation of interaction probability at large couplings.

$$\sigma_{inel,J}\; v_{rel} \leqslant \frac{(2J+1)\,4\pi}{m^2\; v_{rel}}$$

feature of long-range inelastic processes

- **Unitarity realised perturbatively for α ~ 0.5,** i.e. well below the perturbativity limit (α ~ π or 4π).

  At large α ($\alpha \gg v_{rel}$):

$$number_J \times \frac{\pi\alpha^2}{m^2}\,\frac{\alpha}{v_{rel}} \leqslant \frac{(2J+1)\,4\pi}{m^2\; v_{rel}} \quad\Rightarrow\quad \alpha \lesssim 0.5$$

- Massive mediator – Yukawa potential.
- Different interactions, e.g. scalar mediator.
- Non-Abelian non-confining theories,
  e.g. electroweak interactions.

Radiative capture in QFT

[KP, Postma, Wiechers (2015)]

Establish a QFT formalism (instead of QM),
for weakly-coupled theories with long-range interactions.
Take non-relativistic limit relevant for cosmo/astro DM applications.

- Can accommodate non-Abelian interactions, e.g. electroweak interactions.
- Allows systematic inclusion of higher-order corrections in the coupling strength and in the momentum transfer.

Radiative capture in QFT

[KP, Postma, Wiechers (2015)]

In 4 easy steps

$$\mathcal{U}\{\chi_1 + \chi_2\} \to \mathcal{B}\{\chi_1\chi_2\} + \gamma$$

1) Separate the asymptotic states from the interaction part (which includes the radiative vertex).

2) Compute the properties of the asymptotic states.

Scattering state (initial): Long-range interaction between χ_1 & χ_2

⇒ Two-particle state ≠ Two plane waves

⇒ *Sommerfeld effect* (in the non-relativistic regime)

[Sommerfeld (1931), Bethe & Salpeter (1957)]

Bound state (final): One-particle state (pole in scattering amplitude), with the quantum charges of χ_1 & χ_2.

[Bethe & Salpeter (1957)]

3) Compute the interaction part. Feynman diagrams

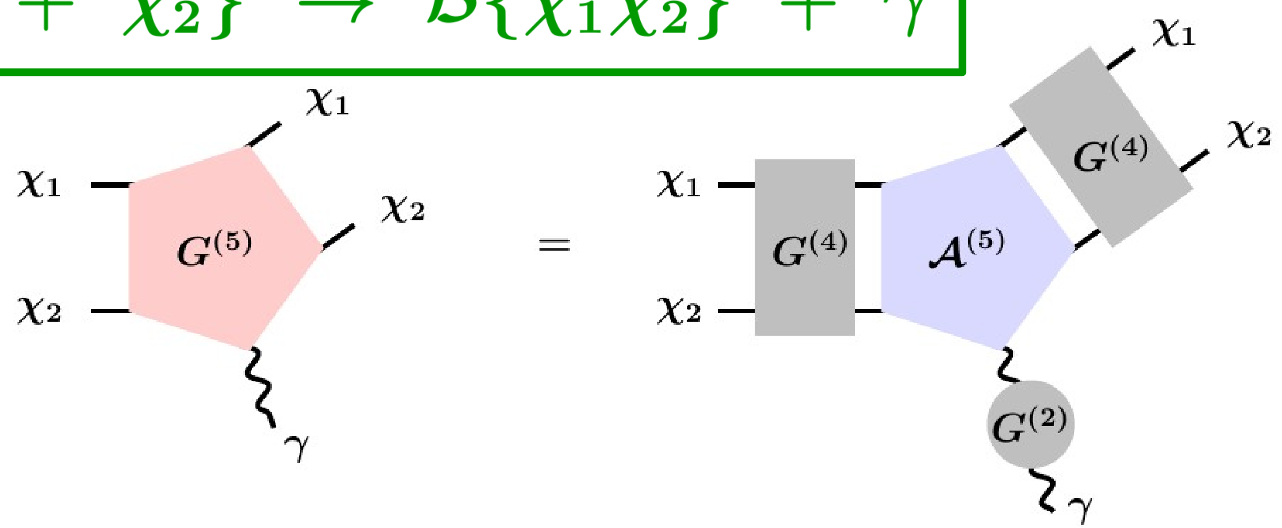
4) Extract the amplitude. LSZ reduction

Radiative capture in QFT

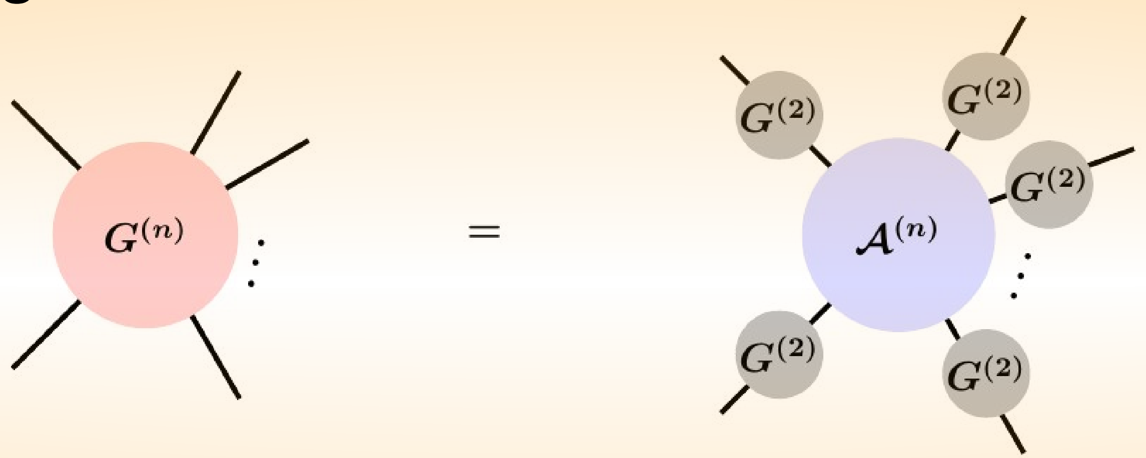
[KP, Postma, Wiechers (2015)]

|1| Identify and separate the asymptotic states

$$\mathcal{U}\{\chi_1 + \chi_2\} \rightarrow \mathcal{B}\{\chi_1\chi_2\} + \gamma$$



No long-range interaction between initial / final state particles



$$G^{(2)} = \frac{iZ_{\vec{p}}}{p^2 - m^2 + i\epsilon}, \qquad Z_{\vec{p}} = |\langle\Omega|\chi(0)|\vec{p}\rangle|^2$$

Radiative capture in QFT

[KP, Postma, Wiechers (2015)]

|1| Identify and separate the asymptotic states

Insert unity in the 4-point function

$$G^{(4)} = \langle \Omega | T \chi_1(x_1) \chi_2(x_2) \chi_1^\dagger(y_1) \chi_2^\dagger(y_2) | \Omega \rangle$$

$$1 \sim \sum_n \int d^3Q \, |\mathcal{B}_{\vec{Q},n}\rangle \langle \mathcal{B}_{\vec{Q},n}| + \int d^3q \, d^3Q \, |\mathcal{U}_{\vec{Q},\vec{q}}\rangle \langle \mathcal{U}_{\vec{Q},\vec{q}}|$$

bound states of energy $\omega_{\vec{Q},n}$

scattering (unbound) states of energy $\omega_{\vec{Q},\vec{q}}$

expectation value of relative momentum

$$G^{(4)}(Q) \sim \sum_n \frac{i\Psi_{\vec{Q},n}(x)\, \Psi^*_{\vec{Q},n}(y)}{Q^0 - \omega_{\vec{Q},n} + i\epsilon} + \int d^3q \, \frac{i\Phi_{\vec{Q},\vec{q}}(x)\, \Phi^*_{\vec{Q},\vec{q}}(y)}{Q^0 - \omega_{\vec{Q},\vec{q}} + i\epsilon}$$

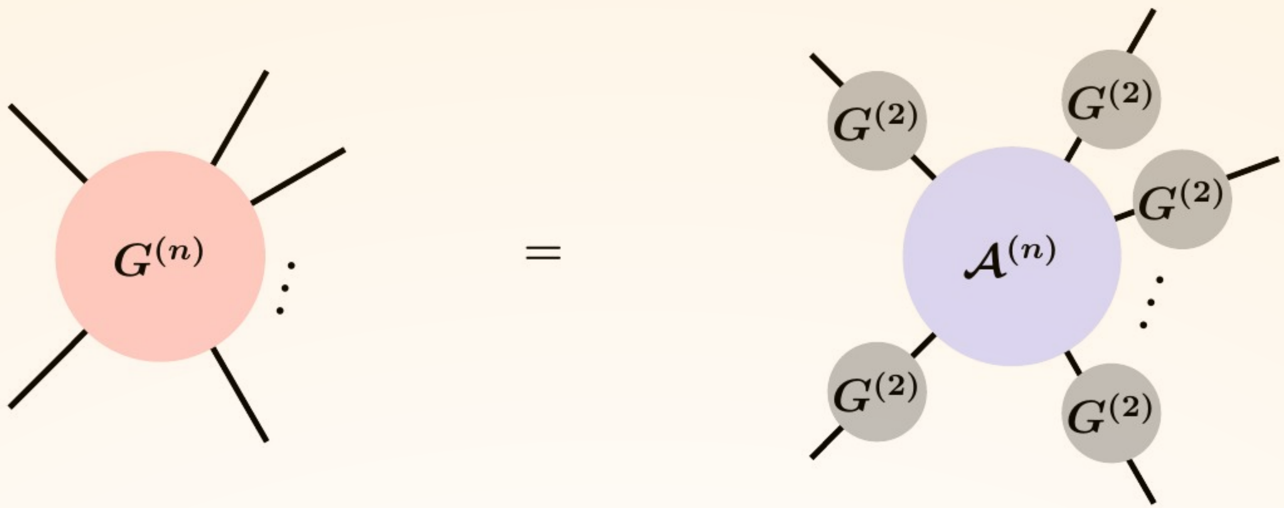
$$\text{where} \begin{cases} \Psi_{\vec{Q},n}(x_1,x_2) \equiv \langle \Omega | T \chi_1(x_1) \chi_2(x_2) | \mathcal{B}_{\vec{Q},n}\rangle = e^{-iQX} \, \Psi_{\vec{Q},n}(x) \\ \Phi_{\vec{Q},\vec{q}}(x_1,x_2) \equiv \langle \Omega | T \chi_1(x_1) \chi_2(x_2) | \mathcal{U}_{\vec{Q},\vec{q}}\rangle = e^{-iQX} \, \Phi_{\vec{Q},\vec{q}}(x) \end{cases}$$

Bethe-Salpeter wavefunctions

$$X \equiv \eta_1 x_1 + \eta_2 x_2$$
$$x \equiv x_1 - x_2$$
$$\eta_{1,2} \equiv \frac{m_{1,2}}{m_1 + m_2}$$

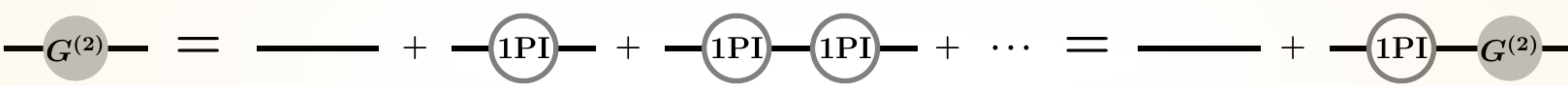
By choosing the energy Q^0, a singularity dominates, and we pick out a state.

No long-range interaction between initial / final state particles



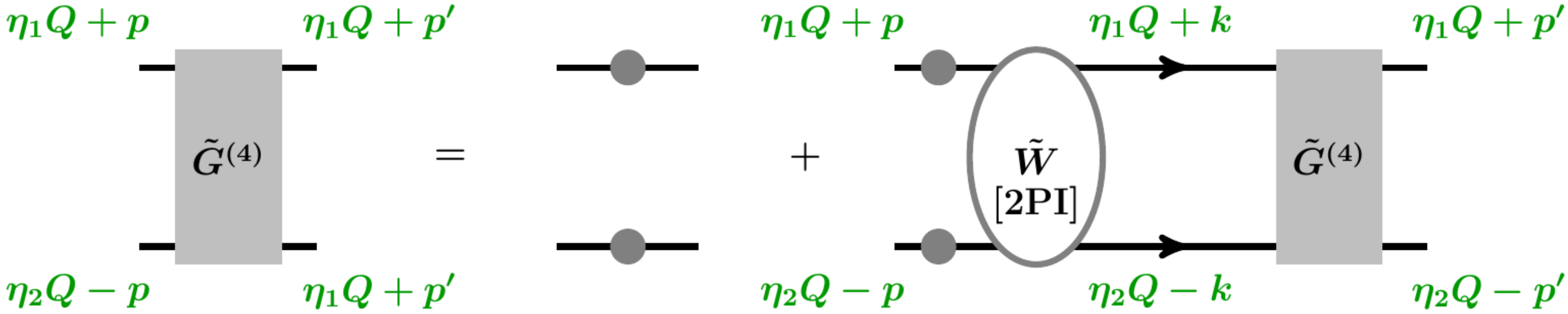
$$= \frac{iZ_{\vec{p}}}{p^2 - m^2 + i\epsilon}, \qquad Z_{\vec{p}} = |\langle\Omega|\chi(0)|\vec{p}\rangle|^2$$

We determine $Z_{\vec{p}}$ and m from the Dyson-Schwinger equation,



after

is specified.



$\eta_{1,2} \equiv m_{1,2}/(m_1 + m_2)$

and

$G^{(4)}$ decomposition into bound & scattering-state contributions

Bethe – Salpeter equations for bound & scattering state WFs

$$\tilde{\Psi}_{\vec{Q},n}(p) = S(p;Q) \int \frac{d^4k}{(2\pi)^4}\, \tilde{W}(p,k;\,Q)\tilde{\Psi}_{\vec{Q},n}(k)$$

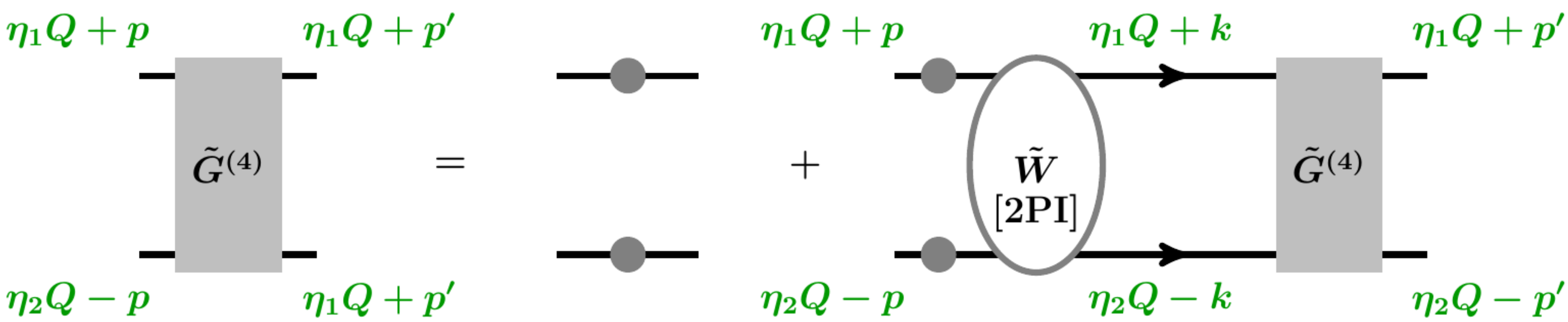
$$\tilde{\Phi}_{\vec{Q},\vec{q}}(p) = S(p;Q) \int \frac{d^4k}{(2\pi)^4}\, \tilde{W}(p,k;\,Q)\tilde{\Phi}_{\vec{Q},\vec{q}}(k)$$

Product of χ_1, χ_2 propagators; contains $\omega_{\bar{Q},n}$ or $\omega_{\bar{Q},\bar{q}}$

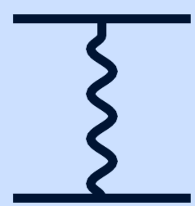
Radiative capture in QFT

[KP, Postma, Wiechers (2015)]

|2| Calculate the properties of the asymptotic states



Specify the perturbative interaction kernel, e.g. one-boson exchange

$\tilde{W} =$ 

$$\tilde{\Psi}_{\vec{Q},n}(p) = S(p;Q) \int \frac{d^4k}{(2\pi)^4}\, \tilde{W}(p,k;\, Q)\tilde{\Psi}_{\vec{Q},n}(k)$$

$$\tilde{\Phi}_{\vec{Q},\vec{q}}(p) = S(p;Q) \int \frac{d^4k}{(2\pi)^4}\, \tilde{W}(p,k;\, Q)\tilde{\Phi}_{\vec{Q},\vec{q}}(k)$$

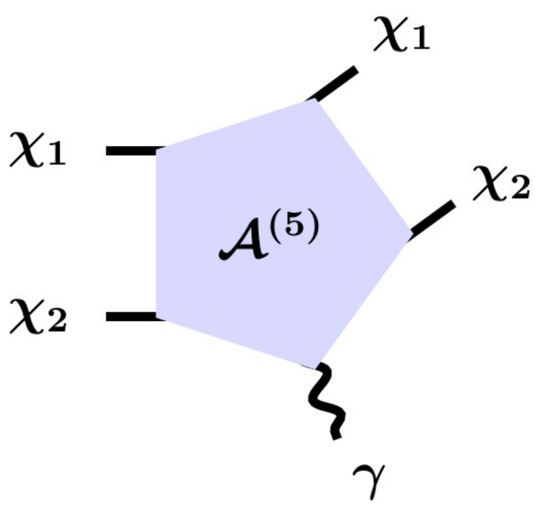
Sommerfeld effect!

Reduce to Schroedinger eqs in non-relativistic regime

Determine $\Psi_{\vec{Q},n}$, $\Phi_{\vec{Q},\vec{q}}$ and $\omega_{\vec{Q},n}$, $\omega_{\vec{Q},\vec{q}}$.

Radiative capture in QFT

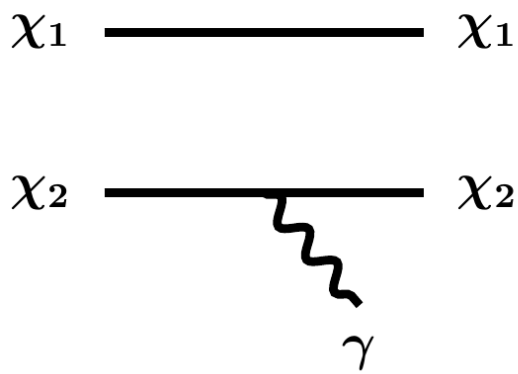
[KP, Postma, Wiechers (2015)]



What diagrams should $\mathcal{A}^{(5)}$ include?

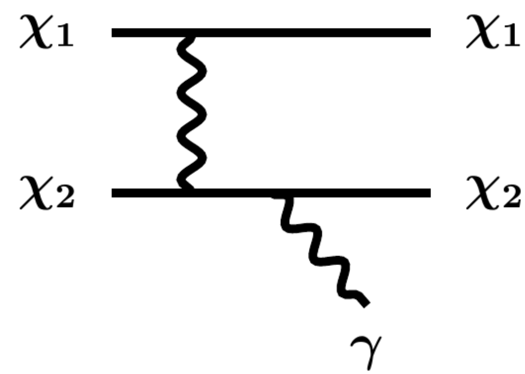
- Amputated by 2-particle-irreducible contributions (otherwise double counting).
- Not "fully" connected diagrams contribute (χ_1, χ_2 legs not on-shell individually).

Yes



not "fully" connected

No

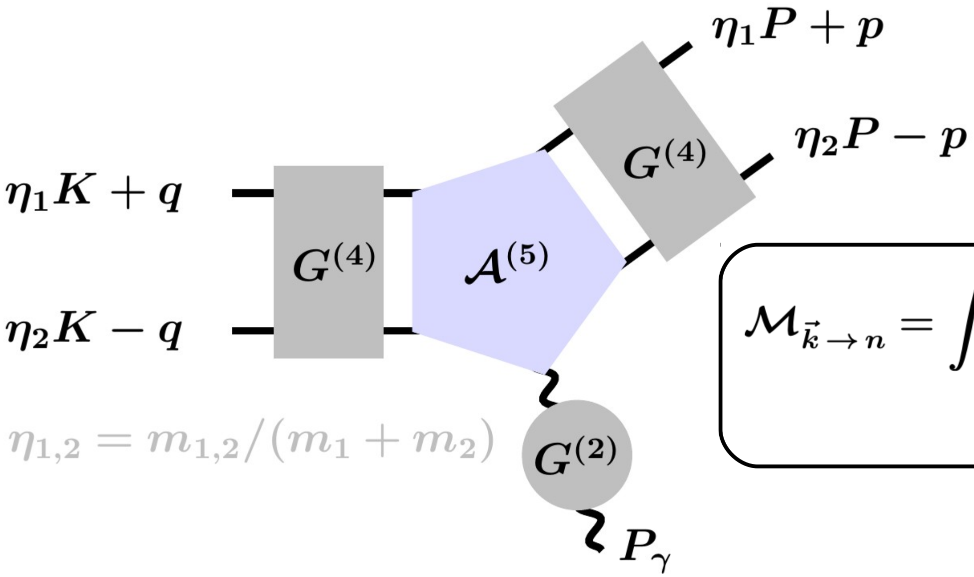


double counting

Radiative capture in QFT

[KP, Postma, Wiechers (2015)]

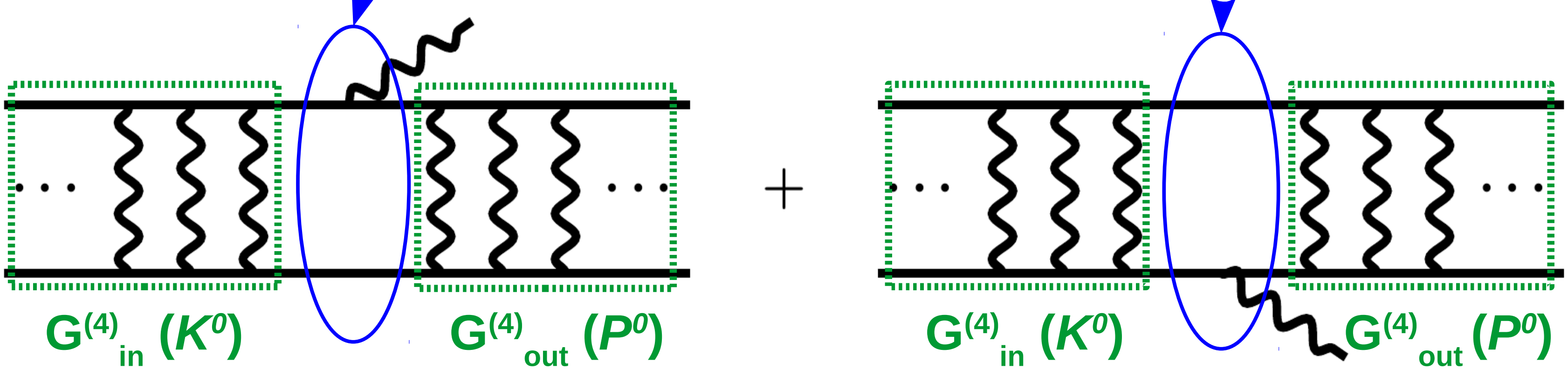
|4| LSZ reduction: Green's function → Amplitude



non-perturbative physics

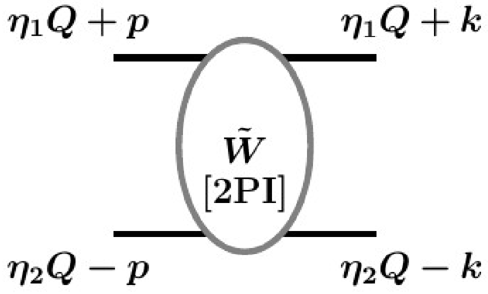
$$\mathcal{M}_{\vec{k}\to n} = \int \frac{d^4p}{(2\pi)^4}\frac{d^4q}{(2\pi)^4}\; \tilde{\Psi}^*_{\vec{P},n}(p)\; \tilde{\Phi}_{\vec{K},\vec{k}}(q) \times \mathcal{A}^{(5)}(P_\gamma, \eta_1 P + p, \eta_2 P - p;\; \eta_1 K + q, \eta_2 K - q)$$

perturbative transition diagrams



[one-boson-exchange kernel]

"Instantaneous" approximation



$$\tilde{W}(p,k;Q) \simeq \tilde{W}(\vec{p},\vec{k})$$

$$\tilde{\Psi}_{\vec{Q},n}(p) = S(p;Q) \times \int \frac{d^4k}{(2\pi)^4}\, \tilde{W}(p,k;\,Q)\, \tilde{\Psi}_{\vec{Q},n}(k)$$

$$\left[\int \frac{dp^0}{2\pi}\, \tilde{\Psi}_{\vec{Q},n}(p)\right] = \left[\int \frac{dp^0}{2\pi} S(p;Q)\right] \times \int \frac{d^3k}{(2\pi)^3}\, \tilde{W}(\vec{p},\vec{k}) \left[\int \frac{dk^0}{2\pi} \tilde{\Psi}_{\vec{Q},n}(k)\right]$$

$$\tilde{\psi}_n(\vec{p}) = \mathcal{S}(\vec{p};Q) \times \int \frac{d^3k}{(2\pi)^3}\, \tilde{W}(\vec{p},\vec{k})\, \tilde{\psi}_n(\vec{k})$$

where

$$\tilde{\psi}_n(\vec{p}) \propto \int \frac{dp^0}{2\pi}\, \tilde{\Psi}_{\vec{Q},n}(p) = \int d^3x\, \Psi_{\vec{Q},n}\left(x^0 = 0, \vec{x}\right)\, e^{-i\vec{p}\cdot\vec{x}}$$

Schrödinger or "equal-time" wavefunction: $x^0 \equiv x_1^0 - x_2^0 = 0$

Generalisation to multiplets

- Multiple interacting quarteta $\chi_a + \chi_b \rightarrow \chi_c + \chi_d$
  with interaction kernels W_{abcd}, described by $G^{(4)}_{abcd}$
  $\Rightarrow$ Coupled Bethe-Salpeter equations for $[\Psi_{\vec{Q},n}]_{ab}$, $[\Phi_{\vec{Q},\vec{q}}]_{ab}$:

$$\left[\tilde{\Psi}_{\vec{Q},n}(p)\right]_{ab} = [S(p;Q)]_{ab} \times \sum_{c,d} \int \frac{d^4k}{(2\pi)^4} \left[\tilde{W}(p,k;\,Q)\right]_{abcd} \left[\tilde{\Psi}_{\vec{Q},n}(k)\right]_{cd}$$

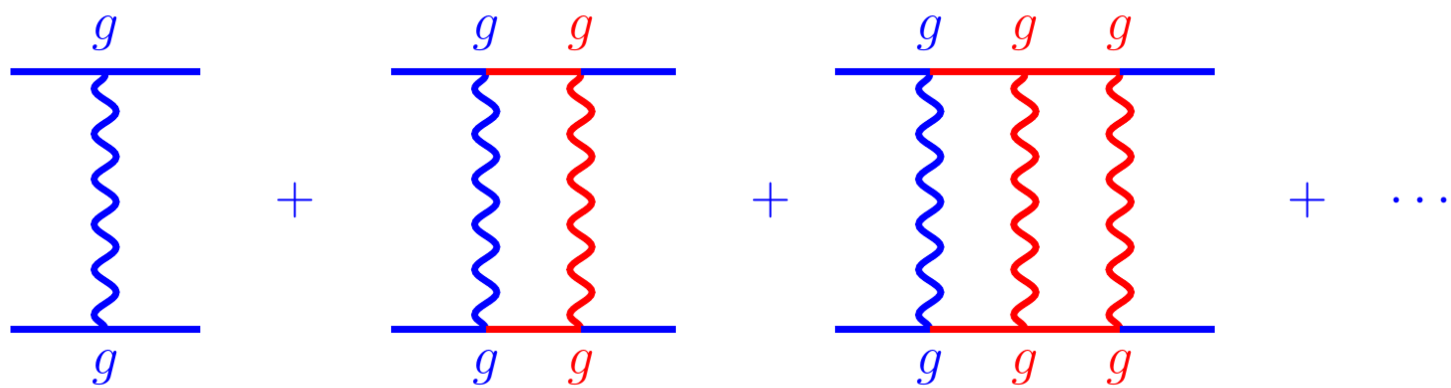
- Multiple radiative vertices $\mathcal{A}_{abcd}$, and contributions $\mathcal{M}_{abcd}$ to the transition amplitude

$$[\mathcal{M}_{\vec{k}\to n}]_{abcd} = \int \frac{d^4p}{(2\pi)^4} \frac{d^4q}{(2\pi)^4} [\tilde{\Phi}_{\vec{K},\vec{k}}(q)]_{ab} \, [\tilde{\Psi}^*_{\vec{P},n}(p)]_{cd} \times \mathcal{A}^{(5)}_{abcd}\left(P_\gamma, \eta_1 P + p, \eta_2 P - p;\; \eta_1 K + q, \eta_2 K - q\right)$$

$$\mathcal{M}_{\vec{k}\to n} = \sum_{a,b,c,d} [\mathcal{M}_{\vec{k}\to n}]_{abcd}$$

Making sense of the ladder diagrams

Every force mediator exchange introduces an additional
$\alpha = g^2/4\pi$ suppression. How do we get the non-perturbative effects?



Energy and momentum transfer scale with α !

Momentum transfer: $\bar{q} \sim \mu\, \alpha$. Energy transfer: $q^0 \sim \bar{q}^2 / \mu \sim \mu\, \alpha^2$.

$$\text{one boson exchange} \sim \alpha\, \frac{1}{k_\gamma^2} \sim \alpha\, \frac{1}{(\mu\alpha)^2} \propto \frac{1}{\alpha}$$

$$\text{each added loop} \sim \alpha \times \int dq^0\, d^3q\, \frac{1}{q_\gamma^2}\, \frac{1}{q_1 - m_1}\, \frac{1}{q_2 - m_2}$$

$$\sim \alpha \times (\mu\alpha^2)(\mu\alpha)^3 \times \frac{1}{(\mu\alpha)^2} \left(\frac{1}{\mu\alpha^2}\right)^2$$

$$\sim 1$$

1/α scaling
responsible for
non-perturbative
effects
(not the largeness
of the coupling)

Are bound states relevant to WIMP dark matter?

Sommerfeld enhancement of annihilation of TeV-scale WIMPs has important effect.
⇒ BSF likely too! For a massless mediator, BSF affects relic density if m_{DM} > TeV.

But WIMP DM candidates couple *to massive* gauge bosons, $W^{\pm}$.

In a Yukawa potential, bound states exist if $m_{mediator} \lesssim (m_X/2)\,\alpha$.
WIMP-onium exists if $m_{WIMP} \gtrsim 5$ TeV. "Natural" scale for WIMP DM!

Radiative BSF necessitates $m_{mediator} \lesssim (m_X/2)\,\alpha^2/2$.
Emission of $W^{\pm}$ possible for $m_{WIMP} \gtrsim 400$ TeV.
⇒ BSF irrelevant for WIMP DM.

Exchange of $W^{\pm}$ converts neutral WIMPs into charged particles.
WIMPonium can form with emission of photons!

OK, but the $W^{\pm}$ exchange surely suppresses the cross-section.

According to the previous scaling argument, there is no suppression! $W^{\pm}$ is part of the ladder.

WIMPonium formation potentially important if $m_{WIMP} \gtrsim 5$ TeV !

And what about sub-TeV WIMP DM?

The Weak interactions manifest as contact type. There is no SE and no WIMPonium.

But Patrick said that in some sub-TeV scenarios, the Sommerfeld effect is important.

In MSSM, charged/coloured NLSP (e.g. stop, stau etc) can be nearly degenerate to the neutralino LSP. This occurs even for light (few × 100 GeV) NLSP/LSP.

Then, the NLSP – LSP co-annihilations are significant for LSP density. But they are also mediated by the heavy $W^{\pm}$ bosons. There is no SE and no bound states.

Right. But because of the mass degeneracy, the NLSP decay is very slow, and may occur *after* LSP freeze-out. Then, the NLSP abundance is itself important.

Aha! For coloured NLSPs, the Sommerfeld effect is important, because the coupling is strong.

And for a charged/coloured NLSPs, there is no threshold for the existence and formation of bound states.

So, bound states may be important even for sub-TeV WIMP DM!

Conclusion

The early universe regulates the DM manifestations today.

Familiar example — Symmetric thermal relics with contact interactions:

$$\Omega_{DM} \propto 1/(\sigma_{ann} v_{rel}) \Rightarrow d\Gamma_{ann}/dV = \rho_{DM}^2 (\sigma_{ann} v_{rel}) \propto 1/(\sigma_{ann} v_{rel})$$

For long-range interactions, the regulator is bound-state formation:

- Symmetric thermal relics: Reduced abundance.
- Asymmetric thermal relics: Neutralises / screens the interaction.

BSF is important for important DM theories,
e.g. WIMPs, self-interacting DM, asymmetric DM.

Need tools to calculate bound-state related processes.

Non-relativistic regime relevant for DM phenomenology.
Nevertheless, QFT formalism preferable or even necessary.

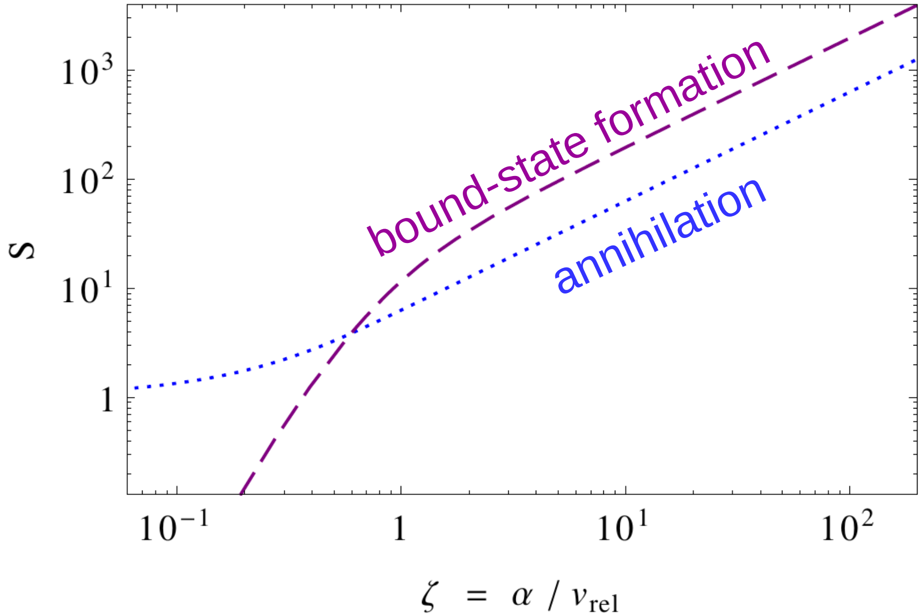
Can accommodate DM residing in multiplets, and multiple interaction kernels. Can reliably yield higher-order corrections.

Extra slides

Relic density of symmetric DM with long-range interactions

[von Harling, KP (2014)]

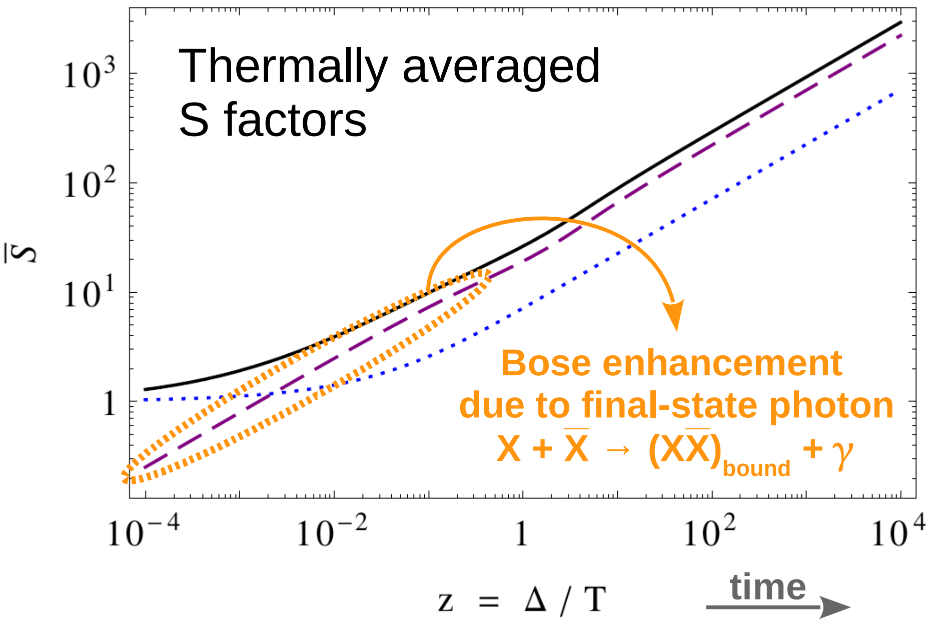
Rates



$$\zeta \equiv \frac{\textit{Bohr momentum}}{\textit{relative momentum}} = \frac{\mu\,\alpha}{\mu\, v_{rel}}$$

(reduced mass μ = m/2)

$\sigma_{BSF}\, v_{rel} > \sigma_{ann}\, v_{rel}$
everywhere the Sommerfeld effect is important (ζ > 1).



Time parameter :

$$z \equiv \frac{\textit{binding energy}\,[\Delta]}{T} \sim \frac{(1/2)\,\mu\,\alpha^2}{(1/6)\,\mu\,\langle v_{rel}^2\rangle} \sim \langle \zeta^2 \rangle$$

$\langle\sigma_{BSF}\, v_{rel}\rangle > \langle\sigma_{ann}\, v_{rel}\rangle$ even at $z \ll 1$,
but

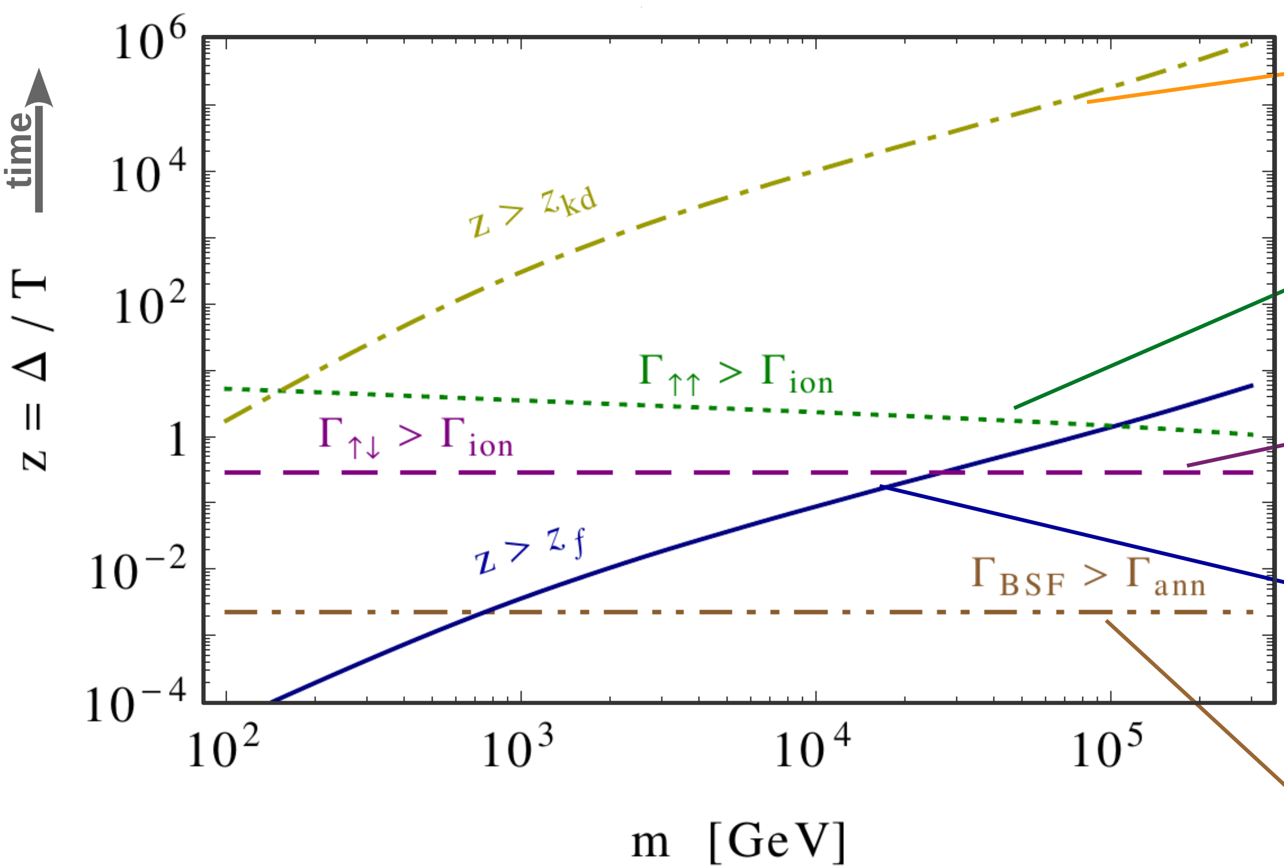
BSF can deplete DM only at $z \gtrsim 1$, when disassociation of bound states becomes unimportant.

Relic density of symmetric DM with long-range interactions

[von Harling, KP (2014)]

Timeline

$\alpha = \alpha(m)$ fixed from relic abundance [see results]



Kinetic decoupling

Ortho-positronium decay faster than ionisation (z ~ 1-10, depending on m)

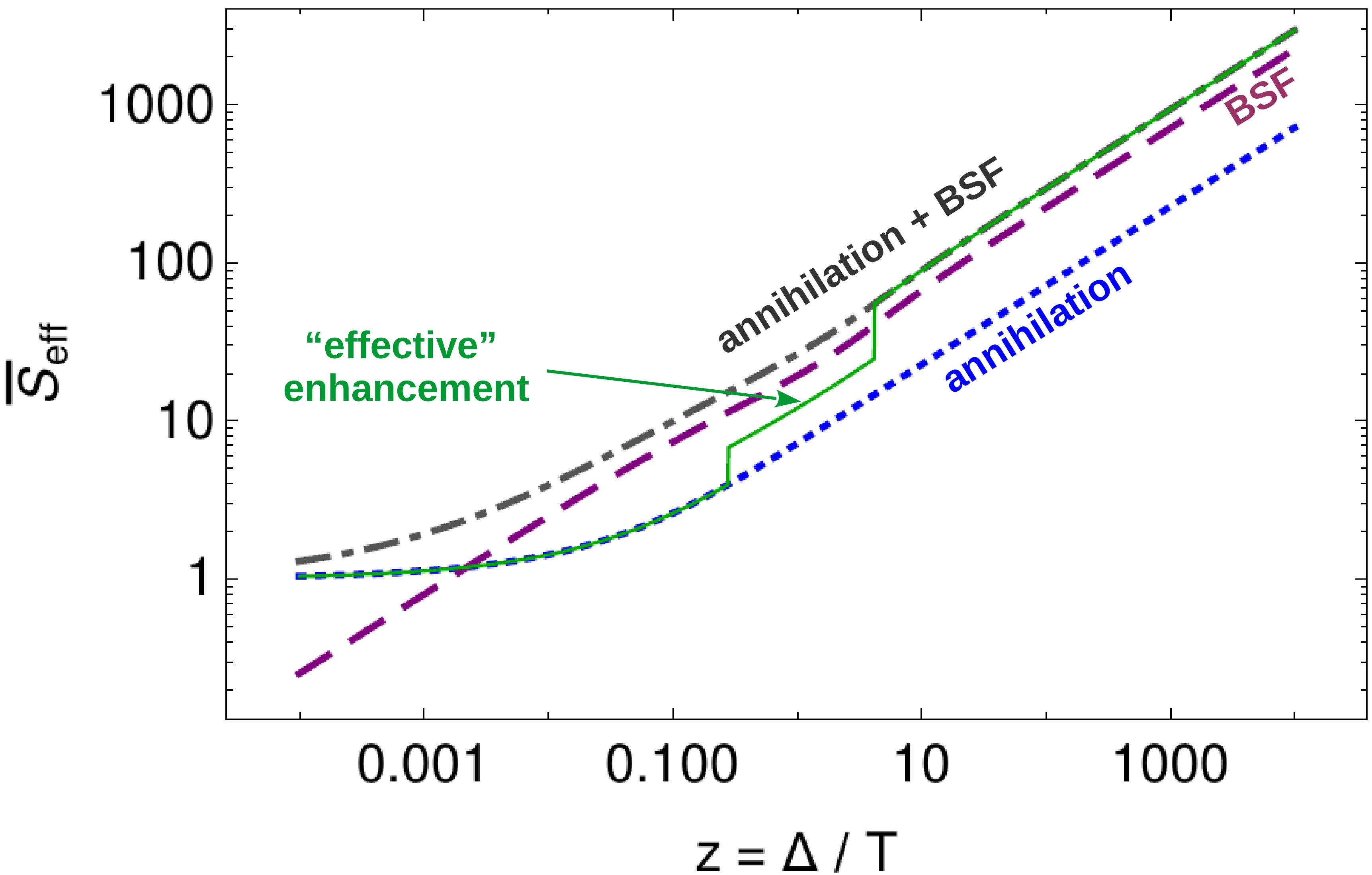
Para-positronium decay faster than ionisation (z=0.28)

Freeze-out:
DM density has departed from equilibrium value by O(1) factor [standard definition]

BSF becomes faster than annihilation: $z = 2.2 \times 10^{-3}$

Relic density of symmetric DM with long-range interactions
[von Harling, KP (2014)]

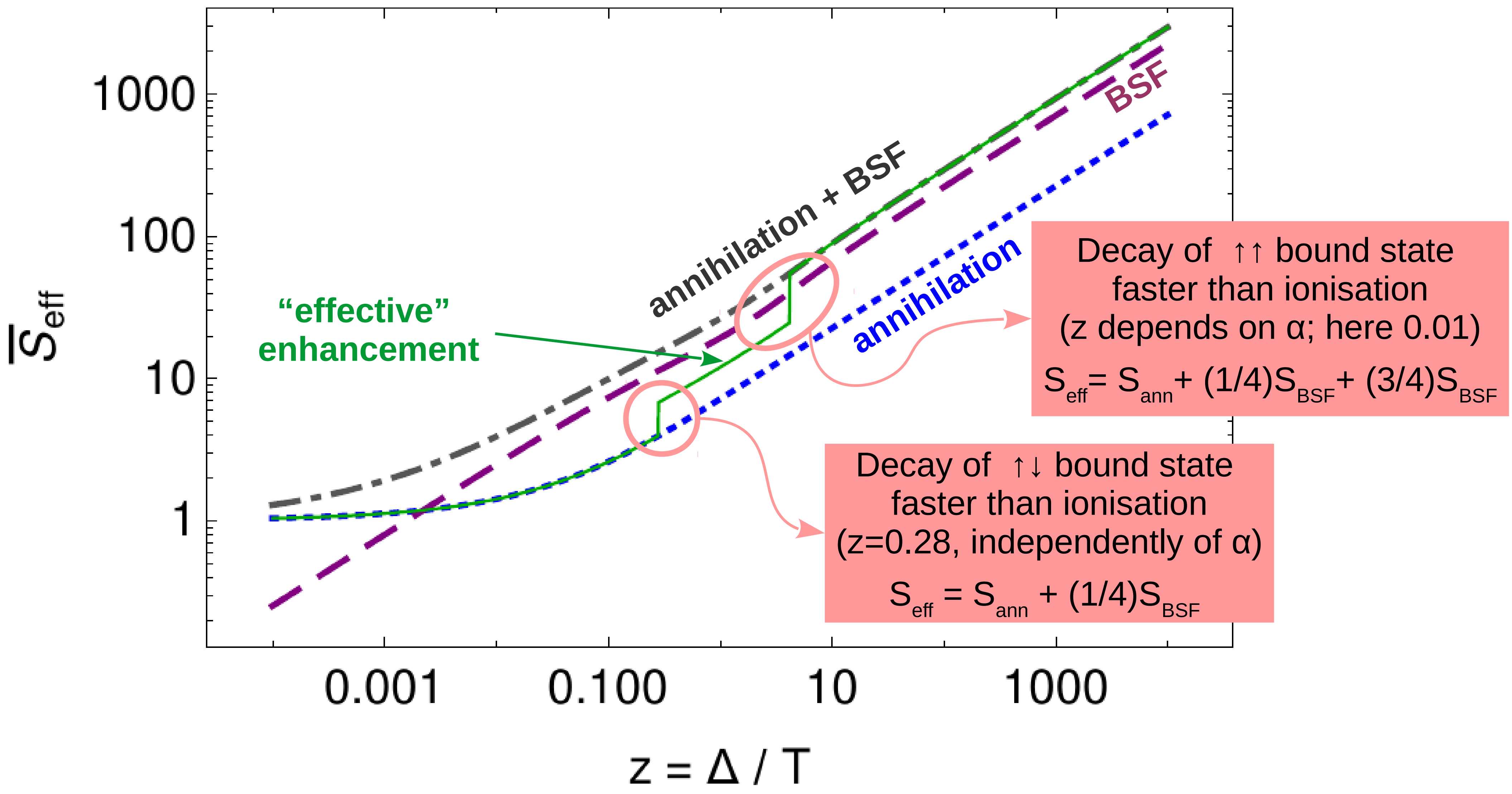
"Effective" enhancement



Relic density of symmetric DM with long-range interactions

[von Harling, KP (2014)]

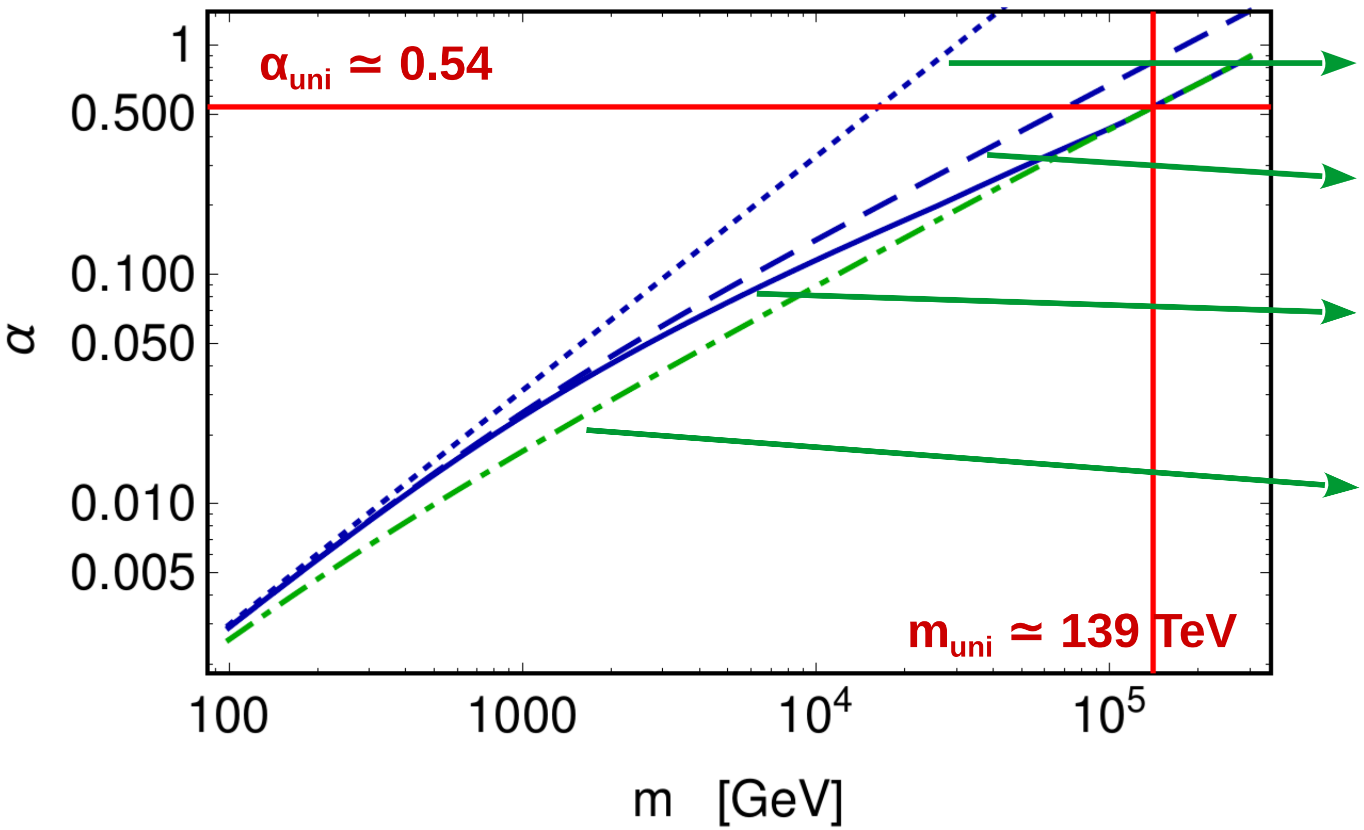
“Effective” enhancement



Relic density of symmetric DM with long-range interactions

[von Harling, KP (2014)]

Determination of $\alpha(m)$ or $m(\alpha)$



- Perturbative annihilation.
- Sommerfeld-enhanced annihilation.
- Sommerfeld-enhanced annihilation and BSF.
- Sommerfeld-enhanced annihilation and BSF, without ionisation.