A strategy for performing non-pert. computations in HQET with dynamical light quarks

ALPHA Collaboration

B. Blossier, M. Della Morte, P. Fritzsch, N. Garron
J. H., H. Meyer, H. Simma, R. Sommer

FlaviA net

Jochen Heitger

institut für
theoretische physik

Universität Münster, Wilhelm-Klemm-Str. 9, D-48149 Münster

LATTICE 2007

The XXV International Symposium on Lattice Field Theory

University of Regensburg, July 30 – August 4, 2007

Outline

Non-perturbative Heavy Quark Effective Theory

Non-perturbative formulation of HQET

Beyond the static approximation

$$S_{\mathrm{HQET}} = a^4 \sum_x \left\{ \mathcal{L}_{\mathrm{stat}}(x) + \sum_{\nu=1}^{n} \mathcal{L}^{(\nu)}(x) \right\}, \quad \mathcal{L}^{(\nu)}(x) = \sum_i \omega_i^{(\nu)} \mathcal{L}_i^{(\nu)}(x)$$

$$\mathcal{L}_{\mathrm{stat}} = \overline{\psi}_h [\nabla_0^* + \delta m] \psi_h \quad \longrightarrow \text{ Eichten-Hill action}$$

$$\mathcal{L}_1^{(1)} = \overline{\psi}_h \left(-\tfrac{1}{2}\, \sigma \cdot \mathbf{B} \right) \psi_h \equiv \mathcal{O}_{\mathrm{spin}} \longrightarrow \left\{ \begin{array}{l} \text{chromomagnetic interaction} \\ \text{with the gluon field} \end{array} \right.$$

$$\mathcal{L}_2^{(1)} = \overline{\psi}_h \left(-\tfrac{1}{2}\, \mathbf{D}^2 \right) \psi_h \equiv \mathcal{O}_{\mathrm{kin}} \longrightarrow \left\{ \begin{array}{l} \text{kinetic energy from heavy} \\ \text{quark's residual motion} \end{array} \right.$$

Non-perturbative formulation of HQET

Beyond the static approximation

$$S_{\mathrm{HQET}} = a^4 \sum_x \left\{ \mathcal{L}_{\mathrm{stat}}(x) + \sum_{\nu=1}^{n} \mathcal{L}^{(\nu)}(x) \right\}, \quad \mathcal{L}^{(\nu)}(x) = \sum_i \omega_i^{(\nu)} \mathcal{L}_i^{(\nu)}(x)$$

$$\mathcal{L}_{\mathrm{stat}} = \overline{\psi}_h \, [\nabla_0^* + \delta m] \, \psi_h \qquad \rightarrow \text{Eichten-Hill action}$$

$$\mathcal{L}_1^{(1)} = \overline{\psi}_h \left(-\tfrac{1}{2}\, \sigma \cdot \mathbf{B} \right) \psi_h \equiv \mathcal{O}_{\mathrm{spin}} \qquad \rightarrow \left\{ \text{chromomagnetic interaction with the gluon field} \right.$$

$$\mathcal{L}_2^{(1)} = \overline{\psi}_h \left(-\tfrac{1}{2}\, \mathbf{D}^2 \right) \psi_h \equiv \mathcal{O}_{\mathrm{kin}} \qquad \rightarrow \left\{ \text{kinetic energy from heavy quark's residual motion} \right.$$

- $\delta m, \omega_i(g_0, m)$ must be determined such that HQET matches QCD
  [At the classical level: $\omega_{\mathrm{spin}} = \omega_{\mathrm{kin}} = 1/m + \mathrm{O}(g_0^2)$, $\delta m = 0 + \mathrm{O}(g_0^2)$]
- Analogously: Composite fields in the effective theory, e.g.

$$A_0^{\mathrm{HQET}}(x) = \underbrace{Z_A^{\mathrm{HQET}}}_{1+\mathrm{O}(g_0^2)} \underbrace{\overline{\psi}_l(x)\gamma_0\gamma_5\psi_h(x)}_{A_0^{\mathrm{stat}}(x)} + \underbrace{c_A^{\mathrm{HQET}}}_{\propto 1/m} \overline{\psi}_l(x)\gamma_j\gamma_5 \overleftarrow{D}_j \psi_h(x) + \ldots$$

Expectation values

Path integral representation at the quantum level

$$\langle O \rangle = \frac{1}{\mathcal{Z}} \int \mathcal{D}[\varphi]\, O[\varphi]\, \mathrm{e}^{-(S_{\rm rel}+S_{\rm HQET})} \qquad \mathcal{Z} = \int \mathcal{D}[\varphi]\, \mathrm{e}^{-(S_{\rm rel}+S_{\rm HQET})}$$

Now the *integrand* is expanded in a *power series* in $1/m$

$$\exp\{-S_{\rm HQET}\} =$$
$$\exp\left\{-a^4 \textstyle\sum_x \mathcal{L}_{\rm stat}(x)\right\} \left\{1 - a^4 \textstyle\sum_x \mathcal{L}^{(1)}(x) + \frac{1}{2}\left[a^4 \textstyle\sum_x \mathcal{L}^{(1)}(x)\right]^2 - a^4 \textstyle\sum_x \mathcal{L}^{(2)}(x) + \ldots\right\}$$

$$\Rightarrow \quad \langle O \rangle = \frac{1}{\mathcal{Z}} \int \mathcal{D}[\varphi]\, \mathrm{e}^{-S_{\rm rel} - a^4 \sum_x \mathcal{L}_{\rm stat}(x)}\, O \left\{1 - a^4 \textstyle\sum_x \mathcal{L}^{(1)}(x) + \ldots\right\}$$

Expectation values

Path integral representation at the quantum level

$$\langle O \rangle = \frac{1}{\mathcal{Z}} \int \mathcal{D}[\varphi]\, O[\varphi]\, e^{-(S_{\rm rel}+S_{\rm HQET})} \qquad \mathcal{Z} = \int \mathcal{D}[\varphi]\, e^{-(S_{\rm rel}+S_{\rm HQET})}$$

Now the *integrand* is expanded in a *power series* in $1/m$

$$\exp\{-S_{\rm HQET}\} =$$
$$\exp\left\{-a^4 \sum_x \mathcal{L}_{\rm stat}(x)\right\} \left\{1 - a^4 \sum_x \mathcal{L}^{(1)}(x) + \tfrac{1}{2}\left[a^4 \sum_x \mathcal{L}^{(1)}(x)\right]^2 - a^4 \sum_x \mathcal{L}^{(2)}(x) + \ldots\right\}$$

$$\Rightarrow \quad \langle O \rangle = \frac{1}{\mathcal{Z}} \int \mathcal{D}[\varphi]\, e^{-S_{\rm rel} - a^4 \sum_x \mathcal{L}_{\rm stat}(x)}\, O \left\{1 - a^4 \sum_x \mathcal{L}^{(1)}(x) + \ldots\right\}$$

Important (but not automatic) implications of this definition of HQET

- $1/m$ – terms appear only as *insertions* of local operators
  $\Rightarrow$ Power counting: Renormalizability at any given order in $1/m$
- $\Leftrightarrow$ Existence of the continuum limit with universality
- Effective theory $=$ Continuum asymptotic expansion in $1/m$

Expectation values

Path integral representation at the quantum level

$$\langle O \rangle = \frac{1}{\mathcal{Z}} \int \mathcal{D}[\varphi]\, O[\varphi]\, e^{-(S_{\rm rel}+S_{\rm HQET})} \qquad \mathcal{Z} = \int \mathcal{D}[\varphi]\, e^{-(S_{\rm rel}+S_{\rm HQET})}$$

Now the *integrand* is expanded in a *power series* in $1/m$

$$\exp\{-S_{\rm HQET}\} =$$

$$\exp\left\{-a^4\sum_x \mathcal{L}_{\rm stat}(x)\right\}\left\{1 - a^4\sum_x \mathcal{L}^{(1)}(x) + \tfrac{1}{2}\left[a^4\sum_x \mathcal{L}^{(1)}(x)\right]^2 - a^4\sum_x \mathcal{L}^{(2)}(x) + \ldots\right\}$$

$$\Rightarrow \quad \langle O \rangle = \frac{1}{\mathcal{Z}} \int \mathcal{D}[\varphi]\, e^{-S_{\rm rel} - a^4 \sum_x \mathcal{L}_{\rm stat}(x)}\, O \left\{1 - a^4\sum_x \mathcal{L}^{(1)}(x) + \ldots\right\}$$

Explicitly:

$$\begin{aligned} \langle \mathcal{O} \rangle &= \langle \mathcal{O} \rangle_{\rm stat} + \omega_{\rm kin}\, a^4 \sum_x \langle \mathcal{O}\, \mathcal{O}_{\rm kin}(x) \rangle_{\rm stat} + \omega_{\rm spin}\, a^4 \sum_x \langle \mathcal{O}\, \mathcal{O}_{\rm spin}(x) \rangle_{\rm stat} \\ &\equiv \langle \mathcal{O} \rangle_{\rm stat} + \omega_{\rm kin} \langle \mathcal{O} \rangle_{\rm kin} + \omega_{\rm spin} \langle \mathcal{O} \rangle_{\rm spin} \end{aligned}$$

$$\langle \mathcal{O} \rangle_{\rm stat} = \frac{1}{\mathcal{Z}} \int_{\rm fields} \mathcal{O} \exp\left\{ - a^4 \sum_x \left[\mathcal{L}_{\rm light}(x) + \mathcal{L}_{\rm h}^{\rm stat}(x) \right] \right\}$$

Mass renormalization pattern in HQET

Already at the level of $\mathcal{L}_{\rm stat}(x) = \overline{\psi}_{\rm h}(x)\,[\,\nabla_0^* + \delta m\,]\,\psi_{\rm h}(x)$:

Linear divergence $\delta m \propto a^{-1}$ originates from mixing of $\overline{\psi}_{\rm h} D_0 \psi_{\rm h}$ with $\overline{\psi}_{\rm h}\psi_{\rm h}$

$$m_{\rm b}^{\overline{\rm MS}} = Z^{\overline{\rm MS}}\{\, m_{\rm bare} + \delta m \,\} \qquad m_{\rm bare} = m_{\rm B} - E_{\rm stat}$$

$$\left[\begin{array}{ll} m_{\rm B}: & \text{(exp.) B-meson mass} \\ E_{\rm stat}: & \text{static binding energy} \end{array}\right]$$

$$\delta m = \frac{c(g_0)}{a} \sim e^{1/(2b_0 g_0^2)} \times \left\{\, c_1 g_0^2 + c_2 g_0^4 + \ldots \right\}$$

Mass renormalization pattern in HQET

Already at the level of $\mathcal{L}_{\rm stat}(x) = \overline{\psi}_{\rm h}(x)\,[\,\nabla_0^* + \delta m\,]\,\psi_{\rm h}(x)$:

Linear divergence $\delta m \propto a^{-1}$ originates from mixing of $\overline{\psi}_{\rm h} D_0 \psi_{\rm h}$ with $\overline{\psi}_{\rm h}\psi_{\rm h}$

$$m_{\rm b}^{\overline{\rm MS}} = Z^{\overline{\rm MS}}\{\,m_{\rm bare} + \delta m\,\} \qquad m_{\rm bare} = m_{\rm B} - E_{\rm stat}$$

$$\left[\begin{array}{ll} m_{\rm B}: & \text{(exp.) B-meson mass} \\ E_{\rm stat}: & \text{static binding energy} \end{array}\right]$$

$$\delta m = \frac{c(g_0)}{a} \sim e^{1/(2b_0 g_0^2)} \times \{\,c_1 g_0^2 + c_2 g_0^4 + \ldots\}$$

- In PT:
  uncertainty $=$ truncation error $\sim e^{1/(2b_0 g_0^2)}\, c_{n+1}\, g_0^{2n+2} \xrightarrow{g_0 \to 0} \infty$!
  $\Rightarrow$ *NP renormalization (resp. matching to QCD) of HQET required for the continuum limit to exist*
- Power-law divergences even worse at the level of $1/m$ – corrections:
  $a^{-1} \to a^{-2}$

NP matching in *finite* volume

[H. & Sommer, JHEP0402(2004)022]

We want: $Lm_b \gg 1$ to apply HQET & $am_b \ll 1$ to keep a–effects small

NP matching in *finite* volume

[H. & Sommer, JHEP0402(2004)022]

We want: $Lm_b \gg 1$ to apply HQET & $am_b \ll 1$ to keep a–effects small

Objection: Need to treat/simulate the b-quark as particle with finite mass

NP matching in *finite* volume

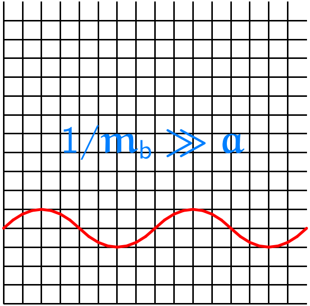
[H. & Sommer, JHEP0402(2004)022]

We want: $Lm_b \gg 1$ to apply HQET & $am_b \ll 1$ to keep a–effects small

Objection: Need to treat/simulate the b-quark as particle with finite mass

$\Rightarrow$ Trick: Start with QCD in a *small* volume $V = L^4$, $L \equiv L_1 \simeq 0.4\,\mathrm{fm}$

QCD



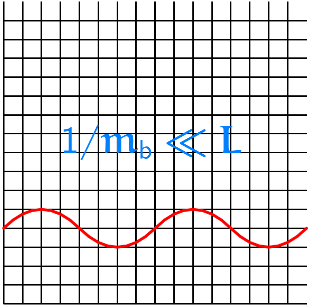
Matching conditions

$$\Phi_k^{\mathrm{QCD}} = \Phi_k^{\mathrm{HQET}}$$

for observables Φ_k

(renormalized quantities, computable for $a \to 0$)

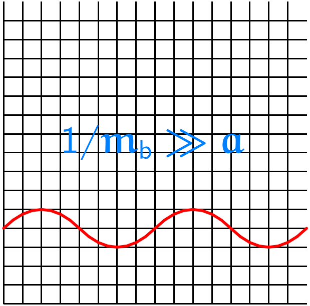
HQET



NP matching in *finite* volume

[H. & Sommer, JHEP0402(2004)022]

QCD



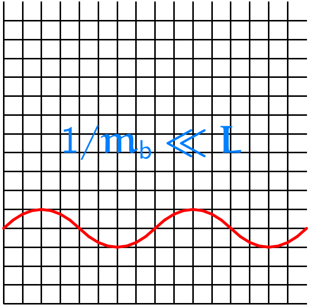
Matching conditions

$$\Phi_k^{\mathrm{QCD}} = \Phi_k^{\mathrm{HQET}}$$

for observables Φ_k

(renormalized quantities, computable for $a \to 0$)

HQET



- HQET parameters fixed by relating them to QCD observables in small V
- Sound approach, because
  - the underlying Lagrangian does not know about the finite V !
  - rather than having a propagating ‘real’ relativistic b-quark, one aims at determining the *NP heavy quark mass dependence* of Φ_k^{QCD}

Connecting small and large volumes

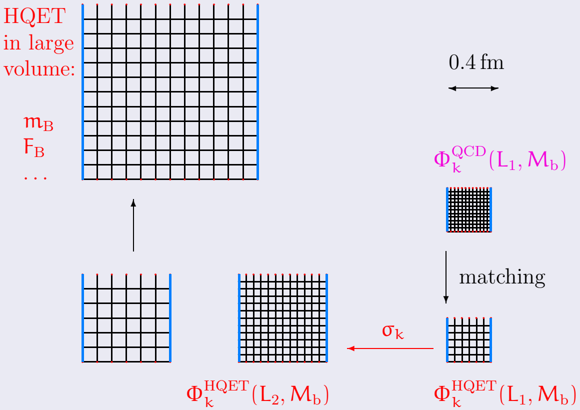
- Matching volume: $L = L_1 \simeq 0.4\,\mathrm{fm}$, very small lattice spacings
- Gap to large volumes and practicable lattice spacings, where physical quantities (e.g. m_B, F_{B_s}) may be extracted, bridged by a . . .

Connecting small and large volumes

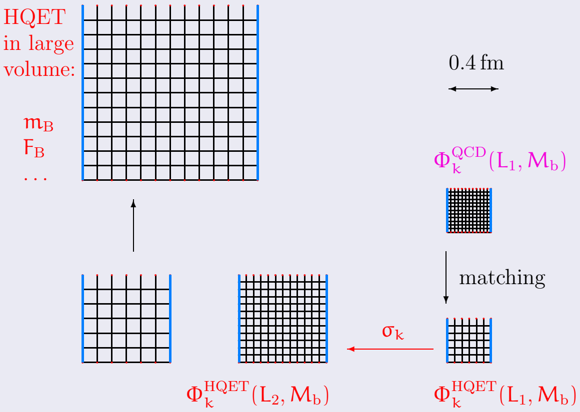
- Matching volume: $L = L_1 \simeq 0.4\,\text{fm}$, very small lattice spacings
- Gap to large volumes and practicable lattice spacings, where physical quantities (e.g. m_B, F_{B_s}) may be extracted, bridged by a . . .

Finite-size scaling step

[Lüscher, Weisz & Wolff, 1991; ALPHA Collaboration, 1993-2006]



Finite-size scaling step [Lüscher, Weisz & Wolff, 1991; ALPHA Collaboration, 1993-2006]



- Fully non-perturbative, continuum limit can be taken everywhere
- Use the *QCD Schrödinger Functional*, $L \to 2L$ via Step Scaling Functions

$$\Phi_k^{HQET}(2L) = \sigma_k\left(\{\Phi_j^{HQET}(L), j = 1, \ldots, N\}\right) \qquad 2L = 2L_1 \simeq 0.8\,\mathrm{fm}$$

$\rightarrow$ Large V ($L \simeq 2$ fm) at same resolution, where a B-meson fits comfortably

Implementation in a concrete example: The bottom quark mass

Computation of M_b

H. & Sommer, JHEP0402(2004)022
Della Morte, Garron, Sommer & Papinutto, JHEP0701(2007)007

Non-trivial matching problem:

$$\{\, m_{\rm bare} + \delta m \,,\ \omega_{\rm kin} \,,\ \omega_{\rm spin} \,\} \qquad \longleftrightarrow \qquad M_b$$

$\Rightarrow$ $N = 3$ matching conditions:

$$\Phi_k^{\rm QCD}(L, M) \;=\; \Phi_k^{\rm HQET}(L, M) \qquad k = 1, 2, 3$$

[M : RGI heavy quark mass]

Computation of M_b

H. & Sommer, JHEP0402(2004)022
Della Morte, Garron, Sommer & Papinutto, JHEP0701(2007)007

Non-trivial matching problem:

$$\{\, m_{\rm bare} + \delta m \,,\; \omega_{\rm kin} \,,\; \omega_{\rm spin} \,\} \qquad \longleftrightarrow \qquad M_b$$

$\Rightarrow$ $N = 3$ matching conditions:

$$\Phi_k^{\rm QCD}(L, M) \;=\; \Phi_k^{\rm HQET}(L, M) \qquad k = 1, 2, 3$$

[M : RGI heavy quark mass]

Basic equation in leading order of HQET (static approximation)

$$m_B \;=\; E_{\rm stat} - E^{\rm sub}_{\rm stat} + E^{\rm sub}_{\rm stat} \quad \text{with} \quad E^{\rm sub}_{\rm stat} = E(L_1, M_b) \quad \text{[matching to QCD]}$$

$$= \underbrace{E_{\rm stat} - E_{\rm stat}(L_2)}_{a \to 0 \text{ in HQET}} + \underbrace{E_{\rm stat}(L_2) - E_{\rm stat}(L_1)}_{a \to 0 \text{ in HQET } (\sigma_m)} + \underbrace{E(L_1, M_b)}_{\stackrel{a\to 0}{\equiv} \Phi_2/L_1 \text{ in QCD}} \quad (\star)$$

- Divergent static quark's self-energy δm cancels in *differences*!
- $\Phi_2(L_1, M)$ carries entire (relativistic) heavy quark mass dependence

Basic equation in leading order of HQET (static approximation)

$$m_B = E_{stat} - E^{sub}_{stat} + E^{sub}_{stat} \quad \text{with} \quad E^{sub}_{stat} = E(L_1, M_b) \quad \text{[matching to QCD]}$$

$$= \underbrace{E_{stat} - E_{stat}(L_2)}_{a \to 0 \text{ in HQET}} + \underbrace{E_{stat}(L_2) - E_{stat}(L_1)}_{a \to 0 \text{ in HQET } (\sigma_m)} + \underbrace{E(L_1, M_b)}_{\overset{a \to 0}{\equiv} \Phi_2/L_1 \text{ in QCD}} \quad (\star)$$

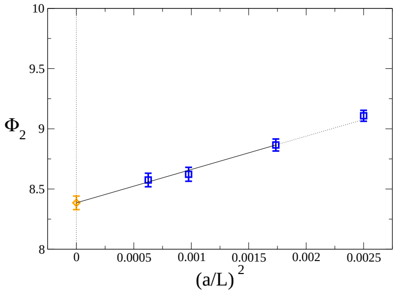
- Divergent static quark's self-energy δm cancels in *differences* !
- $\Phi_2(L_1, M)$ carries entire (relativistic) heavy quark mass dependence

Basic equation in leading order of HQET (static approximation)

$$m_B = E_{stat} - E_{stat}^{sub} + E_{stat}^{sub} \quad \text{with} \quad E_{stat}^{sub} = E(L_1, M_b) \quad \text{[matching to QCD]}$$

$$= \underbrace{E_{stat} - E_{stat}(L_2)}_{a \to 0 \text{ in HQET}} + \underbrace{E_{stat}(L_2) - E_{stat}(L_1)}_{a \to 0 \text{ in HQET } (\sigma_m)} + \underbrace{E(L_1, M_b)}_{\overset{a\to 0}{\equiv} \Phi_2/L_1 \text{ in QCD}} \quad (\star)$$

- Divergent static quark's self-energy δm cancels in *differences*!
- $\Phi_2(L_1, M)$ carries entire (relativistic) heavy quark mass dependence



Quenched result

Della Morte, Garron, Sommer & Papinutto
JHEP01(2007)007

- $L_1 \simeq 0.4\,\text{fm}\,,\; L_2 = 2L_1$
- Use $r_0 m_B^{(exp)}, r_0 = 0.5\,\text{fm}$ & solve $(\star)$

  $\Rightarrow \; M_b^{stat} = (6771 \pm 99)\,\text{MeV}$

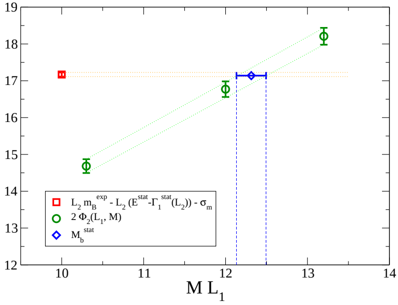
  - NP renormalization & Continuum limit
  - Error dominated by that on Z_M ($\simeq 1\%$) in $LM = Z_M\, Z\,(1 + b_m a m_q) \times L m_q$

Basic equation in leading order of HQET (static approximation)

$$m_B = E_{stat} - E^{sub}_{stat} + E^{sub}_{stat} \quad \text{with} \quad E^{sub}_{stat} = E(L_1, M_b) \quad \text{[matching to QCD]}$$

$$= \underbrace{E_{stat} - E_{stat}(L_2)}_{a \to 0 \text{ in HQET}} + \underbrace{E_{stat}(L_2) - E_{stat}(L_1)}_{a \to 0 \text{ in HQET } (\sigma_m)} + \underbrace{E(L_1, M_b)}_{\overset{a\to 0}{\equiv} \Phi_2/L_1 \text{ in QCD}} \quad (\star)$$

- Divergent static quark's self-energy δm cancels in *differences* !
- $\Phi_2(L_1, M)$ carries entire (relativistic) heavy quark mass dependence



Quenched result

Della Morte, Garron, Sommer & Papinutto
JHEP01(2007)007

- $L_1 \simeq 0.4\,\text{fm}$, $L_2 = 2L_1$
- Use $r_0 m_B^{(exp)}$, $r_0 = 0.5\,\text{fm}$ & solve $(\star)$

$$\Rightarrow \quad M_b^{stat} = (6771 \pm 99)\ \text{MeV}$$

- NP renormalization & Continuum limit
- Error dominated by that on Z_M ($\simeq 1\%$) in $LM = Z_M Z\,(1 + b_m a m_q) \times L m_q$

Inclusion of $1/m$–terms

Della Morte, Garron, Sommer & Papinutto, JHEP0701(2007)007

m_B at next-to-leading order of HQET

$$m_B = m_{bare} + E_{stat} + \omega_{kin} E_{kin} + \omega_{spin} E_{spin}$$

- E_{kin}, E_{spin} associated with $\overline{\psi}_h(-\frac{1}{2}\mathbf{D}^2)\psi_h$ and $\overline{\psi}_h(-\frac{1}{2}\sigma\cdot\mathbf{B})\psi_h$ in $\mathcal{L}^{(1)}$
  $\rightarrow$ Three observables Φ_1, Φ_2, Φ_3 required in the matching step
- Considering the *spin-averaged* B-meson instead, ω_{spin} cancels:

$$m_B^{(av)} = \frac{1}{4} m_B + \frac{3}{4} m_B^* = m_{bare} + E_{stat} + \omega_{kin} E_{kin}$$

$\rightarrow$ Only *two* obervables Φ_1, Φ_2 necessary

Inclusion of $1/m$–terms

Della Morte, Garron, Sommer & Papinutto, JHEP0701(2007)007

m_B at next-to-leading order of HQET

$$m_B = m_{bare} + E_{stat} + \omega_{kin} E_{kin} + \omega_{spin} E_{spin}$$

- E_{kin}, E_{spin} associated with $\overline{\psi}_h(-\frac{1}{2}\mathbf{D}^2)\psi_h$ and $\overline{\psi}_h(-\frac{1}{2}\sigma\cdot\mathbf{B})\psi_h$ in $\mathcal{L}^{(1)}$
  $\rightarrow$ Three observables Φ_1, Φ_2, Φ_3 required in the matching step
- Considering the *spin-averaged* B-meson instead, ω_{spin} cancels:

$$m_B^{(av)} = \frac{1}{4} m_B + \frac{3}{4} m_B^* = m_{bare} + E_{stat} + \omega_{kin} E_{kin}$$

$\rightarrow$ Only *two* obervables Φ_1, Φ_2 necessary

Strategy

Experiment		Lattice with $am_b \ll 1$
$m_B = 5.4\,\text{GeV}$	$L_1 \simeq 0.4\text{fm}, L_2 = 2L_1$ $u_1 = \bar{g}^2(L_1)$	$\Phi_1(L_1, M), \Phi_2(L_1, M)$
$\downarrow$		$\downarrow$
$\Phi_1^{HQET}(L_2), \Phi_2^{HQET}(L_2)$	$\xleftarrow[\sigma_1^{kin}(u_1),\, \sigma_2^{kin}(u_1)]{\sigma_m(u_1)}$	$\Phi_1^{HQET}(L_1), \Phi_2^{HQET}(L_1)$

Matching formula = Static part + $1/m-$correction :

$$L_2 m_B^{(av)} = L_2 m_B^{stat} \Big\} = L_2 \left[E_{stat} - \Gamma_1^{stat}(L_2) \right] + \sigma_m(u_1) + 2\Phi_2(L_1, M_b)$$

$$+ L_2 m_B^{(1)} \Big\} = \sigma_2^{kin}(u_1)\Phi_1(L_1, M_b) + L_2 \left[E_{kin} - \Gamma_1^{kin}(L_2) \right] \omega_{kin}$$

Matching formula $=$ Static part $+$ $1/m-$correction :

$$L_2 m_B^{(av)} = \left. L_2 m_B^{stat} \right\} = L_2 \left[E_{stat} - \Gamma_1^{stat}(L_2) \right] + \sigma_m(u_1) + 2\Phi_2(L_1, M_b)$$

$$+ \left. L_2 m_B^{(1)} \right\} = \sigma_2^{kin}(u_1)\Phi_1(L_1, M_b) + L_2 \left[E_{kin} - \Gamma_1^{kin}(L_2) \right] \omega_{kin}$$

Implementation

- SF boundary conditions, i.e. $T \times L^3$, Dirichlet at $x_0 = 0, T,$ fermion fields periodic in space modulo a phase: $\psi(x + L\hat{k}) = e^{i\theta}\psi(x)$
- Employ (finite-volume) B-meson energies for the matching: Avoid $c_A^{HQET} f_{\delta A}^{stat}-$term in boundary-to-$A_0$ correlator by boundary-to-boundary ones with pseudoscalar or vector quantum numbers

Matching formula $=$ Static part $+$ $1/m-$correction :

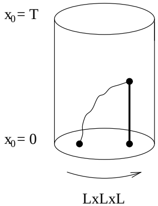
$$L_2 m_B^{(av)} = L_2 m_B^{stat} \} = L_2 \left[E_{stat} - \Gamma_1^{stat}(L_2) \right] + \sigma_m(u_1) + 2\Phi_2(L_1, M_b)$$

$$+ L_2 m_B^{(1)} \} = \sigma_2^{kin}(u_1)\Phi_1(L_1, M_b) + L_2 \left[E_{kin} - \Gamma_1^{kin}(L_2) \right] \omega_{kin}$$

Implementation

- SF boundary conditions, i.e. $T \times L^3$, Dirichlet at $x_0 = 0, T$, fermion fields periodic in space modulo a phase: $\psi(x + L\hat{k}) = e^{i\theta}\psi(x)$
- Employ (finite-volume) B-meson energies for the matching: Avoid $c_A^{HQET} f_{\delta A}^{stat}-$term in boundary-to-$A_0$ correlator by boundary-to-boundary ones with pseudoscalar or vector quantum numbers

$$(f_A)_R(x_0) = Z_A^{HQET} Z_{\zeta_h} Z_\zeta \, e^{-m_{bare}x_0} \left\{ f_A^{stat} + \omega_{kin} f_A^{kin} + \omega_{spin} f_A^{spin} + c_A^{HQET} f_{\delta A}^{stat} \right\}$$



Matching formula $=$ Static part $+$ $1/m-$correction :

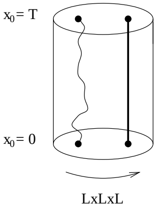
$$L_2 m_B^{(av)} = \left. L_2 m_B^{\rm stat} \right\} = L_2\left[E_{\rm stat} - \Gamma_1^{\rm stat}(L_2)\right] + \sigma_m(u_1) + 2\Phi_2(L_1, M_b)$$

$$+ \left. L_2 m_B^{(1)} \right\} = \sigma_2^{\rm kin}(u_1)\Phi_1(L_1, M_b) + L_2\left[E_{\rm kin} - \Gamma_1^{\rm kin}(L_2)\right]\omega_{\rm kin}$$

Implementation

- SF boundary conditions, i.e. $T \times L^3$, Dirichlet at $x_0 = 0, T$, fermion fields periodic in space modulo a phase: $\psi(x + L\hat{k}) = e^{i\theta}\psi(x)$
- Employ (finite-volume) B-meson energies for the matching: Avoid $c_A^{\rm HQET} f_{\delta A}^{\rm stat}-$term in boundary-to-$A_0$ correlator by boundary-to-boundary ones with pseudoscalar or vector quantum numbers

$$(f_1)_R(T) = Z_{\zeta_h}^2 Z_\zeta^2 \, e^{-m_{\rm bare}T}\left\{f_1^{\rm stat} + \omega_{\rm kin} f_1^{\rm kin} + \omega_{\rm spin} f_1^{\rm spin}\right\}$$



Matching formula $=$ Static part $+$ $1/m-$correction :

$$L_2 m_B^{(av)} = \left. L_2 m_B^{stat} \right\} = L_2 \left[E_{stat} - \Gamma_1^{stat}(L_2) \right] + \sigma_m(u_1) + 2\Phi_2(L_1, M_b)$$
$$+ \left. L_2 m_B^{(1)} \right\} = \sigma_2^{kin}(u_1)\Phi_1(L_1, M_b) + L_2 \left[E_{kin} - \Gamma_1^{kin}(L_2) \right] \omega_{kin}$$

Implementation

- SF boundary conditions, i.e. $T \times L^3$, Dirichlet at $x_0 = 0, T$, fermion fields periodic in space modulo a phase: $\psi(x + L\hat{k}) = e^{i\theta}\psi(x)$
- Employ (finite-volume) B-meson energies for the matching: Avoid $c_A^{HQET} f_{\delta A}^{stat}-$term in boundary-to-$A_0$ correlator by boundary-to-boundary ones with pseudoscalar or vector quantum numbers

$$(f_1)_R(T) = Z_{\zeta_h}^2 Z_\zeta^2 e^{-m_{bare}T} \left\{ f_1^{stat} + \omega_{kin} f_1^{kin} + \omega_{spin} f_1^{spin} \right\}$$

- Φ_1 : Suitable ratios of $f_1(\theta), f_1(\theta')$ such that $Z-$factors drop out
  Φ_2 : *Spin-averaged* energy $\Gamma_1 = -\partial_T \ln f_1^{(av)} = m_{bare} + \Gamma_1^{stat} + \omega_{kin}\Gamma_1^{kin}$
  plus corresponding step scaling functions $\sigma_1^{kin}(u), \sigma_2^{kin}(u)$

A few more details

Boundary-to-boundary SF correlation functions and energies:

$$f_1(\theta) = -\frac{a^{12}}{2L^6} \sum_{\mathbf{u},\mathbf{v},\mathbf{y},\mathbf{z}} \left\langle \bar{\zeta}_l{}'(\mathbf{u})\gamma_5\zeta_b'(\mathbf{v})\,\bar{\zeta}_b(\mathbf{y})\gamma_5\zeta_l(\mathbf{z})\right\rangle$$

$$k_1(\theta) = -\frac{a^{12}}{6L^6} \sum_{\mathbf{u},\mathbf{v},\mathbf{y},\mathbf{z},k} \left\langle \bar{\zeta}_l{}'(\mathbf{u})\gamma_k\zeta_b'(\mathbf{v})\,\bar{\zeta}_b(\mathbf{y})\gamma_k\zeta_l(\mathbf{z})\right\rangle$$

$$F_1(L,\theta) = \frac{1}{4}\Big[\ln f_1(\theta) + 3\ln k_1(\theta)\Big]$$

$$R_1(L,\theta_1,\theta_2) = F_1(L,\theta_1) - F_1(L,\theta_2)\,|_{T=L/2}$$

$$\Gamma_1(L,\theta_0) = -\frac{\partial_T + \partial_T^*}{2} F_1(L,\theta_0)\Big|_{T=L/2}$$

A few more details

From these, form dimensionless observables

$$\begin{aligned}
\Phi_1(L, M_b) &= R_1(L, \theta_1, \theta_2) - R_1^{\mathrm{stat}}(L, \theta_1, \theta_2) \\
\Phi_2(L, M_b) &= L\,\Gamma_1(L, \theta_0) \\
R_1^{\mathrm{stat}}(L, \theta_1, \theta_2) &= \ln\left[f_1^{\mathrm{stat}}(L, \theta_1)/f_1^{\mathrm{stat}}(L, \theta_2) \right]\Big|_{T=L/2}
\end{aligned}$$

that have $1m_b$ – expansions

$$\begin{aligned}
\Phi_1(L, M_b) &= \omega_{\mathrm{kin}} R_1^{\mathrm{kin}}(L, \theta_1, \theta_2) \\
\Phi_2(L, M_b) &= L\left[m_{\mathrm{bare}} + \Gamma_1^{\mathrm{stat}}(L, \theta_0) + \omega_{\mathrm{kin}} \Gamma_1^{\mathrm{kin}}(L, \theta_0) \right]
\end{aligned}$$

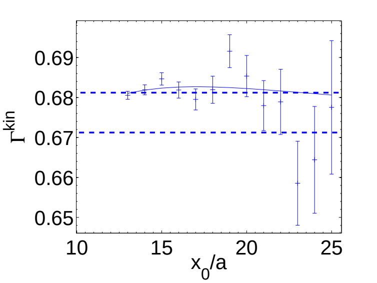
Moreover

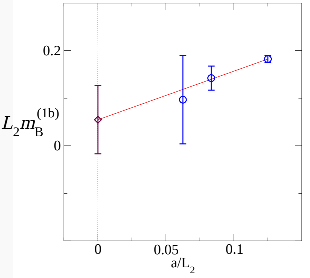
- define SSFs $\sigma_m(u), \sigma_1^{\mathrm{kin}}(u), \sigma_2^{\mathrm{kin}}(u)$ in the effective theory, and
- fit large – x_0 asymptotics of the large-volume B-meson energy to $\Gamma(x_0) = E_{\mathrm{stat}} + \omega_{\mathrm{kin}} E_{\mathrm{kin}} + (A^{\mathrm{stat}} + \omega_{\mathrm{kin}} A^{\mathrm{kin}})\, e^{-\Delta^{\mathrm{stat}} x_0} (1 - \omega_{\mathrm{kin}} x_0 \Delta^{\mathrm{kin}}) + \ldots$

A few more details

Most difficult piece encountered in the quenched calculation:

Large-volume HQET matrix element $[\, E_{kin} - \Gamma_1^{kin}(L_2)\,]$ entering one of the $1/m_b$ – contributions

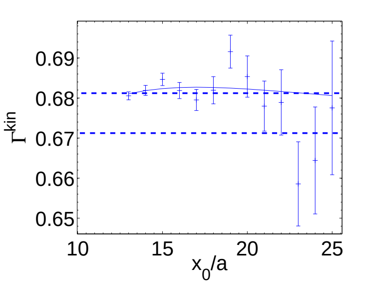


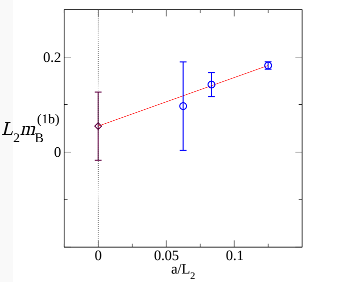


A few more details

Most difficult piece encountered in the quenched calculation:

Large-volume HQET matrix element $[\, E_{kin} - \Gamma_1^{kin}(L_2)\,]$ entering one of the $1/m_b$ – contributions





Result in the quenched approximation

$$\overline{m}_b^{\overline{MS}}(\overline{m}_b) = 4.374(64)\ \text{GeV} \underbrace{-\ 0.027(22)\ \text{GeV}}_{O(\Lambda^2/m_b)} + \underbrace{O(\Lambda^3/m_b^2)}_{\text{negligible}}$$

Sketch & Status
of the computation in two-flavour QCD

Physics goals

- $m_b^{(N_f=2)}$
- $F_{B_s}^{(N_f=2)}$ [$N_f = 0$: talk by N. Garron]

from lattice HQET including $1/m_b$ – corrections

General setup of the dynamical finite-volume simulations:

- QCD with Schrödinger Functional boundary conditions (T, L, θ)
- $N_f = 2$ degenerate massless sea quarks $(m_l \equiv m_{light} = 0)$
- Evaluation of heavy-light correlation functions composed of non-degenerate *quenched* valence quarks (where mostly $m_l^{val} = m_l$)
- Configurations generated on apeNEXT @ DESY-Zeuthen

Elements of the computation

(1) Matching to QCD with a relativistic b-quark

- Choice of the matching volume resp. L_1 :
  - L_1 such that $L_1/r_0 \approx 1$, i.e. $L_1 \approx (0.4 - 0.5)\,\mathrm{fm}$
    $\Rightarrow$ connection to large volume possible after one step scaling step

$$L_1 \quad \xrightarrow{\sigma_k} \quad L_2 = 2L_1 \qquad L_\infty = 4L_1 \approx 2\,\mathrm{fm}$$

  - Based on the knowledge of the $N_f = 2$ running of the renormalized SF coupling $\bar{g}^2 \equiv \bar{g}^2_{\rm SF}$ (resp. its SSF σ) from [ALPHA Collaboration , NPB713(2005)378]
    L_1 is fixed by the condition

$$\bar{g}^2(L_0) = \text{constant} \approx 3.0 \qquad \text{where} \quad L_0 = L_1/2$$

$$\text{such that} \qquad u_1 = \bar{g}^2(L_1) = \sigma\left(\bar{g}^2(L_0)\right) \approx 4.5$$

$\Rightarrow$ $(L_1/(2a), \beta, \kappa_l)$ with $L_1/(2a) = 10, 12, 16, 20$ & $m_l^{\rm PCAC}(L_1/2) = 0$

Elements of the computation

(1) Matching to QCD with a relativistic b-quark

- NP calculation of the heavy quark mass dependence of heavy-light meson observables in (the cont. limit of) finite-volume lattice QCD :
  - $L_1/a = 20, 24, 32, 40$, $T = L_1$, same β's and $m_l^{\mathrm{PCAC}}(L_1) = 0$
  - Fix the RGI heavy quark mass to values around the b-quark via

$$z \;\equiv\; L_1 M \;=\; L_1 \times Z_m \times \underbrace{\frac{M}{\overline{m}(\mu_0)}} \times m_q\,(1 + b_m\,am_q)\;,\quad \mu_0 = \frac{2}{L_1}$$

[ALPHA Collaboration, NPB729(2005)117]

$$am_q \;=\; \frac{1}{2}\left(\frac{1}{\kappa} - \frac{1}{\kappa_c}\right) \qquad Z_m \;=\; \frac{Z(g_0)\,Z_A(g_0)}{Z_P(g_0, L_1/(2a))}$$

$\Rightarrow$ demands to determine $b_m(g_0)$ and $Z(g_0)$ [talk by P. Fritzsch]

- In progress . . .

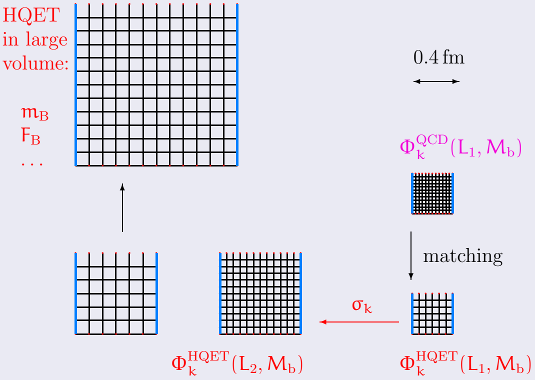
Elements of the computation — 2-Do's & Prospects

- For $1/m$ – corrections :
  - $-\partial_T \ln f_1^{(av)}$ requires additional lattices with $T = L_1/2$, $T = L_1/2 \pm a$
  - however, these simulations will be by a factor $\simeq 4$ less expensive

Elements of the computation — 2-Do's & Prospects

- For $1/m-$corrections :
  - $-\partial_T \ln f_1^{(av)}$ requires additional lattices with $T = L_1/2$, $T = L_1/2 \pm a$
  - however, these simulations will be by a factor $\simeq 4$ less expensive

For the HQET side, remember the general strategy . . .



Elements of the computation — 2-Do's & Prospects

- For $1/m$ – corrections :
  - $-\partial_T \ln f_1^{(av)}$ requires additional lattices with $T = L_1/2$, $T = L_1/2 \pm a$
  - however, these simulations will be by a factor $\simeq 4$ less expensive

(2) Connection to $L_2 = 2L_1$ in HQET

- Through SSFs $\sigma_m(u_1)$, $\sigma_1^{kin}(u_1)$, $\sigma_2^{kin}(u_1)$, after some tuning :
  - $(L_1/a, \beta), (L_2/a, \beta)$ s. th. $\bar{g}^2(L_1), \bar{g}^2(L_2)$ fixed $(L_i/a = (6), 8, 12, 16)$
- Outcome for $5.2 \lesssim \beta \lesssim 5.6$:
  - δm , ω_{kin} , ω_{spin}
  - Z_A^{HQET} , c_A^{HQET}

Elements of the computation — 2-Do's & Prospects

- For $1/m$ – corrections :
  - $-\partial_T \ln f_1^{(av)}$ requires additional lattices with $T = L_1/2$, $T = L_1/2 \pm a$
  - however, these simulations will be by a factor $\simeq 4$ less expensive

(2) Connection to $L_2 = 2L_1$ in HQET

- Through SSFs $\sigma_m(u_1)$, $\sigma_1^{\rm kin}(u_1)$, $\sigma_2^{\rm kin}(u_1)$, after some tuning :
  - $(L_1/a, \beta), (L_2/a, \beta)$ s. th. $\bar{g}^2(L_1), \bar{g}^2(L_2)$ fixed $(L_i/a = (6), 8, 12, 16)$
- Outcome for $5.2 \lesssim \beta \lesssim 5.6$:
  - δm , $\omega_{\rm kin}$, $\omega_{\rm spin}$
  - $Z_A^{\rm HQET}$, $c_A^{\rm HQET}$

(3) HQET observables in physically large volume, $L_\infty = 4L_1 \approx 2\,\text{fm}$

- $E_{\rm stat}$, $E_{\rm kin}$, $E_{\rm spin}$, . . . :
  - periodic BCs, low-mode deflation & all-to-all propagators à la Dublin
  - generate, share & use existing configurations