Lattice simulation of gauge theories with one fermion species: $\mathcal{N} = 1$ super Yang-Mills and one-flavor QCD

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Westfälische Wilhelms-Universität Münster

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- Eur. Phys. J. C52, 305, 2007 (arXiv:0706.1131)
- Eur. Phys. J. C69, 147, 2010 (arXiv:1003.2073)
- PoS (Confinement 2008) 136 (arXiv:0811.1964)
- PoS (LATTICE 2008) 061 (arXiv:0810.0144)
- PoS (LATTICE 2008) 128 (arXiv:0810.0161)
- PoS (LATTICE 2009) 268 (arXiv:0911.0595)
- PoS (LATTICE 2010) (arXiv:1011.0952)

Plan of the talk

1. $\mathcal{N} = 1$ super Yang-Mills
2. One-flavor QCD
3. Lattice formulation and simulation
4. Results on the low-lying bound state spectrum:

   One-flavor QCD and super Yang-Mills with two colors

1. $\mathcal{N} = 1$ super Yang-Mills

$\mathcal{N} = 1$ super Yang-Mills theory (SYM)

Vector superfield:

$$V^a(x) = -\theta\sigma^\mu\bar{\theta}A^a_\mu(x) + \underbrace{i\theta\theta\bar{\theta}\bar{\lambda}^a(x) - i\bar{\theta}\bar{\theta}\theta\lambda^a(x)}_{\text{gluino field}} + \frac{1}{2}\underbrace{\theta\theta\bar{\theta}\bar{\theta}D^a(x)}_{\text{auxiliary field}}$$

Component fields in the adjoint representation of $SU(N_c)$

Spinor field-strength superfield:

$$W_\alpha(x) = -\frac{1}{4}\bar{\mathcal{D}}\bar{\mathcal{D}}\mathrm{e}^{-V(x)}\mathcal{D}_\alpha\mathrm{e}^{V(x)} \qquad (V(x) = V^a(x)T^a)$$

$\rightarrow$ Action:

$$\mathcal{L}_{\mathrm{SYM}} = \frac{1}{2g_0^2}\int d^2\theta\,\mathrm{Tr}(W^\alpha W_\alpha) + c.c.$$

Majorana gluino field: $\Psi^a = \begin{pmatrix} \lambda^a \\ \epsilon\,\lambda^{*\,a} \end{pmatrix} \quad a = 1, \ldots, N_c^2 - 1$

$$\mathcal{L}_{\mathrm{SYM}} = -\tfrac{1}{4} F^a_{\mu\nu} F^{a\,\mu\nu} + \tfrac{i}{2}\,\bar{\Psi}^a \not{D}^{ab}_{adj} \Psi^b$$

Invariance under SUSY transformations / SUSY Ward identities:

$$\begin{aligned} A^a_\mu &\rightarrow A^a_\mu + i\,\bar{\xi}\gamma_\mu \Psi^a \\ \Psi^a &\rightarrow \Psi^a + \tfrac{1}{2}\,F^a_{\mu\nu}\sigma^{\mu\nu}\xi \end{aligned}$$

$$\partial^\mu \underbrace{\left(-\frac{1}{2}\,F^a_{\nu\rho}\sigma^{\nu\rho}\gamma_\mu \Psi^a\right)}_{\text{Supercurrent}} = 0$$

$\mathcal{N} = 1$ super Yang-Mills theory (SYM)

Majorana gluino field: $\Psi^a = \begin{pmatrix} \lambda^a \\ \epsilon\, \lambda^{*\,a} \end{pmatrix} \qquad a = 1, \ldots, N_c^2 - 1$

$$\mathcal{L}_{\rm SYM} = -\tfrac{1}{4} F^a_{\mu\nu} F^{a\,\mu\nu} + \tfrac{i}{2}\, \bar{\Psi}^a \not{\mathcal{D}}^{ab}_{adj} \Psi^b - \tfrac{1}{2}\, m_{\tilde{g}}\, \bar{\Psi}^a \Psi^a$$

Soft breaking

Invariance under SUSY transformations / SUSY Ward identities:

$$\begin{aligned} A^a_\mu &\rightarrow A^a_\mu + i\,\bar{\xi}\gamma_\mu \Psi^a \\ \Psi^a &\rightarrow \Psi^a + \tfrac{1}{2}\, F^a_{\mu\nu}\sigma^{\mu\nu}\xi \end{aligned}$$

$$\partial^\mu \underbrace{\left(-\frac{1}{2}\, F^a_{\nu\rho}\sigma^{\nu\rho}\gamma_\mu \Psi^a \right)}_{\text{Supercurrent}} = -\tfrac{i}{2}\, m_{\tilde{g}}\, F^a_{\mu\nu}\sigma^{\mu\nu}\Psi^a$$

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Non-perturbative aspects:

- Spontaneous breaking of a discrete chiral symmetry
- Color confinement, bound states

Chiral Symmetry $U_R(1)$ (R-Symmetry)

$$\Psi \;\rightarrow\; \mathrm{e}^{i\alpha\gamma^5}\,\Psi$$

i. Anomalous Ward identities:

$$\partial_\mu(\bar\Psi\gamma^\mu\gamma_5\Psi) \;=\; 2N_c\,\tfrac{g_0^2}{32\pi^2}F^a_{\mu\nu}\tilde F^{a\,\mu\nu}$$

$$\boxed{U_R(1) \stackrel{\text{anomaly}}{\longrightarrow} Z_{2N_c}}$$

ii. Spontaneous discrete symmetry breaking:

$$\boxed{Z_{2N_c} \longrightarrow Z_2}$$

- N_c Z_2-invariant vacua, $k = 0, 1, \cdots, N_c - 1$
- Gluino condensate: $\langle\bar\psi\psi\rangle \;=\; C\Lambda^3\mathrm{e}^{i2\pi k/N_c}$

i. Confinement (assumed as in QCD).

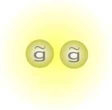
ii. Adjoint fermion: gauge invariant composite fields can be built by taking the color trace of products of any number of gluino and gluon fields:

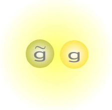
$$O = \mathrm{Tr}[\Psi_{\alpha_1} \cdots \Psi_{\alpha_m} F_{\mu_1\nu_1} \cdots F_{\mu_n\nu_n}] \; .$$

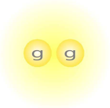
iii. Colorless bound states should form supermultiplets.

Basic degree of freedom at low energies: [Veneziano and Yankielowicz (1982)]

$$S(x,\theta) \sim \underbrace{\bar{\Psi}^a_R \Psi^a_L}_{\text{Guino-Gluino}} + i\theta \underbrace{F^a_{\mu\nu}\sigma^{\mu\nu}\Psi^a_L}_{\text{Gluino-Glue}} + \frac{1}{2}\theta^2 \underbrace{\left(F^a_{\mu\nu}F^{a\mu\nu} + i\tilde{F}^a_{\mu\nu}F^{a\mu\nu}\right)}_{\text{Glue-Glue}} + \cdots$$





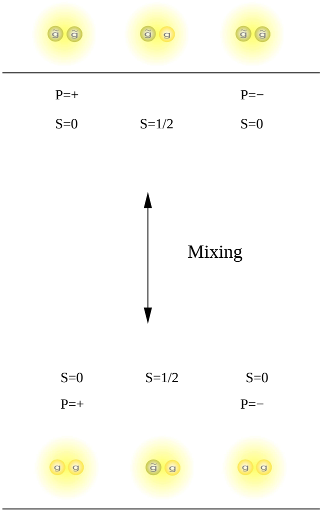


"Mesonic"	"Exotic"	"Glueballs"
S=0	S=1/2	S=0

Adjoint η $(P = -)$

Adjoint σ $(P = +)$

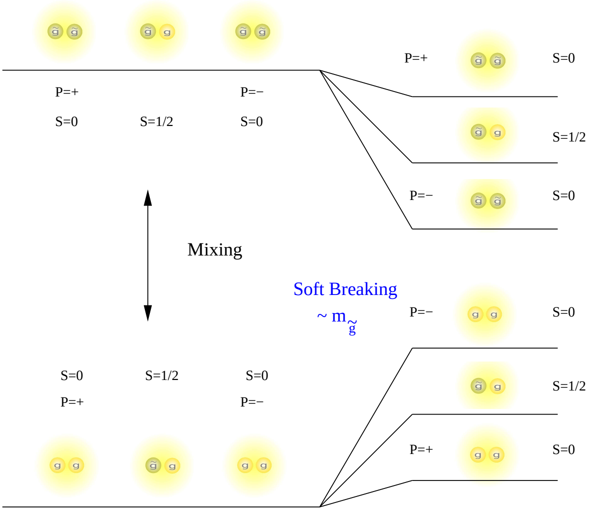
Two Wess-Zumino supermultiplets: [Farrar, Gabadadze and Schwetz (1998,1999)]



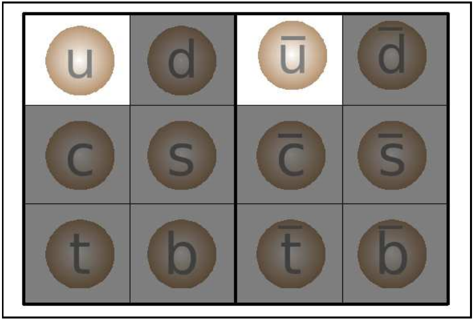
Low-energy mass spectrum

Two Wess-Zumino supermultiplets:

[Farrar, Gabadadze and Schwetz (1998,1999)]



2. One-flavor QCD



One-flavor QCD (1F-QCD)

One (complex) quark field:

$$\psi^i = \left(\begin{array}{c} \eta^i \\ \epsilon\, \xi^{*\,i} \end{array} \right) \qquad i = 1, \ldots, N_c$$

$$\mathcal{L}_{\text{1F-QCD}} = -\frac{1}{4} F^a_{\mu\nu} F^{a\,\mu\nu} + i\, \bar{\psi}^i \not{D}^{ij}_{fun} \psi^i - \; m_q\, \bar{\psi}^i \psi^i$$

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Chiral Symmetry $U_V(1) \times U_A(1)$

$$\left\{ \begin{array}{l} \psi \longrightarrow \mathrm{e}^{i\alpha}\,\psi \\ \bar\psi \longrightarrow \bar\psi\,\mathrm{e}^{-i\alpha} \end{array} \right. \quad \times \quad \left\{ \begin{array}{l} \psi \longrightarrow \mathrm{e}^{i\alpha\gamma_5}\,\psi \\ \bar\psi \longrightarrow \bar\psi\,\mathrm{e}^{i\alpha\gamma_5} \end{array} \right.$$

$$\boxed{U_V(1) \times U_A(1) \stackrel{\text{anomaly}}{\longrightarrow} U_V(1)}$$

Anomalous Ward identities:

$$\partial_\mu(\bar\psi\gamma^\mu\gamma_5\psi) \;=\; 2m_q\bar\psi\gamma_5\psi + 2N_f\,\frac{g_0^2}{32\pi^2}F^a_{\mu\nu}\tilde F^{a\,\mu\nu}$$

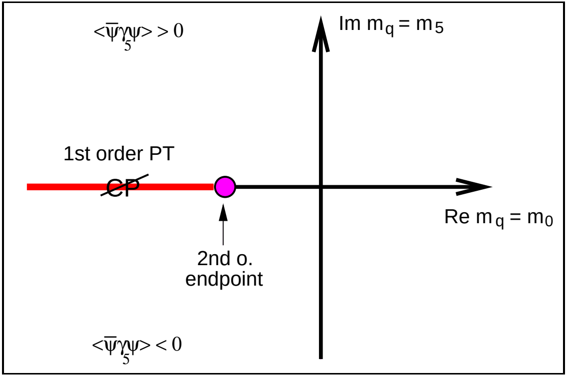
No new symmetry for $m_q = 0$!

$\longrightarrow$ Massless limit physically not well defined ? [M. Creutz (2004)]

Cf. with SYM:

$$U_A(1) \stackrel{\text{anomaly}}{\longrightarrow} Z_{2N_c} \stackrel{\text{condensate}}{\longrightarrow} Z_2$$

Add explicit CP breaking term in the Lagrangian: $\mathcal{L}_{m_5} = im_5 \bar{\psi}\gamma_5\psi$

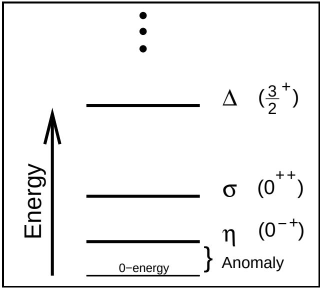


[M. Creutz (1995)]

Gauge invariant projecting operators:

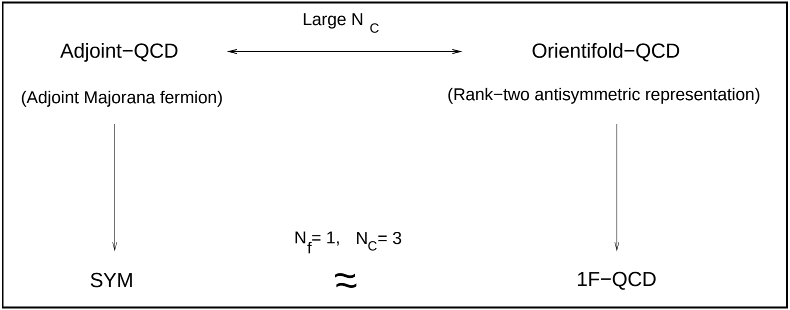
- Two $S = 0$ Mesons (σ,η): $\bar{\psi}\psi$, $\bar{\psi}\gamma_5\psi$
- One $S = 3/2$ Baryon (Δ^{++}): $\epsilon_{c_1 c_2 c_3}[\psi^T_{c_1} C\gamma_i \psi_{c_2}]\,\psi_{c_3}$

t'Hooft large N_c limit: $M^2_{\eta'} = \dfrac{4N_f}{F^2_{\eta'}}\chi_t$ [Witten; Veneziano (1979)]



Orientifold equivalence

[Armoni, Shifman and Veneziano (2003)]



Relics of SUSY in 1F-QCD:

- Scalar condensate

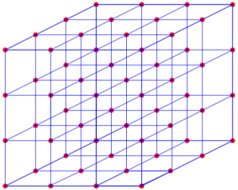
[Armoni, Shifman and Veneziano (2003); DeGrand, Hoffmann, Liu and Schäfer (2006) Armoni, Lucini, Patella and Pica (2008)]

- Meson masses:

$$\frac{m_\eta}{m_\sigma} = \frac{N_c - 2}{N_c} + O\left(\frac{1}{N_c}, \frac{1}{N_c^2}\right)$$

[Armoni and Imeroni (2005)]

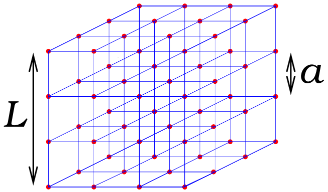
3. Lattice formulation and simulation



Gauge theory: theory of local fields:

$\psi(x)$, $A_\mu(x)$, $F_{\mu\nu}(x)$, $D_\mu\psi(x)$ ...

- Euclidean formulation
- Space-time coordinate restricted to a 4d spacetime-lattice:



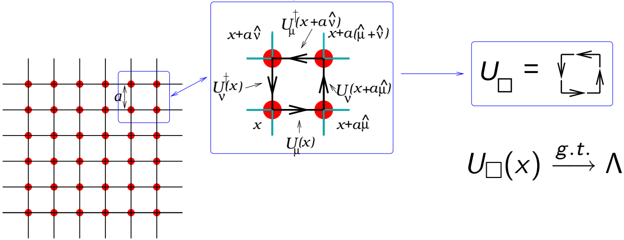
$$\Lambda = \{x_\mu = n_\mu a\,,\ n_\mu \in \mathbf{Z}\}\ , \quad p_\mu < \frac{\pi}{a}$$

Exact gauge invariance: [K. Wilson (1974)]

Gauge field $A_\mu(x) \xrightarrow{lattice} \overset{\text{“Link”}}{U_\mu(x)} = P e^{i g_0 \int_x^{x+a\hat{\mu}} dx_\mu \, A_\mu(x)} \in SU(N_C)$ (Color)

Gauge transformation: $U_\mu(x) \xrightarrow{g.t.} \Lambda(x) U_\mu(x) \Lambda^\dagger(x+\hat{\mu})$

Plaquette



$U_\square(x) \xrightarrow{g.t.} \Lambda(x) U_\square(x) \Lambda^\dagger(x)$

Gauge action: $\boxed{S^{Wil}_{Glue} = \sum_\square \beta \left(1 - \frac{1}{N_c} \mathrm{Tr}_{Color} U_\square\right)}$ $\left(\beta = \frac{2N_c}{g_0^2}\right)$

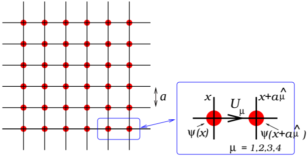
$\xrightarrow{a \to 0} -\frac{1}{4 g_0^2} \int dx (F^a_{\mu\nu}(x))^2$

Wilson action

Covariant derivative (fundamental):

$$D_\mu[A]\psi(x) \xrightarrow{lattice} \nabla_\mu[U]\psi(x) \equiv \frac{1}{a}\left(U_\mu(x)\psi(x+\hat{\mu}) - \psi(x)\right)$$

$$\nabla_\mu[U]\psi(x) \xrightarrow{g.t.} \Lambda(x)\nabla_\mu[U]\psi(x)$$



SYM: $U \rightarrow V$ (Adjoint link):

$$V_\mu^{ab}(x) \equiv 2\,\mathrm{Tr}(U_\mu^\dagger(x)T^a U_\mu(x)T^b) \in SO(N_c^2-1)$$

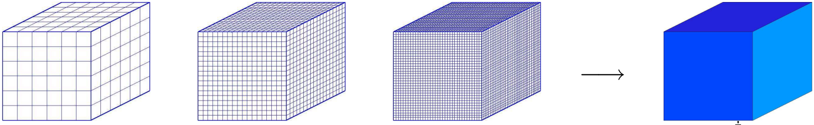
Fermion action:

$$S_{Quark}^{Wil} = a^4 \sum_x \bar{\psi}(x)\gamma_\mu \nabla_\mu^{sym}\psi(x) + m_0\,\bar{\psi}(x)\psi(x) - \underbrace{\frac{a}{2}\bar{\psi}(x)\nabla^2\psi(x)}_{\text{"Wilson term"}}$$

Wilson action

- Wilson term needed to decouple unphysical modes ("doublers") for $a \to 0$
- Chiral symmetry broken at classical level
  - additive renormalization of quark/gluino mass ("tuning problem")
  - $O(a)$ discretization errors in physical quantities
- SYM: SUSY also broken by discretization

Both symmetries restored after continuum limit is taken:



Alternative formulations for lattice SUSY

- "Exact lattice SUSY" [Catterall (2003); Kaplan, Katz and Unsal (2003); Sugino (2004)]
  - Only works with extended SUSY
- Domain-Wall Fermions [Kaplan and Schmaltz (2000)]
  - Classical chiral symmetry recovered in the limit of an infinite fifth dimension: no tuning problem, positive measure
  - First large-scale numerical simulations starting right now [Fleming, Kogut and Vranas (2000); Endres (2008); Giedt, Brower, Catterall, Fleming and Vranas (2008)]
  - Problem: large residual chirality breaking for finite fifth dimension

Simulation

On a lattice path integral $\longrightarrow$ ordinary integral:

$$\langle O^L \rangle = Z_L^{-1} \int \prod_x d\bar{\psi}_x d\psi_x \prod_\mu dU_{x\mu} \, \mathrm{e}^{-\bar{\psi}_y M_{yz} \psi_z} \mathrm{e}^{-S^{Wil}_{Glue}[U]} \, O^L[U; \bar{\psi}, \psi]$$

Integrate out fermion fields:

$$\langle O^L \rangle = Z_L^{-1} \int \prod_{x\mu} dU_{x\mu} \underbrace{|\det M[U]|}_{\mathrm{e}^{-S^{(q)}_{eff}[U]}} \, \mathrm{sgn}(\det M[U]) \, \mathrm{e}^{-S^{Wil}_{Glue}[U]} \, \tilde{O}^L[U]$$

SYM: $\det M[U] \rightarrow \mathrm{Pf}\, M[V] = |\det M[V]|^{\frac{1}{2}} \, \mathrm{sgn}(\mathrm{Pf}\, M[V])$

Simulation

Importance sampling: MC $\rightarrow$ $\{U_1, U_2, \ldots, U_N\}$

$$P[U] \sim e^{-S_{Glue}^{Wil}[U]} \underbrace{e^{-S_{eff}^{(q)}[U]}}_{|\det M[U]|^{\alpha}}$$

Applied here: two-step Polynomial Hybrid Monte Carlo

[Forcrand and Takaishi; Frezzotti and Jansen (1997); Montvay and Scholz (2005)]

$$\langle O^L \rangle = \frac{\langle \tilde{O}[U]\,\mathrm{sgn}[U]\rangle|_{P[U]}}{\langle \mathrm{sgn}[U]\rangle|_{P[U]}} \simeq \frac{\sum_i \tilde{O}[U_i]\,\mathrm{sgn}[U_i]}{\sum_i \mathrm{sgn}[U_i]}$$

$\sum_i \mathrm{sgn}[U_i] \approx 0 \rightarrow$ possible “sign problem”

Otherwise: $\delta\langle O^L \rangle \sim 1/\sqrt{N}$

Bound state masses

$$G_{hh}(\vec{k},t) \;=\; a^3 \sum_x e^{i\vec{k}\cdot\vec{x}} \langle O_h(\vec{x},t) O_h^\dagger(\vec{0},0)\rangle \;=$$

$$\sum_n \frac{1}{2E_{h,n}(\vec{k})} \, |\langle \Omega | O_h(\vec{0},0) | h, n, \vec{k}\rangle|^2 \; e^{-E_{h,n}(\vec{k})t}$$

$$\sim \; e^{-M_{h,0}t} \quad \text{for } \vec{k} = 0 \text{ , large } t$$

4. Results on the low-lying bound state spectrum:

One-flavor QCD and super Yang-Mills with two colors

Simulation program

L/a	Link	β	a (fm)	L (fm)	M_π (MeV)
SYM ($N_c = 2$)					
16	Thin	1.6	0.1	1.6	440
24	Thin	1.6	0.1	2.4	990
24	Stout	1.6	0.09	2.2	400
32	Stout	1.6	0.09	2.9	400
1F-QCD					
12	Thin	3.8	0.19	2.2	340
16	Thin	4.0	0.13	2.2	394
16	Stout	4.0	0.13	2.2	300
24	Stout	4.0	0.13	3.1	290
32	Stout	4.2	0.1	3.2	240

- Lattice scale fixed by the Sommer scale r_0 (QCD units)
- "Pion" defined in the theory with additional valence flavors
- Stout: smeared link in the lattice Dirac operator

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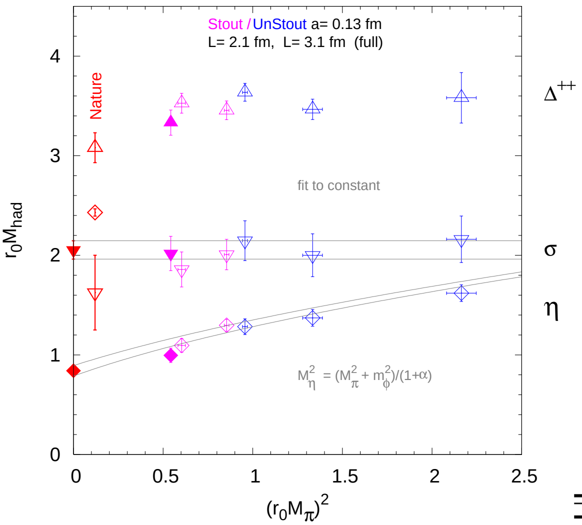
1F-QCD bound state spectrum



Massless quark extrapolation

- $m_q \to 0$ extrapolated masses:

$$r_0 m_\sigma = 2.05(9)\ [810(35)\text{MeV}]$$

$$r_0 m_\eta = 0.84(5)\ [330(20)\text{MeV}] \qquad \text{Cf. } m'_\eta = 958\text{MeV} \propto \sqrt{N_f}$$

- Orientifold planar equivalence (with $N_C = 3$):

$$\frac{m_\eta}{m_\sigma} = \frac{1}{3}\,[1+\Delta]\,, \quad \Delta = O(\frac{1}{N_c}, \frac{1}{N_c^2})$$

[Armoni, Shifman and Veneziano (2003); Armoni and Imeroni (2005)]

We find: $\frac{m_\eta}{m_\sigma} = 0.410(32)(25) \to \Delta = 0.23(12)$

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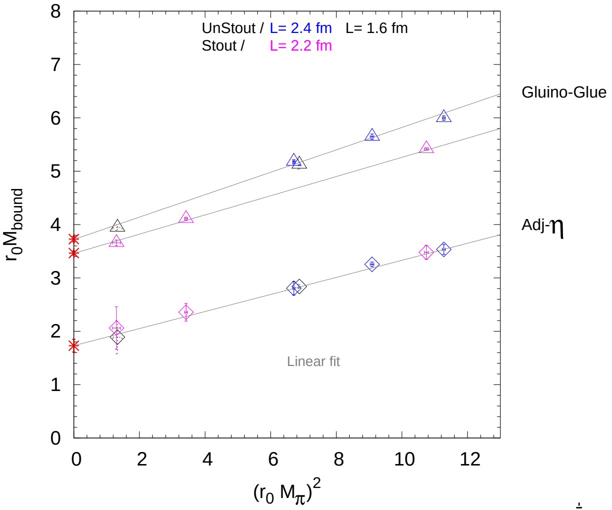
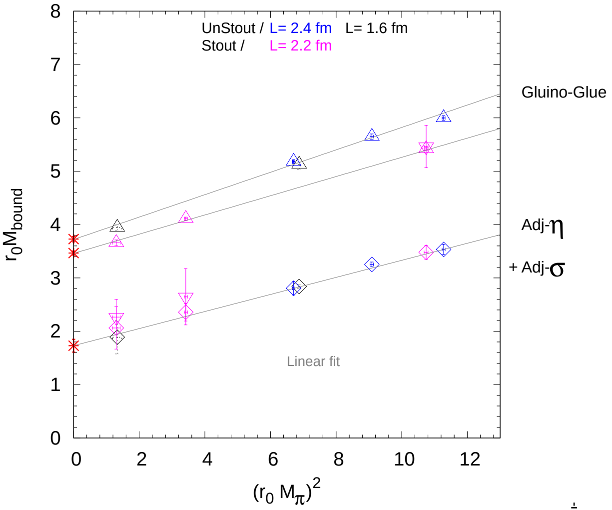
SYM bound state spectrum



SYM bound state spectrum



Comparison 1F-QCD/$N_c = 2$ SYM

Bound state masses for vanishing quark/gluino mass

In units of the Sommer scale:

Model	m_σ	m_η	$m_{\Delta^{++}/\tilde{g}g}$
1F-QCD	2.05(9)	0.84(5)	3.48(7)
$N_c = 2$ SYM	2.27(33)	1.73(12)	3.47(4)

In MeV:

Model	m_σ	m_η	$m_{\Delta^{++}/\tilde{g}g}$
1F-QCD	810(35)	330(20)	1370(30)
$N_c = 2$ SYM	890(130)	680(50)	1370(16)

Conclusions and perspectives

- 1F-QCD: η considerably lighter than in nature, σ and Δ^{++} in rough agreement
- 1F-QCD: hierarchy of masses for η and σ from orientifold equivalence roughly reproduced
- $N_c = 2$ SYM: expected supermultiplet structure not reproduced

- Put these conclusions on more solid basis:
  - Second smaller lattice spacing: 0.07 fm ($L \approx 3$ fm)
  - lighter gluino masses ($M_\pi \lesssim 300$ MeV)

- Study of the spontaneous breaking of CP in 1F-QCD and of the discrete chiral symmetry in SYM