Phase structure of tmQCD at finite temperature

Lars Zeidlewicz

institut für theoretische physik

WWU Münster

FS QFT Nov 24, 2008

The plan of this talk

Overview

Twisted mass lattice QCD (tmQCD):

- Introduction of the twisted mass term.
- $\mathcal{O}(a)$ improvement for maximal twist.

Lattice phase structure:

- What is already known from Wilson fermions?
- Phase structure of tmQCD at $T = 0$.

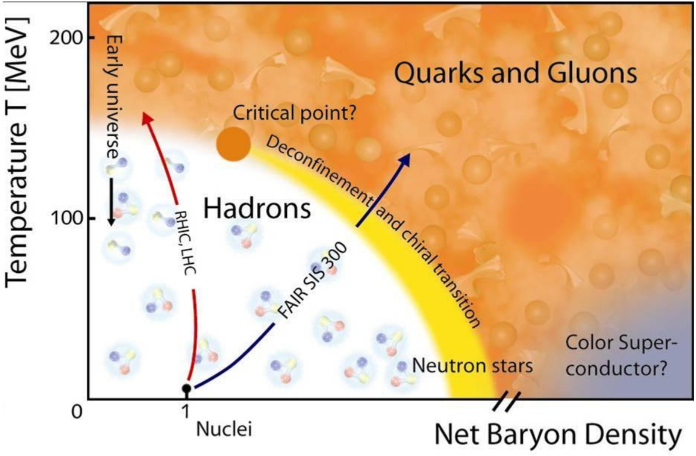
Phase structure of tmQCD at finite temperatures:

- Expectations from theoretical conjectures/predictions.
- Results from our MC simulations.

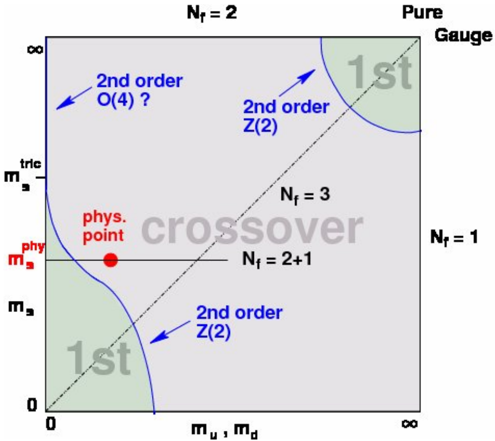
tmfT collaboration
PoS(Lat2008)206 [arXiv:0809.5228 [hep-lat]]

Physical context

Transition to QGP: What can we learn from lattice QCD?



$N_f = 2 + 1$ mass plane



- Transition depends on quark mass values.
- Behaviour in the $N_f = 2$ chiral limit is an open question.

Twisting the mass in the continuum

From physical to twisted basis:

$$\psi \longrightarrow \chi = e^{i\gamma_5\tau^3\omega/2}\,\psi$$
$$\overline{\psi} \longrightarrow \overline{\chi} = \overline{\psi}\,e^{i\gamma_5\tau^3\omega/2}$$

- Kinetic term is left invariant.
- Mass term:

$$\overline{\psi} m_q \psi \longrightarrow \overline{\chi}\left(\underbrace{m_q\cos(\omega)}_{m'} + i\gamma_5\tau^3\,\underbrace{m_q\sin(\omega)}_{\mu}\right)\chi$$

- Non-anomalous transformation: same physics in different bases

Lattice action

$$S_{\text{tm}} = S_{\text{naive}} + S_{\text{Wilson}} + i\mu \sum_x \overline{\psi}\gamma_5\tau^3\psi$$

Naive discretization:

- Replace derivatives by finite differences:

$$\nabla_\nu\psi(x) = \frac{1}{2a}\left(\psi(x+a\hat{\nu}) - \psi(x-a\hat{\nu})\right)$$

Wilson term: $\sim \overline{\psi}\Box\psi$

- Removes doublers...
- ...but breaks chiral symmetry.
- The twist rotation is no longer a symmetry of the action.

Automatic $\mathcal{O}(a)$-improvement

Maximal twist:

- Tune untwisted quark mass to its critical value $m_c(\beta)$. $(\omega = \pi/2)$
- The quark mass is solely determined by the twisted mass parameter μ.

Improvement:

- Lattice artefacts are a function of the twist angle:

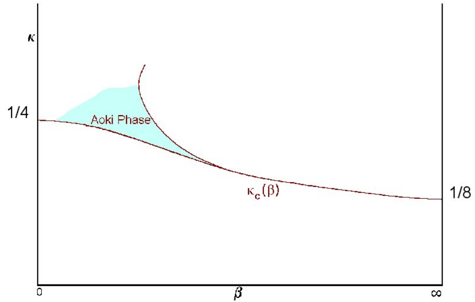
$$O_{\text{Lat}}(\omega) = O_{\text{Cont}} + aO_1(\omega) + a^2 O_2(\omega) + \dots$$

- At maximal twist $O_1(\omega = \pi/2) = 0$ (as long as $O_{\text{Cont}} \neq 0$)

in fact $O_{2n+1}(\pi/2) = 0$ for all $n \in \mathbb{N}$

Aoki phase

- The twisted mass term introduces an explicit breaking of (ordinary) parity and flavour symmetry due to an external field (μ).
- Aoki phase: SSB in the (κ, β) plane (i. e. for Wilson fermions $\mu = 0$).

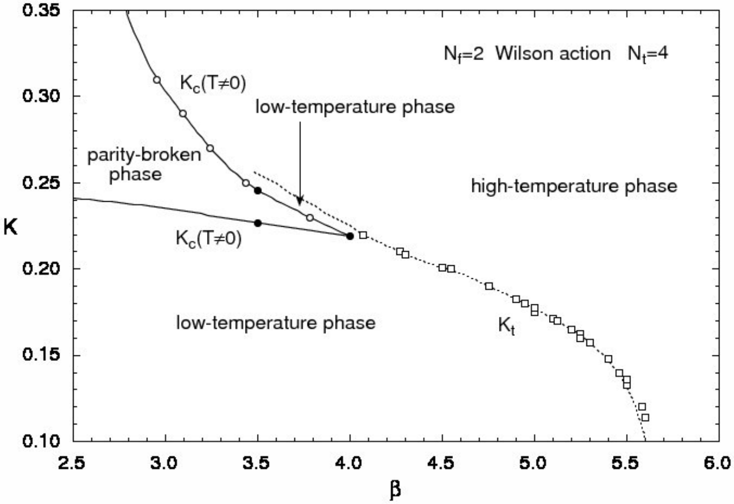


$$\lim_{h \to 0} \lim_{V \to \infty} \left\langle i\bar{\psi}\gamma_5\tau^3\psi \right\rangle \neq 0$$

$$\kappa = 1/(2am + 8)$$
$$\beta = 6/g^2$$

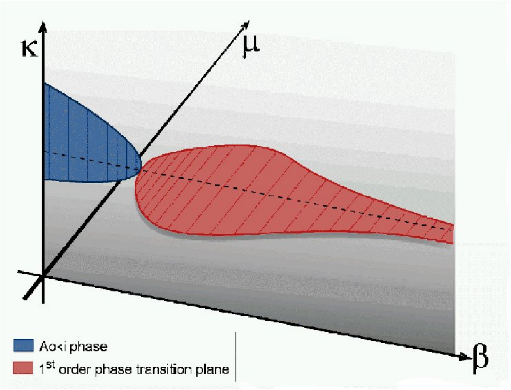
Wilson fermions at finite T

Thermal confinement → deconfinement phase transition:

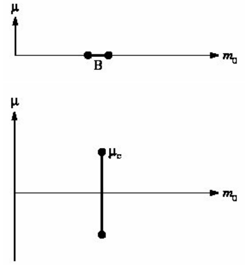


[arXiv:hep-lat/9508008]

Zero temperature phase structure of tmQCD



[arXiv:hep-lat/0509131]



JHEP 09:035, 2004

- 3d phase space: (κ, β, μ)
- $\kappa_c(\beta)$ is part of a transition surface extending to $\mu \neq 0$.
- Both phase transition scenarios are predicted by χPT.

How does the phase structure of tmQCD look like at finite T?

- Temperature is given by finite time direction: $T = \frac{1}{aN_t}$
- Phase space is three dimensional: (κ, β, μ)
- Expect "leftovers" of zero temperature phase structure: Aoki phase and 1^{st}-order scenario (Sharpe/Singleton)

Starting points:

- Theoretical arguments that provide some expectations.
- Results of numerical simulations.
- Knowledge of finite T phase structure of Wilson fermions.

Simulation details

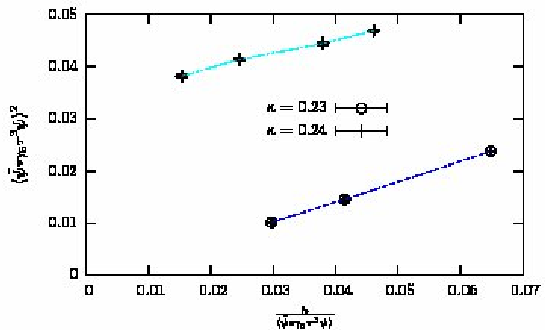
Regions of investigation:

- $\beta = 1.8$ and $\beta = 3.0$: strong coupling *(Aoki)*
- $\beta \approx 3.45$: intermediate coupling *(Sharpe/Singleton)*
- $\beta \gtrsim 3.65$: weak coupling/thermal transition

- Algorithm: Generalized Hybrid MonteCarlo
  see Comput. Phys. Commun., 174:87-98, 2006
- Tree-level Symanzik improved gauge action (plaquettes and rectangles).
- Lattice extent: $N_s^3 \times N_t = 16^3 \times 8$

Small β regime

- Order parameter for Aoki phase: $\langle\overline{\psi} i\gamma_5\tau^3\psi\rangle$
- Fisher plot: (o.p.)^2 vs. external field normalized to o.p.

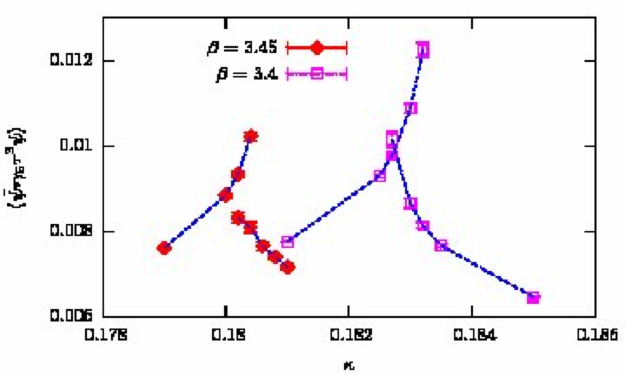


[arXiv:0809.5228 [hep-lat]]

For $\beta \leq 3.0$ there is an Aoki phase of nonvanishing width!

Intermediate couplings

- For $\beta \sim 3.4$ there are signs of a 1^{st} order transition plane.
- Metastabilities:



[arXiv:0809.5228 [hep-lat]]

Towards the continuum limit

The thermal transition/crossover:

- $\beta \gtrsim 3.65$
- How does the transition depend on κ, μ, β?
- Where do we find it?

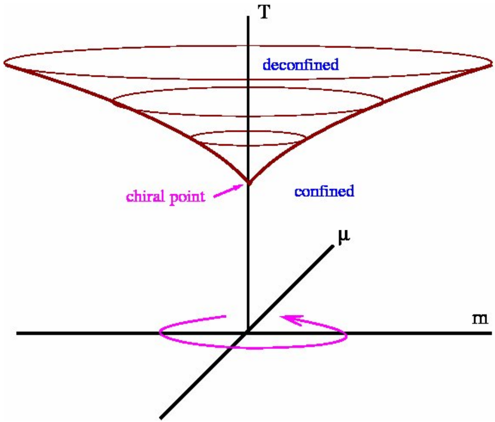
Two approaches:

1. Looking for a thermal transition line as a function of β for fixed $\mu(= 0.005)$.
2. Looking for the behaviour of the transition depending on κ and μ for fixed coupling ($\beta = 3.75$).

Creutz's continuum argument

Phys. Rev. D76 054501 (2007)

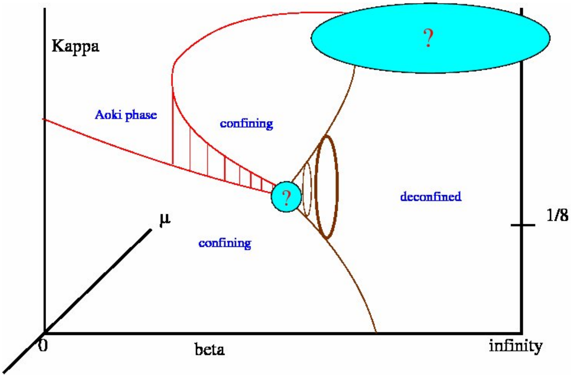
- Continuum tree-level relation: $m_q^2 = m'^2 + \mu^2$
- $dT_c/dm_q > 0$



- On the lattice: continuum-like behaviour for $\beta \to \infty$.

Creutz's proposal

- Aoki phase persists at $T > 0$ up to a critical β_c.
- The thermal transition line of Wilson fermions is part of a conical thermal transition surface.



Phys. Rev. D76 054501 (2007)

Lattice Observables I

Polyakov loop

$$L(\mathbf{x}) = \frac{1}{N_c} \mathrm{Tr} \prod_{n_4=0}^{N_t-1} U_4(\mathbf{x}, x_4)$$

- The Polyakov loop expectation value is related to the free energy of a single quark:

$$\langle L \rangle = e^{-\beta F}$$

- In pure gauge theory $\langle L \rangle$ is also the order parameter for the breaking of the Z_3 symmetry.
- A rise in $\langle L \rangle$ indicates the deconfinement transition.

Lattice Observables II

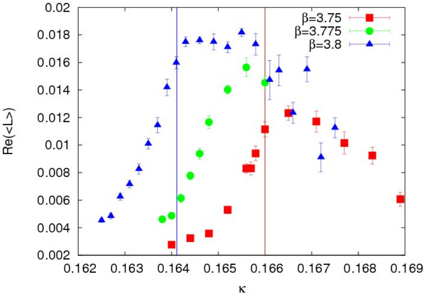
Pionnorm

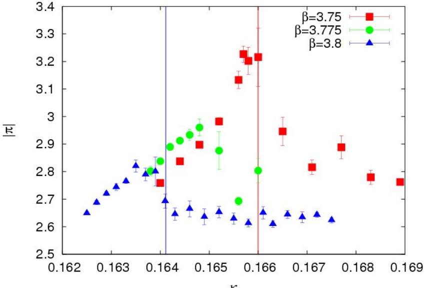
$$\|\pi\|^2 = \sum_x \left\langle \overline{d}(x)\gamma_5 u(x)\overline{u}(0)\gamma_5 d(0) \right\rangle$$

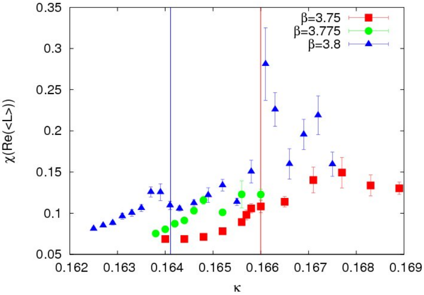
- Pionnorm is the zero momentum pion correlator.
- The pion correlator peaks at the phase transition or crossover.
- $\|\pi\|^2$ is invariant under twist rotations of the flavour doublets $\psi = (u, d)$:

$$\psi \to e^{\frac{i}{2}\omega\gamma_5\tau^3}\psi \qquad \overline{\psi} \to \overline{\psi}e^{\frac{i}{2}\omega\gamma_5\tau^3}$$

Small twist angles: $\mu = 0.005$







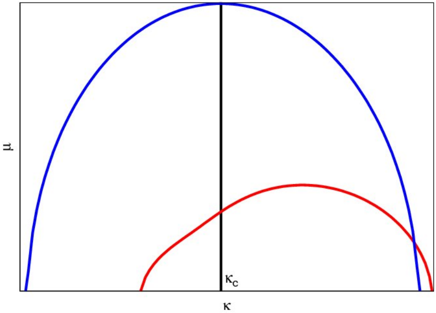
- Two transitions but...
- asymmetric situation:
- second transition ($m_q < 0$) is very broad; no signal in chiral observables.

Distortions by the lattice: χPT in NLO

NLO LχPT relation:

$$m_q^2 = \left(\frac{1}{Z_P^2}\mu^2 + \frac{1}{Z_S^2}\frac{1}{4}\left(\frac{1}{\kappa} - \frac{1}{\kappa_c}\right)^2 \right)(1 + K\cos\omega)$$

- Tree level:
  $m_q^2 = \mu^2 + \frac{1}{4}\left(\frac{1}{\kappa} - \frac{1}{\kappa_c}\right)^2$
- Known from zero temperature simulations (ETMC):
  - $Z_P(\beta)$, $Z_S(\beta)$ translate to continuum parameters
  - $\kappa_c(\beta)$
- K: unknown $\mathcal{O}(a)$ parameter from χPT



Transition surface: NLO χPT prediction

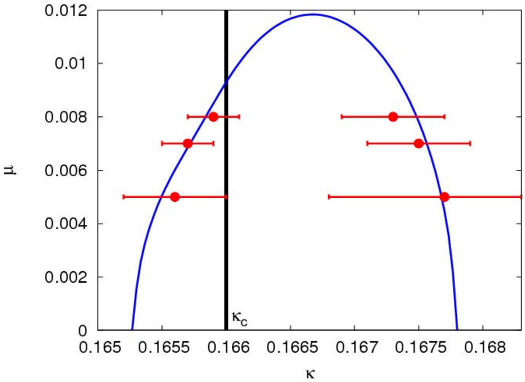
Check of reliability at $\beta = 3.75$: $\mu = 0.005, 0.007, 0.008, 0.010$

Transition surface: NLO χPT prediction

Check of reliability at $\beta = 3.75$: $\mu = 0.005, 0.007, 0.008, 0.010$

Results:

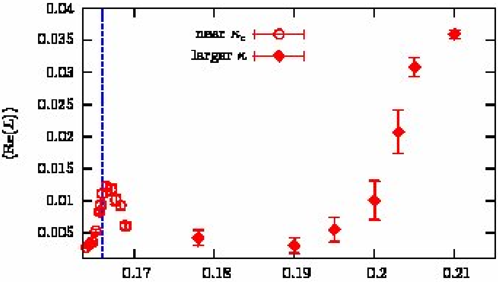
- Behaviour of observables is qualitatively unaltered.
- Transitions move closer towards each other.



fitted parameters:
$K = 0.42(8)$ and
$m_q = 0.031(4)$

(but χ^2 is tiny)

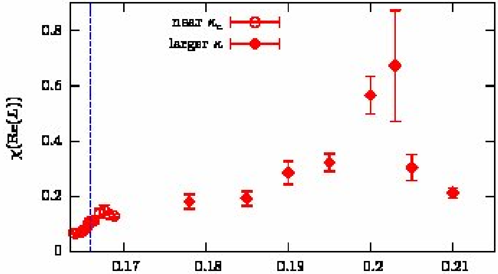
Doubler signal



Polyakov loop

What happens for very large values of κ?

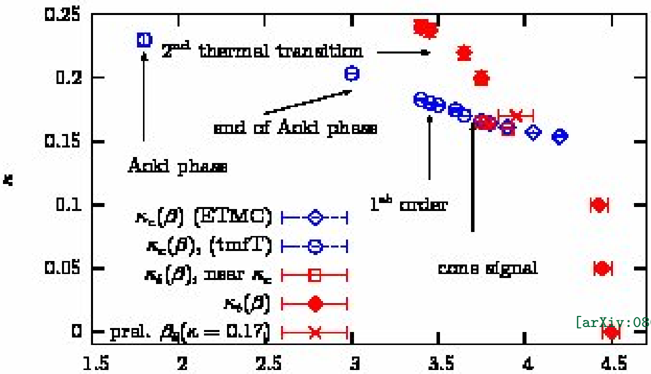
[arXiv:0809.5228 [hep-lat]]



Susceptibility of the Polyakov loop

Collection of results

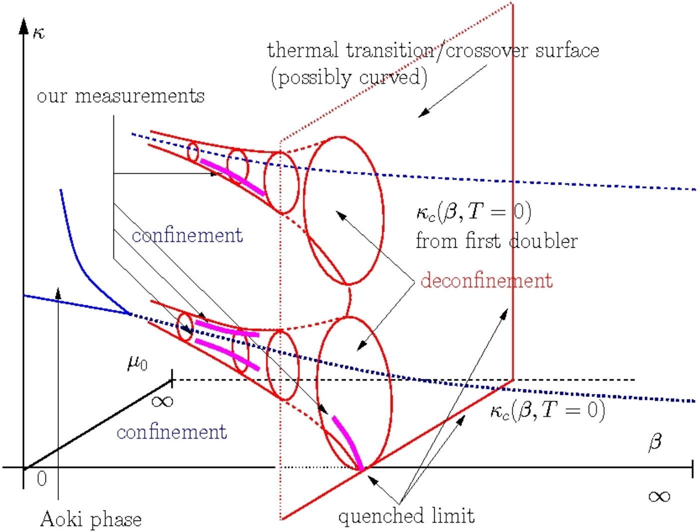
- Aoki phase for small $\beta \lesssim 3.0$
- 1^{st} order behaviour for $\beta \sim 3.4$
- thermal transition surface surrounding $\kappa_c(\beta)$ for $\beta \gtrsim 3.7$
- signals of doubler physics for large $\kappa \gg \kappa_c$



[arXiv:0809.5228 [hep-lat]]

Artistic view

[arXiv:0809.5228 [hep-lat]]



What's next?

- finish study at $\beta = 3.75$ by “closing the cone”
- scan in β-direction at maximal twist
- zero temperature simulations to set the scale
- physical questions: Thermal transition, EoS, order of the transition in the chiral limit,. . .