Real-Time Static Octet Potential in hot QCD

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Heavy Quarkonia

Definition

Quarkonium, a flavour-neutral meson, consists of a heavy quark and its antiquark.

- *just* for heavy quarks
- high masses (much larger than Λ_{QCD})
  - non-relativistic speed
  - assumption: movement of quarks in a **static potential**
- analogue: hydrogen atom

Non-Relativistic Schrödinger Equation

- discovery of heavy quarkonium
  - description of bound states by Schrödinger Equation:

Non-Relativistic Schrödinger Equation

$$\left(-\frac{\Delta}{M} + V(r)\right)\psi = E\psi$$

- M: heavy quark mass
  $V(r)$: static potential
  E: binding energy
- goal: derivation of a finite-temperature potential by generalising SE
- spectra of potential not purely Coulombic

$q\bar{q}$-Potential

Popular potential model:

Cornell Potential

$$V(r) = a/r + br$$

- r: effective radius
  a, b: parameters
- first part:
  - $1/r$ form equivalent to Coulomb potential
  - exchange of gluons
  - $r \to 0$: pure Coulombic
- second part:
  - potential increases linearly, development of flux tube in between $q\bar{q}$-pair
  - non-perturbative effects of QCD
  - $r \to \infty \Rightarrow V \to \infty$ confinement!

$q\bar{q}$-Potential

Experimentally obtained potential:

Potential

$$V(r) = -\frac{4}{3}\frac{\alpha_s(r)}{r} + kr$$

- α_s: strong coupling constant (r dependence!)
  k: string tension
- string tension of $c\bar{c}$ is of magnitude $k \approx 1\,\mathrm{GeV/fm}$
- asymptotic behaviour

Decay Mechanisms at vanishing temperature

Decay of Quarkonia

1. Decay through weak interaction
2. String Breaking: production of a light $q\overline{q}$-pair resulting in a break up of original bound state
    - strong interaction
3. Annihilation of heavy q and $\overline{q}$
    - electromagnetic and strong interaction

Decay Mechanisms at vanishing temperature

Decay of Quarkonia

1. **Decay through weak interaction**

- strong and electromagnetic much faster than weak decay

Decay Mechanisms at vanishing temperature

Decay of Quarkonia

2. **String Breaking: production of a light $q\bar{q}$-pair resulting in a break up of original bound state**
    - strong interaction

- for static heavy $q\bar{q}$-pair just strong interaction
- minimum excitation energy necessary
    - creation of light $q\bar{q}$-pair

Decay Mechanisms at vanishing temperature

Decay of Quarkonia

3. **Annihilation of heavy q and $\overline{q}$**
   - electromagnetic and strong interaction

- strong: conservation of colour and parity $\longrightarrow$ 3 gluons
- electromagnetic: real or virtual photons
  - production of dilepton pairs

J/ψ Suppression

Quark-Gluon Plasma(QGP)

- hot temperatures
- quarks, gluons move freely over distances more than $1\,\mathrm{fm}$
  - **deconfinement**

J/ψ Suppression by QGP[1]

- J/ψ suppression: unambigous (model-independent) indicator of QGP-formation
- quarkonium dissociation due to colour screening
  - also J/ψ suppression in nuclear matter phase
- *assumption*: medium effects can be explained by temperature-dependent potential

Real-Time Static Singlet Potential

- potential in Minkowski time at finite temperature
- first non-trivial order in Hard Thermal Loop (HTL) resummed perturbation theory
- defined by the time evolution of mesonic correlator

Mesonic Correlator

$$iC^{21}(t,\mathbf{r}) \equiv \int d^3\mathbf{x} \left\langle \bar{\psi}(t,\mathbf{x}+\frac{\mathbf{r}}{2})\gamma^{\mu} W \psi(t,\mathbf{x}-\frac{\mathbf{r}}{2})\bar{\psi}(0,\mathbf{0})\gamma_{\mu}\psi(0,\mathbf{0}) \right\rangle$$

Generalising Schrödinger equation:

Modified Schrödinger Equation

$$i\partial_t C^{21}(t,\mathbf{r}) = \left[2M - \frac{\triangle_{\mathbf{r}}}{M} + V(t,\mathbf{r})\right] C^{21}(t,\mathbf{r})$$

Real-Time Static Singlet Potential

- infinitely heavy quark propagators represented by Wilson lines
  - apart from phase factor e^{-iMt}

Wilson Line (series expansion)

$$W[z_1, z_0] = \mathbf{1} + ig \int_{z_0}^{z_1} \mathrm{d}x^\mu A_\mu(x) + (ig)^2 \int_{z_0}^{z_1} \mathrm{d}x^\mu \int_{z_0}^{x} \mathrm{d}y^\mu A_\mu(x) A_\nu(y) + \cdots$$

- gauge fields: $A_\mu = A_\mu^a T^a$
- Wilson loop in temporal gauge:

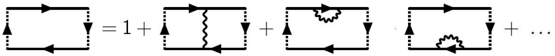


Figure: Symbolic representation of the Wilson loop to $\mathcal{O}\left(g^2\right)$[5]

- potential is complex quantity
  - real part: screened Coulomb potential
  - imaginary part: medium-induced damping effects (Landau damping)
    - classical non-perturbative approximation
- definition of potential between two static sources
  - derivation of energy and thermal decay width of static quarks

Real-Time Static Singlet Potential

$$V(\mathbf{r}) = \lim_{t\to\infty} \frac{i\partial_t C_{21}(t,\mathbf{r})}{C_{21}(t,\mathbf{r})}$$

Spectral Function

- modification of spectral function by imaginary part of potential

Spectral Function

$$\rho_V(Q) \equiv \int_{-\infty}^{\infty} \mathrm{d}t \int \mathrm{d}^3\mathbf{x} e^{iQx} \left\langle \frac{1}{2} \left[\hat{\mathcal{J}}^\mu(x), \hat{\mathcal{J}}_\mu(0) \right] \right\rangle$$

- colour current: $\hat{\mathcal{J}}^\mu \equiv \hat{\bar{\psi}} \gamma^\mu \hat{\psi}$
  heavy quark field operator: $\hat{\psi}$
  thermal expectation value: $\langle \ldots \rangle \equiv \mathcal{Z}^{-1} \,\mathrm{Tr} \left[(\ldots) \, e^{-\beta \hat{H}} \right]$
  inverse temperature: $\beta \equiv 1/T$

Melting of Heavy Quarkonium

- at threshold ($Q^2 \cong 2M^2$): resonance peak
- height and width functions of temperature (in collision)
- expected widening of peak with increasing temperature

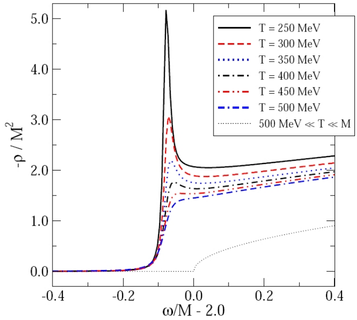


Figure: Phenomenological results for the spectral function ρ_V[6]

Dilepton Production

- leptons: small cross section with QGP
- consider $\mu^+\mu-$-pairs

Production Rate

$$\frac{\mathrm{d}N_{\mu^-\mu^+}}{\mathrm{d}^4x\mathrm{d}^4Q} = \frac{-2e^4Z^2}{3\left(2\pi\right)^5 Q^2}\left(1+\frac{2m_\mu^2}{Q^2}\right)\left(1-\frac{4m_\mu^2}{Q^2}\right)^{\frac{1}{2}} n_B\left(q^0\right)\rho_V(Q)$$

- Z: heavy quark electric charge
  n_B: Bose-Einstein distribution function
  $Q = P_{\mu^+} + P_{\mu^-}$: total four-momentum

Dilepton Production Rate

- M: pole mass of bottom quark

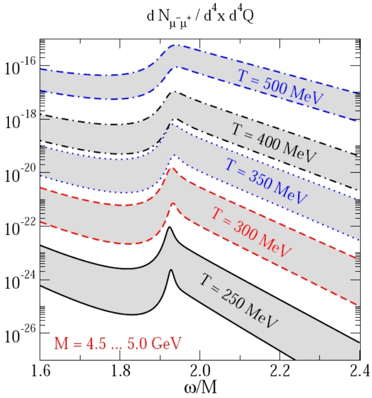
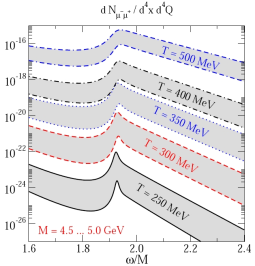


Figure: Phenomenological results for dilepton production rate $\frac{\mathrm{d}N_{\mu^-\mu^+}}{\mathrm{d}^4x\mathrm{d}^4Q}$ [7]

Melting of Heavy Quarkonium

- interval for bottom quark mass
  - uncertainties of several hundred MeV
- practical limitations
  - non-equilibrium features
  - background effects
  - energy resolution of detector
- **but** complete vanishing of peak still recognizable



Real-Time Static Octet Potential

- above deconfinement region
  - imaginary part of potential from thermal fluctuations
  - break up of colour singlet bound state into octet $q\overline{q}$ state + gluons
  - forbidden at zero temperature

Static Octet Potential

$$V^{o}(\mathbf{r}) = \lim_{t\to\infty} \frac{i\partial_t C_{21}^{o}(t,\mathbf{r})}{C_{21}^{o}(t,\mathbf{r})}$$

Octet Meson

$$\overline{\psi}\left(-\frac{\mathbf{r}}{2}\right) W\left(0,-\frac{\mathbf{r}}{2}\right) T^{a} W\left(0,\frac{\mathbf{r}}{2}\right) \psi\left(\frac{\mathbf{r}}{2}\right)$$

- singlet meson interrupted by generator T^{a}

Real-Time Static Octet Potential

Correlation Function

$$
\begin{aligned}
C^{o}_{21}(t,\mathbf{r}) \;=\; & \frac{1}{N_c}\,\mathrm{Tr}\Big\langle \hat{\mathcal{T}}\, W\left[\left(t,\frac{\mathbf{r}}{2}\right),\left(0,\frac{\mathbf{r}}{2}\right)\right] \\
& \times W\left[(t,\mathbf{0}),\left(t,\frac{\mathbf{r}}{2}\right)\right] T^a W\left[\left(t,-\frac{\mathbf{r}}{2}\right),(t,\mathbf{0})\right] \\
& \times W\left[\left(0,-\frac{\mathbf{r}}{2}\right),\left(t,-\frac{\mathbf{r}}{2}\right)\right] \\
& \times W\left[(0,\mathbf{0}),\left(0,-\frac{\mathbf{r}}{2}\right)\right] T^b W\left[\left(0,\frac{\mathbf{r}}{2}\right),(0,\mathbf{0})\right]\Big\rangle
\end{aligned}
$$

- $\hat{\mathcal{T}}$: time-ordering
  T^a: Hermitean generators of $SU(N_c)$
  - normalised by $\mathrm{Tr}[T^a T^b] = \frac{1}{2}\delta_{ab}$

Ansatz

Wilson Line

$$W[z_1, z_0] = \mathbf{1} + ig \int_{z_0}^{z_1} \mathrm{d}x^\mu A_\mu(x) + (ig)^2 \int_{z_0}^{z_1} \mathrm{d}x^\mu \int_{z_0}^{x} \mathrm{d}y^\mu A_\mu(x) A_\nu(y) + \cdots$$

- Fourier transform of time-ordered gluon propagator $\tilde{G}^{11}_{\mu\nu}(k)$:

$$\langle \hat{\mathcal{T}} \{ A^a_\mu(x) A^b_\nu(y) \} \rangle = \delta_{ab} i G^{11}_{\mu\nu}(x-y) = \delta_{ab} \int \frac{\mathrm{d}^4 k}{(2\pi)^4} e^{-ik(x-y)} i \tilde{G}^{11}_{\mu\nu}(k)$$

- related to retarded propagator $\tilde{G}^R(k)$ and Wightman propagator $\tilde{G}^{21}(k)$ by:

$$\tilde{G}^{11}_{33}(k) = \tilde{G}^R(k) + \tilde{G}^{12}(k) = \tilde{G}^R(k) + e^{-\beta\omega} \tilde{G}^{21}(k)$$

Resumed Propagator

Schwinger-Dyson Equation

$$\tilde{G} = G + G \cdot \Pi \cdot \tilde{G}$$
$$\tilde{G}^{-1} = G^{-1} - \Pi$$

$$\Rightarrow \tilde{G} = \frac{1}{G^{-1} - \Pi} \cong \frac{1}{k^2 - \Pi}$$

- $G \cong \frac{1}{k^2}$: bare gluon propagator
  $\tilde{G}$: resummed gluon propagator

Resummed Propagator

Kubo-Martin-Schwinger (KMS) Condition

$$i\tilde{G}^{21}(k) = -2\left(n_B(\omega)+1\right) Im\tilde{G}_R(k)$$

- static case: $n_B(\omega) = \frac{1}{e^{\beta\omega}-1} \overset{\omega\to 0}{\approx} \frac{1}{\beta\omega}$

Gluon Propagator

$$\tilde{G}^R_{ij}(k) = -\frac{1}{k^2-\Pi_T(k)}A_{ij}(k) - \frac{1}{\mathbf{k}^2-\Pi_L(k)}\frac{\mathbf{k}^2}{\omega^2}B_{ij}(k)$$

- Π: gluon self-energy
- A, B: transversal and longitudinal projectors

Debye Screening

Transversal and Longitudinal Projectors

$$A_{ij}(k) = \delta_{ij} - \frac{k_i k_j}{\mathbf{k}^2}$$

$$B_{ij}(k) = \frac{k_i k_j}{\mathbf{k}^2}$$

Transversal and Longitudinal Self Energies

$$\Pi_T(k) = \frac{m_D^2}{2}\frac{\omega^2}{k^2}\left[1 - \frac{\omega^2 - k^2}{2\omega k}\log\frac{\omega + k}{\omega - k}\right]$$

$$\Pi_L(k) = m_D^2\left[\frac{\omega}{2k}\log\frac{\omega + k}{\omega - k} - 1\right]$$

Static Limit

$$\lim_{\omega \to 0} \Pi(\omega, \mathbf{k}) = m_D^2 \sim g^2 T^2$$

Scales of Thermal QCD

Bound states at finite temperature are characterized by

Scales of Thermal QCD

- inverse distance $1/r$
- temperature scale T
- Debye mass
  - inverse length scale of chromoelectric screening
- lower energy scales (gT, g^2T)
- characteristic QCD scale Λ_{QCD} (as for $T = 0$)

- heavy quarkonia: M large in comparison to Λ_{QCD}
- near threshold: momenta ν small compared to masses M
  - $\nu/M \ll 1$
- perturbation expansion breaks down when $\alpha_s \sim \nu$

Scales of Thermal QCD

Characterisation of heavy quarkonia by three widely seperated scales:

Scales

- M - hard scale
  - mass M of heavy quarks
  - description in perturbation theory possible $\longrightarrow$ asymptotic freedom
- $M\nu$ - soft scale
  - relative momentum of heavy $q\bar{q}$
- $M\nu^2$ - ultrasoft scale
  - typical kinetic energy of heavy $q\bar{q}$

- Effective Field Theories
  - describing observables of particular energy range
  - integrating out degrees of freedom

Non-relativistic QCD

- heavy quarkonium $\rightarrow$ possibility to use NR picture
  - interested in physics at low energy scale E
  - integrating out energy scales larger than E

Degrees of Freedom

- energy scales smaller than masses of static : no further pairs
- $\psi^\dagger(x)$ creates a quark
  $\chi(x)$ creates an antiquark
- gluon fields in covariant derivative D_μ

Leading-order Lagrangian density

$$\mathcal{L} = \psi^\dagger \left(iD_0 + \frac{1}{2m}\mathbf{D}^2 \right) \psi + \chi^\dagger \left(iD_0 - \frac{1}{2m}\mathbf{D}^2 \right) \chi$$

- *principle*: consistently integrating out mass from QCD and expanding order by order in $1/M$

potential Non-Relativistic QCD

Integrating out all energy scales but $M\nu^2 \rightarrow$ pNRQCD

Differences from NRQCD

- for NRQCD m must be larger than remaining scales ($|\mathbf{p}|, E, \Lambda_{QCD}, \ldots$)
- at scale of binding energy E: unphysical degrees of freedom
  - light degrees of freedom and heavy quarks with fluctuation of energy $\sim |\mathbf{p}| \gg E$
- integrating out $\longrightarrow$ new effective theory: pNRQCD
  - depends on relative size of Λ_{QCD} compared to $|\mathbf{p}|$ and E

pNRQCD - Weak-Coupling Regime

Weak-Coupling Regime

- $|\mathbf{p}| \gg \Lambda_{QCD}$
- perturbation theory applicable
- $Q - \overline{Q}$ state can be decomposed into singlet and *octet* state
  - colour gauge transformation
  - in QED: no analogue to octet state
- if $E \gtrsim \Lambda_{QCD}$: EFT in weak coupling regime
- if $E \ll \Lambda_{QCD}$: necessary to integrate out energy scale Λ_{QCD} and corresponding momentum scale $\sqrt{\Lambda_{QCD} m}$

pNRQCD - Strong-Coupling Regime

Strong-Coupling Regime

- $\Lambda_{QCD} \gg E$
- $|\mathbf{p}| \gtrsim \Lambda_{QCD}$
  - cannot integrate out energy degrees of freedom at scale $|\mathbf{p}|$ perturbatively in α_s
- ultrasoft scales $\longrightarrow$ integrating out scales of larger energies
- ground state energy: static QCD potential

Leading Thermal Effects to Potential[4]

$T \leq E$

- no thermal contributions to potential
  - Coulomb potential
- thermal effects: loop corrections induced by low-energy gluons
- thermal width due to thermal fluctuation
  - at short range: colour singlet $q\bar{q}$-state breaks up into octet state and gluons
  - dominant contribution for $T = E$

- T: temperature
  E: bound state energy

Leading Thermal Effects to Potential[4]

$1/r > T > E$ and $m_D \leq E$

- potential develops real and imaginary part
- contribution to imaginary part
  - singlet and octet thermal breakup
  - imaginary part of gluon self-energy
    - induced by Landau damping
- $m_D < E$
  - dominant contribution: singlet to octet thermal breakup

- $1/r$: inverse distance
  m_D: Debye mass

$1/r > T > E$ and $m_D > E$

- additional contribution to potential
  - HTL resummed gluon propagators

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Thank you for your attention!