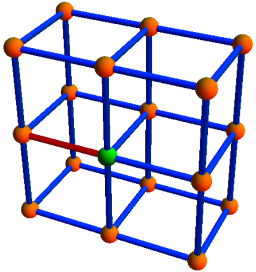
The challenge of supersymmetry on the lattice and the $\mathcal{N} = 1$ SuperYang-Mills theory



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4 November, 2013

Supersymmetry

The project

DESY-Münster collaboration:
$\mathcal{N} = 1$ supersymmetric Yang-Mills theory on the lattice *(Investigation of non-perturbative aspects of a supersymmetric Yang-Mills theory)*

Members of the collaboration:

- G. Münster (WWU), I. Montvay (DESY)
- G. Bergner (Frankfurt)
- P. Giudice (WWU)
- Phd students: U. D. Özugurel, S. Piemonte, D. Sandbrink (WWU)

Supersymmetry

The project

Recent publications:

- *G. Bergner, T. Berheide, I. Montvay, G. Münster, U. D. Özugurel, D. Sandbrink*: The gluino-glue particle and finite size effects in supersymmetric Yang-Mills theory, *JHEP 09 (2012) 108, arXiv:1206.2341 [hep-lat]*
- *S. Musberg, G. Münster, S. Piemonte*: Perturbative calculation of the clover term for Wilson fermions in any representation of the gauge group SU(N), *JHEP 05 (2013) 143, arXiv:1304.5741 [hep-lat]*
- *G. Bergner, I. Montvay, G. Münster, U.D. Özugurel and D. Sandbrink*: Towards the spectrum of low-lying particles in supersymmetric Yang-Mills theory, *arXiv:1304.2168 [hep-lat]*

Supersymmetry

Why do we study a supersymmetric theory?

Supersymmetry (SUSY) relates boson particles to fermion particles:

$$\begin{aligned} Q|\text{boson}\rangle &= |\text{fermion}\rangle \\ Q|\text{fermion}\rangle &= |\text{boson}\rangle \end{aligned}$$

SUSY appears as a challenge/solution to problems in:

1. Particle physics: hierarchy problem, Ads/Cft, ...
2. Astrophysics/thermodynamics: cosmological constant problem, quasi-conformal regime of QCD, ...
3. Theoretical aspect of field theory: divergences in QFT, formulation of SUSY on the lattice, ...

We try to study the limit of exact low energy SUSY on the lattice, but exact supersymmetry *is not* realized in nature ...

Supersymmetry

Why do we study a supersymmetric theory?

> *"Although supersymmetry holds the promise of being a fundamental symmetry in physics, we will study these theories not because they have an immediate application to particle physics, but because they provide a fascinating laboratory in which one can probe the limits of quantum field theory."*
>
> Michio Kaku in *Quantum Field theory, A modern introduction*, chapter 2 p. 57

Lattice simulations add to this "fascinating laboratory" an important non-perturbative tool.

Supersymmetry

The action

We study on the lattice the $\mathcal{N}=1$ SUSY based on gauge group $SU(2)$. In the continuum the action can be written as:

$$S=\int d^4x\left\{\frac{1}{4}(F^a_{\mu\nu}F^a_{\mu\nu})+\frac{1}{2}\bar{\lambda}_a\gamma^\mu D^{ab}_\mu\lambda_b\right\}$$

where λ is a Majorana fermion in the adjoint representation:

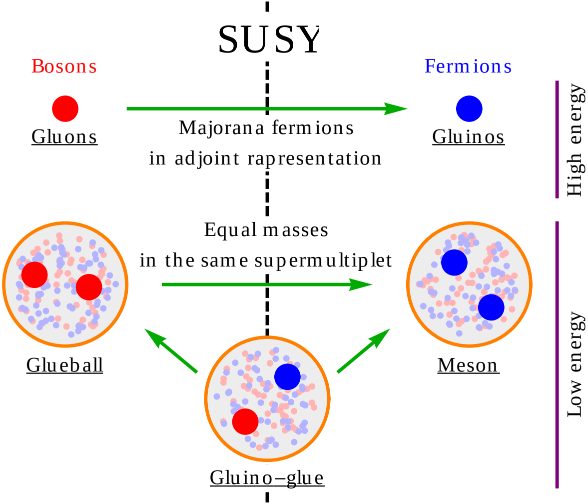
$$\begin{aligned}\bar{\lambda}_a &= \lambda_a^T C\\ D^{ab}_\mu\lambda_b &= \partial_\mu\lambda_a+igA^c_\mu(T^A_c)^{ab}\lambda_b\end{aligned}$$

The global supersymmetry interchanges boson gauge fields with fermion fields:

$$\begin{aligned}A_\mu &\rightarrow A_\mu-2i\bar{\lambda}\gamma_\mu\epsilon\\ \lambda^a &\rightarrow \lambda^a-\sigma_{\mu\nu}F^a_{\mu\nu}\epsilon\end{aligned}$$

Supersymmetry

The particle content



Supersymmetry

The particle content

The spectrum of low-lying bound states of the theory can be organized in two supermultiplets:

1. A higher energy chiral supermultiplet:[1]
   - $a-\eta'$: $\bar{\lambda}_a \gamma_5 \lambda_a$ a pseudoscalar meson 0^{-+}
   - $a-f_0$: $\bar{\lambda}_a \lambda_a$ a scalar meson 0^{++}
   - $g\tilde{g}$: $F^a_{\mu\nu}\sigma_{\mu\nu}\lambda_a$ the gluino-glue, a spin $1/2$ Majorana fermion
2. A lower energy chiral supermultiplet:[2]
   - gg: $F^{\mu\nu}F_{\mu\nu}$ a scalar glueball 0^{++}
   - gg: $\epsilon_{\mu\nu\rho\sigma}F^{\mu\nu}F^{\rho\sigma}$ a pseudoscalar glueball 0^{-+}
   - $g\tilde{g}$: $F^a_{\mu\nu}\sigma_{\mu\nu}\lambda_a$ a lower energy gluino-glue state

If exact supersymmetry is realized, then the masses in the two supermultiplets must be degenerate.

[1] G. Veneziano, S. Yankielowicz, Phys. Lett. B113 (1982) 231.

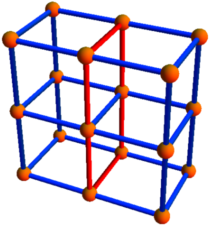
[2] G. R. Farrar, G. Gabadadze, M. Schwetz, Phys. Rev. D58 (1998) 015009 [arXiv:hep-th/9711166]

The lattice

Wilson loops and Dirac-Wilson operator

For performing Monte Carlo simulations, we introduce a finite lattice spacing a, but it breaks explicitly SUSY:

$$\{Q_\alpha, Q_\beta\} = (\gamma^\mu C)_{\alpha\beta} P_\mu$$



The propagation of a gluino in the space-time is related to the inverse of the Dirac-Wilson operator D_W:

$$S_f = \frac{1}{2}\bar{\lambda}(D_W[V_\mu] + m)\lambda$$

The mass $m \neq 0$ is a renormalization parameter of the theory. The link V_μ in D_W are in the adjoint representation:

$$V_\mu(x)_{ab} = 2\mathrm{Tr}(U_\mu^\dagger(x) T_a^F U_\mu(x) T_b^F)$$

The lattice
Questions

Is supersymmetry restored in the continuum limit?

Fine tuning of (g, m) to the critical point in order to recover both Lorentz symmetry and zero quark mass needed by SUSY.

In the computer simulations, we perform this limit for:

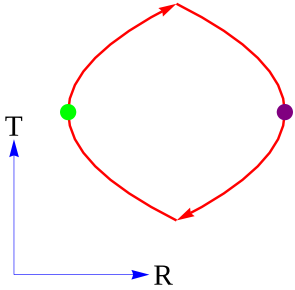
- Different couplings: $\beta = \frac{2N_c}{g^2} = \{1.6, 1.79, 1.9\}$
- Different masses: $\kappa = \frac{1}{2(m+4)} = \{0.135 - 0.137\}$
- Different lattice sizes: $N_s^3 \times N_t = \{16^3 \times 32, 24^3 \times 48, 32^3 \times 64\}$

looking for the degeneracy of the masses in the supermultiplets.

The lattice

Wilson loops and Dirac-Wilson operator

A meson can be constructed "naively" from a separation in space-time of a quark-antiquark pair in a quantum gauge background field:



The arrow represents the parallel transporters $W[\gamma] \in SU(N)$:

$$W[\gamma] = \prod_{x \in \gamma} (1 + A_\mu(x) dx) \equiv \exp\left\{ \oint_\gamma A_\mu(x) dx \right\}$$

The lattice

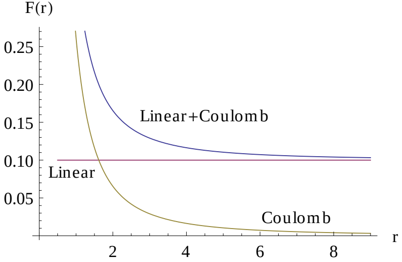
Wilson loops and Dirac-Wilson operator

Setting the scale

Pion mass and static potential

In order to extrapolate to continuum limit, the lattice spacing a and the gluino mass m_g have to be estimated.

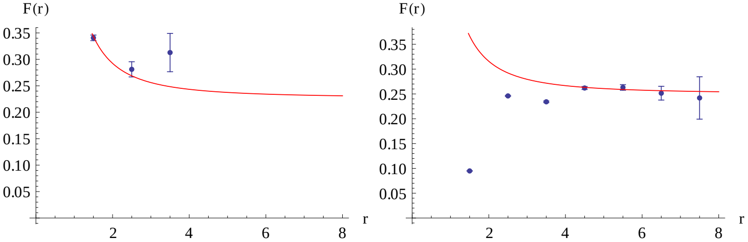
1. The gluino mass m_g is extracted from the pion mass m_π, defined in a partially quenched approach, as: $m_g \propto m_\pi^2$
2. The lattice spacing is extracted from the static quark potential $V(r) = -\sigma r + \mu + \frac{c}{r} + \frac{d}{r^2} \ldots$. From the effective string model $c \simeq \frac{\pi}{12}$.



Setting the scale

Static potential

On the lattice the static potential has noisy signal and big systematical errors:



(a) Without smearing, lattice 16^4 $\beta = 1.65$ $\kappa = 0.16$, $a^2\sigma = 0.227(5)$

(b) With 5 level stout-smearing, $\epsilon = 0.25$, $a^2\sigma = 0.250(4)$

Sommer scale r_0 defined as the point where $r_0^2 F_L(r_0) = 1.5$.[3]

[3] R. Sommer,: Nucl. Phys. B411 (1994) 839

Setting the scale

Can we do better? Wilson/Symanzik flow

Diffusion equations can be written naturally on the gauge fields using the action:[4]

$$\frac{\partial}{\partial \tau} V(x,\tau) = -g_0^2 \left(\frac{\partial S_g(V_\mu(x,\tau))}{\partial V_\mu(x,\tau)} \right) V_\mu(x,\tau)$$

in terms of an additional fifth temporal dimension τ, with boundary conditions $V(x,\tau)|_{\tau=0} = U(x)$. The diffusion process provides quantity "automatically renormalized" for every $\tau > 0$.

The idea is to set the scale using the time t_0 when the "gauge energy" reaches some specified value:

$$t_0^2 \langle E(t_0) \rangle = t_0^2 \langle F_{\mu\nu}^2(t_0) \rangle = 0.3$$

[4] M. Luscher: JHEP 1008 (2010) 071, [arXiv:1006.4518]

Setting the scale

Can we do better? Wilson/Symanzik flow

The calculation of t_0 requires only a linear fit of the flow equations, less systematical and less statistical errors!

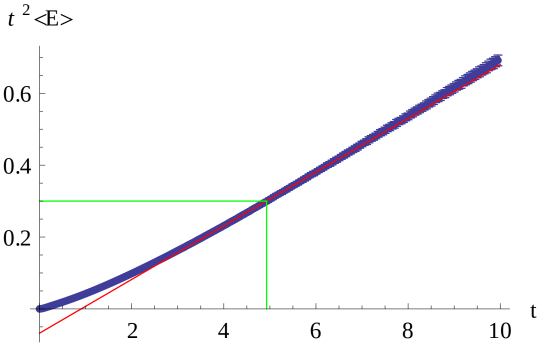


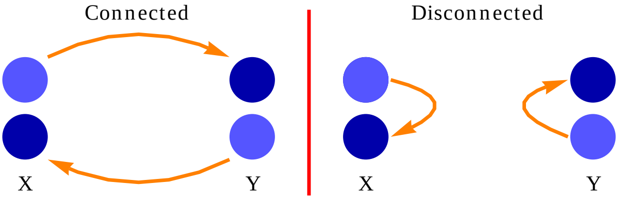
Figure : Symanzik flow for 16^4 $\beta = 1.65$ $\kappa = 0.16$: $\sqrt{t_0}/a = 2.22(1)$

Lattice SUSY

Physical particles - Mesons

The meson masses are extracted from the correlators like:

$$
\begin{aligned}
C_{a-\pi} &= \frac{1}{L^3}\sum_x \langle \mathrm{Tr}\left(\gamma_5 (D_W^{-1})_{xy}\gamma_5 (D_W^{-1})_{yx}\right)\rangle \\
C_{a-\eta} &= \frac{1}{L^3}\sum_x \langle \mathrm{Tr}\left(\gamma_5 (D_W^{-1})_{xy}\gamma_5 (D_W^{-1})_{yx}\right)\rangle \\
&\quad -\frac{1}{2L^3}\sum_x \langle \mathrm{Tr}\left(\gamma_5 (D_W^{-1})_{xx}\right) \mathrm{Tr}\left(\gamma_5 (D_W^{-1})_{yy}\right)\rangle
\end{aligned}
$$

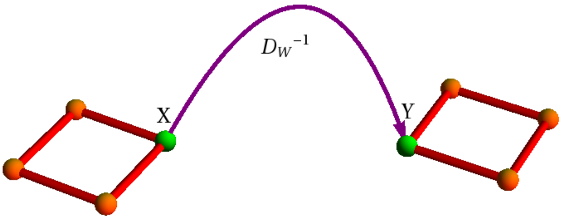


Lattice SUSY

Physical particles - Gluino-glue

The gluino-glue particles is not present in standard QCD, typical only of adjoint models:

$$C_{g\tilde{g}} = -\frac{1}{4}\sum\langle\sigma_{ij}^{\alpha\beta}\mathrm{Tr}(P_{ij}(x)\tau^{a})(D_W^{-1})_{xa\beta;yb\rho}\mathrm{Tr}(P_{kl}(y)\tau^{b})\sigma_{kl}^{\alpha\rho}\rangle$$



There are no disconnected contributions, good signals.

Lattice SUSY

Physical particles - Glueballs

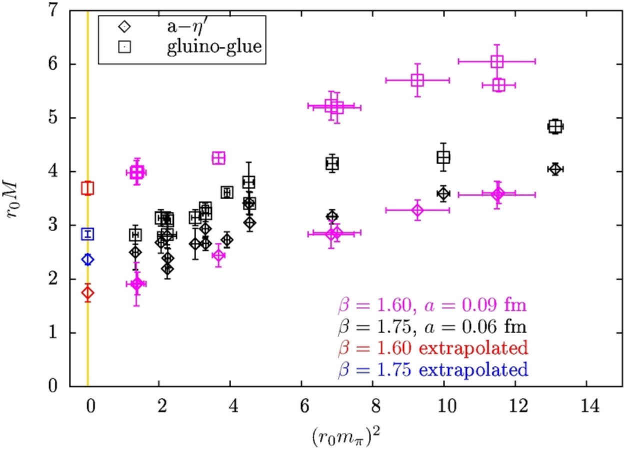
Glueballs are extracted from Wilson loop connected correlators:

$$C_{gg} = \langle W(x)W(y)\rangle - \langle W(x)\rangle\langle W(y)\rangle$$

They usually have bad signals, smearing is used to reduce the noise and the excited states contribution.

Lattice SUSY

Physical particles - The supersymmetric limit[5]



[5] G. Bergner, T. Berheide, I. Montvay, G. Münster, U. D. Özugurel, D. Sandbrink: JHEP 09 (2012) 108, arXiv:1206.2341 [hep-lat]

Lattice SUSY

Source of Supersimmetry breaking

What are the possible sources of SUSY breaking in our simulations?

1. Finite volume - under control
2. Boundary conditions - to be tested
3. Finite lattice spacing - to be further reduced

The Wilson fermion action has an discretization error proportional to $O(a)$:

$$\lim_{a \to 0} \frac{m_{a-\eta'}(a)}{m_{a-f_0}(a)} = \frac{m^c_{a-\eta'}}{m^c_{a-f_0}}(1 + O(a))$$

in contrast to the pure gauge theory with Wilson action, where the error scales as $O(a^2)$.

Symanzik program

The clover term

The Symanzik program reduces the discretization error $O(a)$ order by order in perturbation theory adding irrelevant operators to the Lagrangian:

$$\begin{aligned} \mathcal{L} &= \mathcal{L}_0 - a\frac{c_{SW}}{4}O_{CL} \\ O_{CL} &= \bar{\lambda}\sigma_{\mu\nu}F^{\mu\nu}\lambda \end{aligned}$$

The Sheikholeslami-Wohlert coefficient:[6]

$$c_{SW} = c_{SW}^0 + c_{SW}^1 g^2 + \dots$$

can be tuned similarly to the QCD action, requiring no $O(a)$ errors for on-shell quantities (like the gluino-gluino scattering cross section).[7]

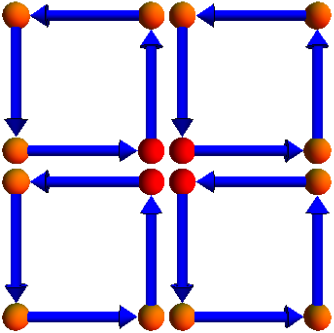
[6] B. Sheikholeslami and R. Wohlert: Nucl. Phys. B 259 (1985) 572.

[7] S. Aoki and Y. Kuramashi: Phys. Rev. D 68 (2003) 094019 [hep-lat/0306015]

Symanzik program

The clover term

The operator $\bar{\lambda}\sigma_{\mu\nu}F^{\mu\nu}\lambda$ is "called clover" term from the shape depicted by the lattice version of $F^{\mu\nu}$:



Symanzik program

The clover term

The gluino-gluino-gluon vertex operator has the generic form:

$$\Lambda(p,p')^a_{\mu;cd} = g\left(i\gamma_\mu A + g\frac{a}{2}(p+p')_\mu(B-c_{SW}) + \right.$$
$$\left. + O(p^2,p'^2) + O(a^2)\right)(T^a_R)_{cd}$$

At tree level $A = 1 + O(g^2)$ and $B = 1 + O(g^2)$, the terms proportional to a vanish if the clover term is set one:

$$c^0_{SW} = 1$$

independent from the representation of the fermions and from their number of degrees of freedom (Dirac/Majorana).[8]

[8] This is the unique choice that ensures the cancellation of the infrared divergences from the gauge theory.

Symanzik program

The clover term

At one loop, in the Feynman diagrams traces and sums of the group generators will introduce a representation dependent contribution to B:

$$
\begin{aligned}
B &= 1 + g^2(0.16764(3)C_R + 0.01503(3)N) \\
C_R\delta_{bc} &= \sum_a (T_a^R T_a^R)_{bc}
\end{aligned}
$$

Therefore the clover term has to be fixed to:

$$c_{SW}^1 = 0.16764(3)C_R + 0.01503(3)N$$

independently from the number of degrees of freedom of the fermion (Dirac/Majorana).

Symanzik program

The clover term

The perturbative result is in agreement with the non-perturbative determination of the clover term for $SU(2)$ adjoint model:[9]

$$c_{SW}(g) = \frac{1 + 0.032653 g^2 - 0.002844 g^4}{1 - 0.314153 g^2}$$

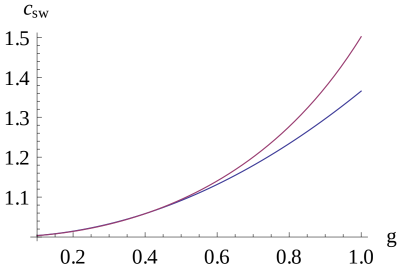


Figure: Comparison of the perturbative (blue line) vs non-perturbative (purple line) estimation of c_{sw}

[9] T. Karavirta, K. Tuominen, A. Mykkanen, J. Rantaharju and K. Rummukainen: PoS(Lattice 2010)064 [arXiv:1011.1781]

SUSY Thermodynamic

Why should we study SUSY at finite temperature?

"It is plausible that finite temperature QCD and finite temperature supersymmetric gauge theory are not all that different."

Barton Zwiebach in *An introduction to String Theory*

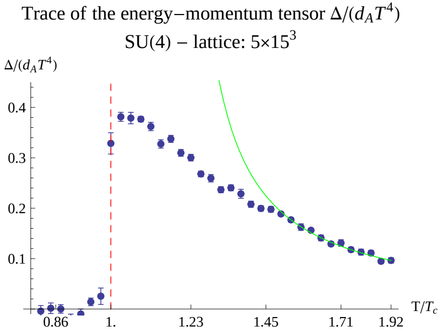
SUSY is broken at finite temperature, fermions and gluons "feel differently" the temperature ...

... that's why it could be possible that SUSY and QCD share the same properties at finite temperature.

Finite temperature pure-gauge SU(N)

The quasi-conformal regime

There are evidence for a strongly interacting quark-gluon plasma above T_c in QCD:



How can we explain it? Limits of the Ads/Cft correspondence.

Finite temperature SUSY

The Polyakov loop

The Polyakov loop P_L:

$$P_L = \frac{1}{V_s} \sum_x \prod_{t=1}^{N_t} U_4(x, t)$$

is the order parameter of the deconfinement transition. Unlike QCD, in $\mathcal{N} = 1$ SUSY the center simmetry of P_L is exact for any value of the gluino mass m_g:

$$U_4(x, t) \to \exp\left(i\frac{2\pi}{N_c}n\right) U_4(x, t)$$

since the fermion part of the action contains on links in the adjoint representation.

The deconfinement transition

The order of the deconfinement transition

The Svetitsky-Yaffe conjecture[10] could be still valid in SUSY, therefore for the gauge group $SU(2)$ we would espect a second order phase transition.

How can this be tested?

1. Finite size scaling: check the behaviour of the correlation length ξ
2. Is there a coexistence of many phases? If yes, the transition is of first order
3. Behaviour of the binder cumulant $B_4 = 1 - \frac{\langle P_L^4 \rangle}{3\langle P_L^2 \rangle^2}$: parameter that contains information about the shape of the distribution of the order parameter.

[10] B.Svetitsky and L.G.Yaffe, Nucl. Phys. B210 (1982) 423

The deconfinement transition

Landau effective potential for a second order phase transition in SU(2)

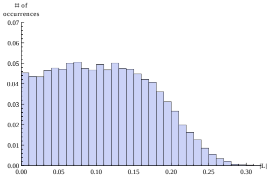
The deconfinement transition

Landau effective potential for a first order phase transition in SU(2)

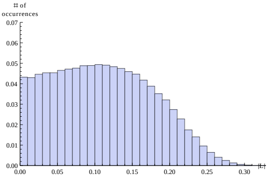
The deconfinement transition

The results for the Polyakov loop distribution

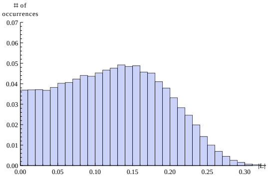
Polyakov loop distribution for the lattice $8^3 \times 4$ and $\beta = 1.65$:



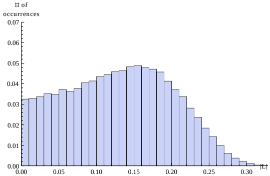
(a) $\kappa = 0.135$



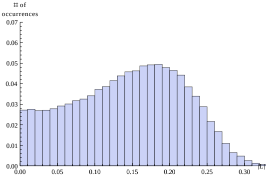
(b) $\kappa = 0.140$



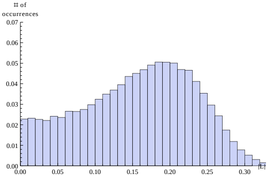
(c) $\kappa = 0.145$



(d) $\kappa = 0.150$



(e) $\kappa = 0.155$

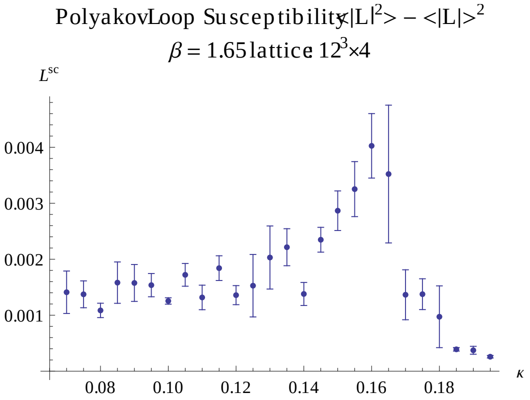


(f) $\kappa = 0.160$

The deconfinement transition

The results for the Polyakov loop distribution

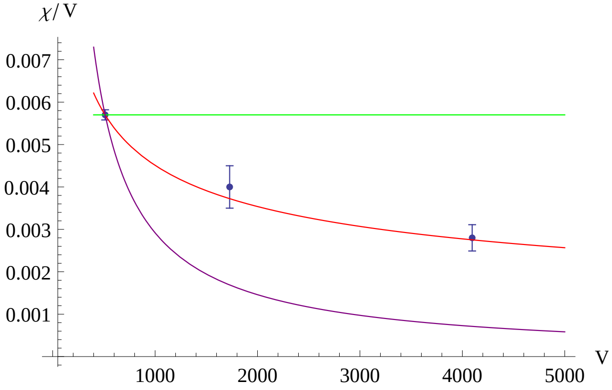
Polyakov loop susceptibility for the lattice $12^3 \times 4$ and $\beta = 1.65$:



The deconfinement transition

The finite size scaling

The finite size scaling is in good agreement with a Ising Z_2 second order phase transition:



Supersymmetry

The chiral symmetry

The action for $\mathcal{N} = 1$ SUSY:

$$S = \int d^4x \left\{ \frac{1}{4}(F^a_{\mu\nu} F^a_{\mu\nu}) + \frac{1}{2}\bar{\lambda}_a \gamma^\mu D^{ab}_\mu \lambda_b - \frac{\Theta}{16\pi} \epsilon_{\mu\nu\rho\sigma} F^{\mu\nu} F^{\rho\sigma} \right\}$$

has also a "classical" QCD-like $U(1)_A$ axial symmetry:

$$\bar{\lambda}\gamma_\mu\lambda = \bar{\lambda} e^{(-i\omega\gamma_5)} \gamma_\mu e^{(-i\omega\gamma_5)} \lambda = \bar{\lambda}\gamma_\mu e^{(+i\omega\gamma_5)} e^{(-i\omega\gamma_5)} \lambda = \bar{\lambda}\gamma_\mu\lambda$$

referred in SUSY as $U(1)_R$. At quantum level chiral symmetry is anomalously broken:

$$\partial_\mu J^\mu_5 = \partial_\mu(\bar{\lambda}\gamma^\mu\gamma_5\lambda) = N_c \frac{g^2}{32\pi^2} \epsilon_{\mu\nu\rho\sigma} F^{\mu\nu} F^{\rho\sigma}$$

Supersymmetry

The Θ parameter

The Θ parameter is periodic:

$$\Theta = \Theta + 2n\pi$$

and related to the topological charge, usually it is assumed $\Theta = 0$.
There is still a Z_{2N_c} unbroken by anomaly, because the transformation:

$$\begin{aligned} \lambda &\rightarrow e^{(-i\omega\gamma_5)}\lambda \\ \Theta &\rightarrow \Theta - 2N_c\omega \end{aligned}$$

leaves the partition function invariant if $\omega = \frac{n\pi}{N_c}$, $n : 0...2N_c - 1$, due to the Θ periodicity.

Supersymmetry

The chiral condensate

The Z_{2N_c} symmetry is broken spontaneously to Z_2, it is observed a non zero expectation of the gaugino chiral condensate:

$$\langle \bar{\lambda}\lambda \rangle = \left\langle \mathrm{Tr}(D^{-1}) \right\rangle$$

$\langle \bar{\lambda}\lambda \rangle$ is not symmetric under the transformation $\lambda \to e^{\left(-i\frac{n\pi}{N_c}\gamma_5\right)}\lambda$, but only under a remaining "sign flip" $\lambda \to -\lambda$. Therefore the full phase structure is:[11]

$$U(1)_A \to Z_{2N_c} \to Z_2$$

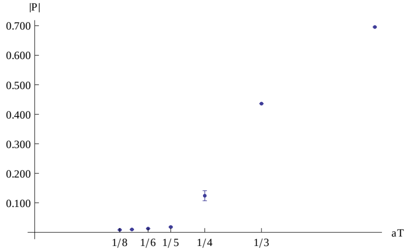
This picture is strictly true only for $M_g = 0$, otherwise a simple crossover is expected.

[11] R. Kirchner, S. Luckmann, I. Montvay, K. Spanderen, J. Westphalen: Phys. Letters B 446 (1999) 209, [arXiv:hep-lat/9810062]

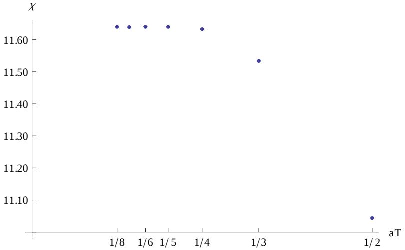
Supersymmetry

The chiral condensate

Up to the maximal temperature available, there is no signal for a chiral phase transition:



(g) Polyakov Loop

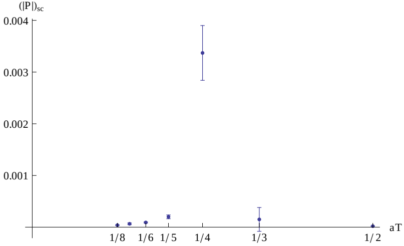


(h) Bare Chiral Condensate

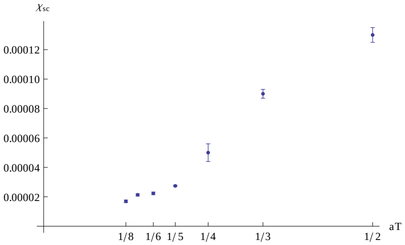
Supersymmetry

The chiral condensate

Up to the maximal temperature available, there is no signal for a chiral phase transition:



(i) Polyakov Loop Susceptibility



(j) Disconnected Chiral Susceptibility

There is a clear deconfinement transition at $T_{dec} \simeq 234(3)MeV$, but $T_{chi} > 467MeV$.

Conclusions

The challenge of supersymmetry on the lattice

For the $\mathcal{N} = 1$ supersymmetric Yang-Mills theory on the lattice:

- The lowest masses of the spectrum are already estimated with reasonable precision
- A previously observed mass splitting between the gluino-glue and its possible superpartners is significantly reduced in the latest results
- The clover coefficient c_{sw} is computed to one-loop order
- At finite temperature, there is a clear signal of a deconfinement phase transition

Future works:

- Small pion mass thermodynamics
- Possible procedure for non-perturbative improvements for SUSY?
- Test the energy as the order parameter for SUSY breaking