Gluelumps and Hybrid Mesons

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Outline

What is a Gluelump?

Abbildung: Gluelump http://www.hanisauland.de/zoom/hihu1

Hybrid Mesons and Gluelumps

Definition

Hybrid mesons are states consisting of a quark-antiquark pair bound by an excited gluon field.

Definition

A system containing a valence gluon connected by an adjoint string to an adjoint source is called *gluelump*.

- adjoint source $\widehat{=}$ infinitely heavy gluon $\longrightarrow$ Gluino?
- no physical object, cannot be measured experimentally
- resemblance to glueballs and heavy-light mesons

Why to consider Gluelumps?

- gluelump masses define decay of field strength correlators in QCD vacuum

$$G^{gluelump}(x, y) \leftrightarrow \left\langle F^{a}_{\mu\nu}(x) F^{b}_{\lambda\sigma}(y) \right\rangle$$

- inverse mass of lowest gluelump equals the gluon correlation length T_g
- testing ground for models of low energy QCD (MIT bag model, flux tube model)
- needed for getting insights in non-perturbative features of QCD (heavy quarkonia) $\rightarrow$ idealized system

Representations of SU(N)

Definition of representation

A *representation* is a specific realization of the multiplication of the group elements by matrices.

1. fundamental (defining) representation $\stackrel{SU(3)}{\Rightarrow}$ quarks, antiquarks
2. adjoint (regular) representation $\stackrel{SU(3)}{\Rightarrow}$ gluons

remember $SU(2)$ Spin-systems:

$$\left|j,m;j_1;j_2\right\rangle = \sum_{m_1+m_2=m} C\left(j_1,m_1;j_2,m_2\middle|j,m;j_1;j_2\right)\left|j_1,m_1;j_2,m_2\right\rangle.$$

Fundamental Representation

- complete set of hermitian traceless 3×3 matrices
  $\lambda_k, \qquad k = 1, ..3^2 - 1$
  $$\lambda_k = \lambda_k^\dagger, \qquad Tr(\lambda_k) = 0, \qquad Tr(\lambda_k \lambda_l) = 2\delta_{kl}$$
- fulfil
  $$[\lambda_k, \lambda_l] = 2i f_{klm} \lambda_m$$
  where f_{klm} totally antisymmetric structure constants of the group
- fundamental representation:
  $$t_k = \frac{1}{2}\lambda_k, \qquad k = 1, ..8$$
- exponential parameterization
  $$U = e^{i\alpha^k t_k}, \qquad U \in SU(3)$$

Adjoint Representation

- changing between representations → arbitrary unitary transformation
- adjoint representation R, carried by generators

$$U^\dagger t_k U = R_{kl} t_l, \qquad U \in SU(3)$$

- explicit representation in terms of group elements

$$R_{kl} = 2Tr(U^\dagger t_k U \ t_l)$$

- in terms of parameters α^k

$$U(y) = e^{iy\alpha^p t_p}, \qquad R_{kl} = 2Tr(U^\dagger(y) t_k U(y) t_l)$$

Adjoint Representation

- consider derivative

$$\frac{\partial}{\partial y} R_{kl} = -i\alpha^p 2 Tr(U^\dagger(y)[t_p, t_k] U(y) t_l)$$
$$= i\alpha^p (F_p)_{kn} R_{nl}$$

where $(F_p)_{mn} \equiv -if_{pmn}$

- in matrix-notation:

$$\frac{\partial}{\partial y} R(y) = i\alpha^p F_p R(y)$$
$$\Rightarrow R(y) = e^{i\alpha^p F_p}, \qquad \text{using} \qquad R(0) = 1$$

$\Rightarrow$ F_p are the generators in the adjoint representation.

$\Rightarrow$ The structure constants generate the adjoint representation.

Mass Determination

Extract ground state mass of a meson state with quantum numbers α, denote $O_\alpha \equiv \overline{\Psi}_\alpha S \Psi_\alpha$:

1. consider vacuum expectation value

$$C_\alpha = \langle 0 | O_\alpha(t, \mathbf{x}) O_\alpha(0, \mathbf{x}) | 0 \rangle$$

2. time development (Eucl.): $O(\tau) = e^{H\tau} O e^{-H\tau}$
3. insert complete set of eigenstates of the Hamiltonian:

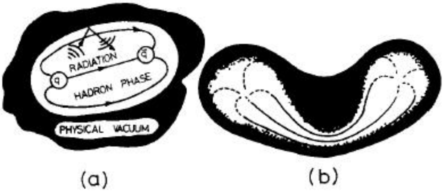
$$\begin{aligned} C_\alpha(\tau) &= \sum_n | \langle 0 | O_\alpha | n_\alpha \rangle |^2 e^{-(E_n^\alpha - E_0^\alpha)\tau} \\ &\overset{\tau\to\infty}{\longrightarrow} e^{-(E_1^\alpha - E_0^\alpha)\tau} | \langle 0 | O_\alpha | 1_\alpha \rangle |^2 \end{aligned}$$

- $\Rightarrow$ lowest energy eigenstate dominates sum, when states have good overlap with each other
- otherwise change $O \Rightarrow$ smearing techniques

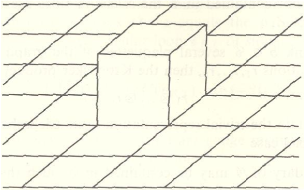
The Problem

- Hybrid meson $\hat{=}$ meson with excited 'constituent' glue
  $\longrightarrow$ exotic quantum numbers
- What is 'constituent' glue? $\Rightarrow$ *QCD does not distinguish*
- even quenched approximation cannot neglect 'sea' gluons
- distinguish between hybrids and standard quark model states

$\Rightarrow$ bag models, strong coupling lattice model or flux tube model

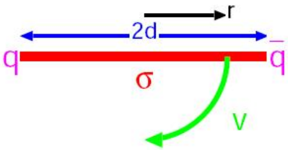


P. Hasenfratz and J. Kuti: Quark Bag Model



Lowest non-trivial graph in the strong-coupling expansion G. Münster, I. Montvay

The Flux Tube Model



Gunnar S. Bali

- large separation $d \rightarrow$ Nambu-Goto string
- Cornell potential:
  $V(r) = -\frac{e}{r} + \sigma r \qquad e = \frac{4}{3}\alpha_s$
- for $d \rightarrow 0$ gluelumps
- system rotationally invariant
  $\Rightarrow J^{PC}$ quantum numbers

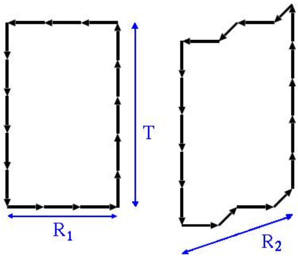
- distinction between hybrid meson and ordinary meson not well defined
- hybrid potential clearly distinct from ground state potential Σ_g^+ or its radial excitations

$\Rightarrow$ need hybrid potential

Wilson Loop

Wilson loop

$$W(C) = Tr\Big\{\mathcal{P}\Big[\exp\big(i\int_{\delta C} dx_\mu A_\mu(x)\big)\Big]\Big\}$$
$$= Tr\Big(\prod_{(x,\mu)\in\delta C} U_{x,\mu}\Big)$$



- definition in fundamental representation
- require Wilson loop in adjoint representation
- use relation: $A^{ab}_{x,\mu}[U_{x,\mu}] = \frac{1}{2} Tr(\sigma^a U_{x,\mu} \sigma^b U^\dagger_{x,\mu})$

Wilson Loop and Static Quark Potential

- notion of static quarks $\leftrightarrow$ behaviour of states under gauge transformation
- Yang-Mills action (Minkowski): $S = -\frac{1}{4}\int d^4x F^a_{\mu\nu} F^{\mu\nu a}$
- canonically conjugated momentum: $\pi^a_i = \frac{\delta S}{\delta(\partial_4 A^a_i)} = \frac{1}{g^2} F^a_{4i} = -\frac{1}{g} E^a_i$
- temporal gauge: $A^a_4 = 0 \qquad \Rightarrow \qquad \pi^a_\mu = -i\frac{\partial}{\partial^a_\mu}$
- construct Hamiltonian (acts onto $\Psi[A_\mu]$):

$$H = \int d^3x(\pi^a_\mu \partial_4 A^a_\mu - \frac{1}{4g^2} F^a_{\mu\nu} F^a_{\mu\nu}) = \frac{1}{2}\int d^3x(E^a_i E^a_i - B^a_i B^a_i)$$

- consider time independent gauge transformation $\Lambda(x)$, represented by operator $R(\Lambda)$:

$$(R(\Lambda)\Psi)[A_\mu] = \Psi[\Lambda^{-1}A_\mu\Lambda + \Lambda^{-1}\partial_\mu\Lambda]$$

- for infinitesimal gauge transformation $\Lambda = \mathbf{1} + i\omega^a T_a$ one obtains

$$R(\Lambda) = \mathbf{1} - i\omega^a \frac{1}{g} D^i E_i^a + \mathcal{O}(\omega^2).$$

$\Rightarrow D_i E_i$ generator of local gauge transformations

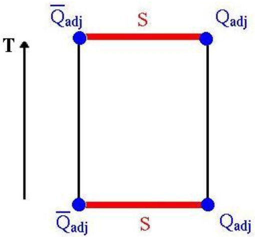
- remember Gauß law $D^i E_i = 0$

Gauß law	$\Leftrightarrow$	gauge invariance of wave functional
external charges	$\Leftrightarrow$	consider non-invariant wave functionals

- external charge: $D^i E_i^a \Psi = -g\delta^3(\mathbf{r}) T^a \Psi$
- with definition of static quark potential $V(R) \equiv E_0$ we obtain

$$\langle \Psi | e^{-TH} | \Psi \rangle = \sum_n |\langle \Psi^{(n)} | \Psi \rangle|^2 e^{-TE_n}$$

$$\overset{T\to\infty}{\sim} |\langle \Psi^{(0)} | \Psi \rangle|^2 e^{-TV(R)}$$



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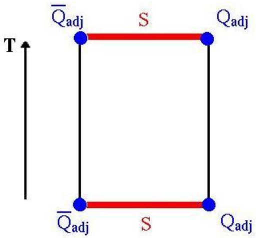
- taking adoint Wilson loop $W_{adj} = |W(R,T)|^2 - 1$ results in

Adjoint Potential and String Tension

$$V^{(adj)}(R) \equiv -\lim_{T\to\infty} \frac{1}{T} \ln W_{adj}(R,T)$$

$$\sigma^{(adj)} \equiv -\lim_{R\to\infty} V^{(adj)}(R) = -\lim_{R,T\to\infty} \frac{1}{RT} \ln W_{adj}(R,T)$$



Hybrid Potentials

- classify as excitations of homonuclear diatomic molecules: heavy quarks $\Rightarrow$ nuclei, gluon field $\Rightarrow$ electrons
- cylindrical symmetry group $D_{\infty h} = D_\infty \times Z_2$
- angular momentum $\Lambda_{\hat{\mathbf{r}}}$ about molecular axis $\Rightarrow \Lambda = 0, 1, 2.. = \Sigma, \Pi, \Delta..$
- state might transform evenly (g) or oddly (u) $\Rightarrow$ qn η
- $\Lambda = 0$: additional qn σ_v parity index
- determine energy-levels of the gluon field as function of $Q\overline{Q}$ separation r: adiabatic potentials $V_{Qg\overline{Q}}(r)$
- solving Schrödinger equation for each potential leads to possible hybrid charmonia and bottomonia levels

Schrödinger Equation

- total spin $\mathbf{J} = \mathbf{K} + \mathbf{S}$, where $\mathbf{K} = \mathbf{L} + \mathbf{S_g}$
- rotationally symmetric potential: $\psi_{nll_3}(\mathbf{x}) = \frac{u_{nl}(r)}{r} Y_{ll_3}(\theta, \phi)$
- Schrödinger equation:

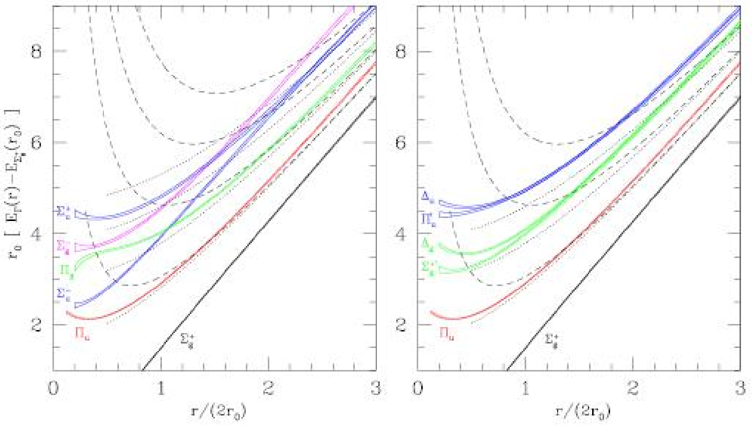
$$\left(\frac{\mathbf{p}^2}{2\mu_R} + V_{Qg\overline{Q}}(r)\right)\psi_{nll_3}(\mathbf{x}) = E_{nl}\psi_{nll_3}(\mathbf{x})$$

- leads to radial Schrödinger equation:

$$u''_{nl}(r) + 2\mu_R\left[E_{nl} - V_{Qg\overline{Q}}(r) - \frac{1}{2\mu_R r^2}\left(k(k+1) - 2\Lambda^2 + \left\langle \mathbf{S}_g^2 \right\rangle\right)\right] u_{nl}(r) = 0$$

where $\left\langle \mathbf{S}_g^2 \right\rangle = 0$ for Σ_g^+ and $\left\langle \mathbf{S}_g^2 \right\rangle = 2$ for Π_u and Σ_u^-

Hybrid Excitations of Static SU(3) Potential



C. J. Morningstar, K. J. Juge, J. Kuti, 1999

Singlet and Octet States

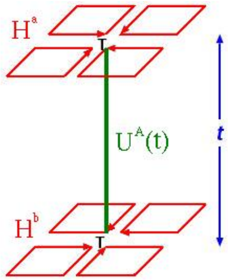
- consider limit $r \longrightarrow 0$: full rotational group $D_{\infty h} \subset O(3) \otimes C$
- classify states as singlets and octets according to their local gauge transformation properties:

singlet state	decouples from temporal transporters within $r = 0$ 'Wilson loop'
octet state	couples to temporal Schwinger line in adjoint representation

- in limit $m \longrightarrow \infty$ spin can be neglected
  $\Rightarrow$ temporal transporter $\widehat{=}$ static gluino propagator

singlet state	$\Leftrightarrow$	glueball
octet state	$\Leftrightarrow$	glueballino/gluelump

Correlation Function



The gluelump correlation function G. S. Bali

- start with correlation function

$$C(t) = \frac{1}{2N}\left\langle H^a_{\mathbf{0}t}[U^A_{\mathbf{0}}(t)]^{ab} H^b_{\mathbf{0}0}\right\rangle$$

- rewrite in fundamental representation using completeness relation
$2\sum_a T^a_{\alpha\beta} T^a_{\gamma\delta} = \delta_{\alpha\beta}\delta_{\beta\gamma} - \frac{1}{N}\delta_{\alpha\beta}\delta_{\gamma\delta}$
and identity:

$$C(t) = \left\langle Tr_F\left[H_{\mathbf{0},t} U_{\mathbf{0}}(t) H^{F,\dagger}_{\mathbf{0},0} U^{\dagger}_{\mathbf{0}}(t)\right]\right\rangle$$

'hybrid' Wilson loop in $\lim r \to 0$

factorise into singlet and octet components

$$\left\langle W_\Psi(r,t)\right\rangle = c_1 e^{-m_{gl}(a)t} + c_2 e^{-(m_{gb}+V_{\Sigma_g^+}(r,a))t} + \ldots \qquad (r \to 0)$$

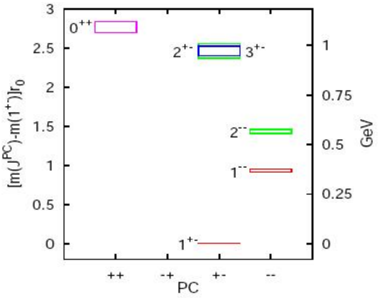
Gluelump Masses

$$\left\langle W_\Psi(r,t)\right\rangle = c_1 e^{-m_{gl}(a)t} + c_2 e^{-(m_{gb}+V_{\Sigma_g^+}(r,a))t} + \dots$$

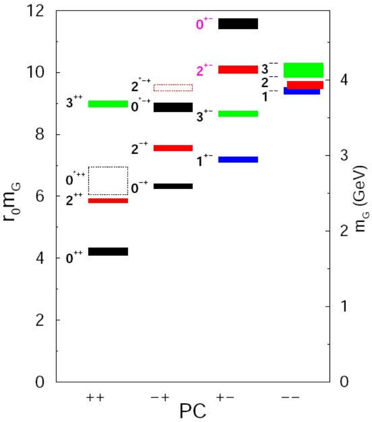
- on the lattice: $V_{\Sigma_g^+}(0,a) = V_{\Sigma_g^+}(0) + V_{self}(a) = 0$
- $r \gg a$: $V_{\Sigma_g^+}(r)$ approaches continuum potential
- assume mass of lightest gluelump within sector of allowed J^{PC} qn that have overlap with $\Psi^\dagger$
- gluelump mass will contain a finite and a self-energy contribution
$$m_{gl}(a) = m_{finite} + m_{self}(a)$$

$$\boxed{m_{self}(a) = \frac{C_A}{C_F}\frac{V_{self}(a)}{2} = \frac{N^2}{N^2-1}V_{self}(a) > V_{self}(a)}$$

Spectra



The lowest six gluelump states
M. Foster and C. Michael



The glueball spectrum of SU(3) gauge theory
C. J. Morningstar and M. Peardon

Short Review

1. defined gluelump
2. know how to determine masses in general
3. derived hybrid potential by using Wilson loop
4. computed gluelump mass

What can we do with this exactly?
Determine bottom quark mass!

Motivation of NRQCD

- heavy mesons not computable in full QCD
- heavy quark systems have characterising scale:
  m_Q is much larger than any dynamical scale in the problem
  $\Rightarrow$ use effective field theory (EFT)
- afterwards: matching to QCD (Foldy-Wouthuysen-Trafo, renormalon ambiguities, tree-level of S-matrix elements,..)
- heavy-light sector: Heavy quark effective theory (HQET)
- heavy-heavy sector: Non-relativistic QCD (NRQCD) and potential non-relativistic QCD (pNRQCD)
- derive and solve QCD Schrödinger - like equation

The Method

Definition of heavy quark: $m \gg \Lambda_{QCD}$
Characterising scales:

1. hard scale: mass m of heavy quark
2. soft scale: relative momentum of heavy quark-antiquark
   $|\mathbf{p}| \equiv p \sim mv, \qquad v \ll 1$
3. ultra soft scale: typical kinetic energy $E \sim mv^2$ of heavy quark and antiquark

where $E, p, \Lambda_{QCD} \ll m$.
Integrating out hard scale $m \rightarrow$ NRQCD
$\Rightarrow$ Expand the Lagrangian of QCD in powers of $1/m$!

pNRQCD

- still problems with NRQCD (power counting, perturbative calculations)
- solution: potential NRQCD: containing only degrees of freedom (DOF) relevant for Q-$\bar{Q}$ systems
- namely: integrate out soft scale

$$\mathcal{L}_{pNRQCD} = \Phi^\dagger(\mathbf{r})\Big(i\partial_0 - \frac{\mathbf{p}^2}{2m} - V^{(0)}(r) \\ + \text{corrections to the potential} \\ + \text{interactions with other low-energy DOF}\Big)\Phi(\mathbf{r})$$

where $V^{(0)}$ are the static potentials (singlet and octet), used as matching coefficients.

Bottom Quark Mass from $\Upsilon(1S)$ System

- consider $\mathcal{L}_{pNRQCD}$
- compute heavy quarkonium spectrum

$$M = 2m + \sum_m A^m \alpha_s^m + \delta M$$

- δM contains gluonic correlator (cannot use perturbation theory, need non-perturbative non-local condensate)
- A. Pineda obtained (2001)

$$m_{b,\overline{MS}}(m_{b,\overline{MS}}) = 4\ 210^{+90}_{-90}(\text{theory})^{-25}_{+25}(\alpha_s) MeV$$

QCD Field Strength Correlator at the next-to-leading Order

Two point correlation function of the QCD field strength tensor $F^a_{\mu\nu}(x)$ in the adjoint representation

$$\mathcal{D}_{\mu\nu\lambda\omega}(z) \equiv \left\langle 0 \right| T\left\{ F^a_{\mu\nu}(y) \mathcal{P} e^{gf^{abc} z^\tau \int_0^1 d\sigma A^c_\tau(x+\sigma z)} F^b_{\lambda\omega}(x) \right\} \left| 0 \right\rangle$$

where $F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu$ and $z = y - x$.

- important role in non-perturbative approaches to QCD
- basic quantity in the stochastic model of the QCD vacuum
- spectrum of heavy quark bound states: governs effect of gluon condensate on level splittings
- useful for determination of spin dependent parts in heavy quark potential

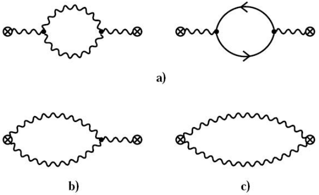
Field Strength Correlator

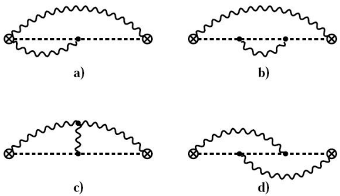
Lorentz structure: parametrise correlator in terms of two scalar functions $\mathcal{D}(z^2)$ and $\mathcal{D}_1(z^2)$

$$\mathcal{D}_{\mu\nu\lambda\omega}(z) = \left[g_{\mu\lambda}g_{\nu\omega} - g_{\mu\omega}g_{\nu\lambda}\right]\left(\mathcal{D}(z^2) + \mathcal{D}_1(z^2)\right) + \left[g_{\mu\lambda}z_\nu z_\omega - g_{\mu\omega}z_\nu z_\lambda - g_{\nu\lambda}z_\mu z_\omega + g_{\nu\omega}z_\mu z_\lambda\right]\frac{\partial \mathcal{D}_1(z^2)}{\partial z^2}$$

Leading order:

next-to-leading order:





Field Strength Correlator from HQET

Consider two-point correlator of hybrid current $(h^a F^a_{\mu\nu})(x)$:

$$\tilde{\mathcal{D}}_{\mu\nu\lambda\omega}(z) \equiv \left\langle 0 \right| T\left\{(h^a F^a_{\mu\nu})(y)(\bar{h}^b F^b_{\lambda\omega})(x)\right\}\left| 0 \right\rangle$$

where $h^a(x)$ is an octet of heavy quark fields (in adjoint representation)

$\Rightarrow$ gluelumps

Retransformation using path integral representation

$$\tilde{\mathcal{D}}_{\mu\nu\lambda\omega}(z) = S(z)\mathcal{D}_{\mu\nu\lambda\omega}$$

where $S(z)$ coordinate space propagator of a heavy quark field defined by

$$T\{h^a(y)\bar{h}^b(x)\} = \delta^{ab} S(z).$$

Future Work

- QCD-FSC calculated in perturbation theory by M. Eidemüller and M. Jamin, 1997
- QCD-FSC calculated on lattice by A. Di Giacomo, E.Meggiolaro, H. Panagopoulos and M. D'Elia, 1997
- direct comparison difficult: $m_{self}(a)$ divergent, dependend on cut-off $\Rightarrow$ need relation between the two schemes: $m_{self}(a) \leftrightarrow m_{self}(\overline{MS})$

A Perturbative calculation in the lattice regularisation scheme is required.

...and a happy New Year!