Inverse scattering-theory approach to the exact large-n solutions of $O(n)$ ϕ^4 models on films and semi-infinite systems bounded by free surfaces.

S. B. Rutkevich

In 1977, Bray & Moore [1] solved exactly the d-dimensional n-component ϕ^4 model in the half-space $z > 0$ with free boundary condition in the limit $n = \infty$ directly at the bulk critical point T_c. We have obtained the exact solution of the same model in the half-space geometry at $d = 3$ in the whole critical region $T \gtrless T_c$ at zero magnetic field. The logarithmic singularity of the surface free energy $f_s(t)$ as the function of the temperature parameter $t \sim (T - T_c)/T_c$ was found, and the explicit expression for the surface spin-spin correlation function in the whole critical region was obtained. The presence of the free boundary at $z = 0$ in the ϕ^4 model breaks explicitly its the translation symmetry in z-direction. As the result, the generalized spherical model, to which the $O(n)$ ϕ^4 model reduces at $n \to \infty$, contains the non-uniform effective potential $V(z,t)$. At a given temperature t, the latter should be determined from the z-dependent self-consistency equation [1, 2]. Since it turns out impossible to solve this equation analytically with respect to $V(z,t)$ at $t \neq 0$, a different strategy has been used, which applied the methods of the inverse scattering theory. We reformulated the self-consistency equation in terms of the scattering data of the associated one-dimensional Schrödinger operator $-\partial_z^2 + V(z,t)$ in the half-line $0 < z < \infty$, and then determined these scattering data in the explicit form at all temperatures $t \gtrless 0$ in the scaling region. Afterwords, the desired physical quantities were calculated in the scaling region directly from the scattering data without explicit knowledge of the self-consistent potential $V(z,t)$.

In the slab geometry, the finite-size scaling arguments imply the following expansion for the free energy (normalized by the unit area of the boundary surface) of the slab of width L in the scaling region: $f(t,L) = Lf_b(t) + 2\,f_s(t) + \Theta(x)/L^2 + \ldots$, where $f_b(t)$ is the bulk free energy density, $f_s(t)$ is the surface free energy, and $\Theta(x)$ is the universal Casimir free energy scaling function, which depends on the scaling parameter $x = tL$. Based on the results of the inverse scattering solution of the half-space problem, we have calculated the asymptotics of the scaling function $\Theta(x)$ at $x \to \pm\infty$, and described its singular behavior at $x \to \pm 0$. Obtained analytical results were confirmed by recent precise numerical calculations of the Casimir force and Casimir free energy scaling functions [3].

References

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